ABSTRACTS

Second Annual Upstate Number Theory Conference
April 28-29, 2012

Joel Dodge. "Refined conjectures on special values of L-functions."
Abstract: The theory of special values of L-functions aims to make a link between the analytic and algebraic invariants associated to global fields. I will recall some of the classical conjectures in this field, discuss what is meant by a "refined" version of such a conjecture and go over some progress in characteristic p.

Joel Dreibelbis. "The dynamical Mordell-Lang conjecture for linear maps."
Abstract: For a self-map f from S to S (one may take S to be g-tuples of complex numbers) and a point q in S, the orbit set of q under f is the set of points $\{q, f(q), f(f(q)), \dots\}$. What can be said about the intersection between the orbit set and a hypersurface H? Results will be discussed in the case where f is a linear map which culminates in a uniform bound for the dynamical Mordell-Lang conjecture for linear maps.

Paul Fili. "Numbers of small height in extensions with splitting conditions."
Abstract: It is a theorem of Schinzel that the Weil height is essentially bounded away from zero in the maximal totally real extension of the rationals, and a more recent result of Bombieri and Zannier that the same is true for the maximal totally p-adic extension of the rationals. We will discuss how these results naturally relate to results on equidistribution and dynamical heights and discuss current work on improving the best known lower bounds via potential theoretic techniques.

Hui Gao (Purdue University). "Galois lattices and strongly divisible lattices."
Abstract: Integral p-adic Hodge theory is the study about integral lattices in Galois representations, and it has proved to be extremely useful in all kind of applications. In this talk, we will discuss about Breuil's classification of integral lattices in semi-stable Galois representations via the so called "strongly divisible lattices". By generalizing Tong Liu's work to the "borderline" case, we now have a complete proof of Breuil's conjecture.

Steve Gonek. "Zeros of partial sums of the Riemann zeta-function."
Abstract: We will discuss the distribution of the zeros of partial sums of the Riemann zeta-function,

$$F_X(s) = \sum_{n \le X} n^{-s},$$

estimating the number of zeros up to height T, the number of zeros to the right of a given vertical line, and other aspects of their horizontal distribution. this is joint work with Andrew Ledoan.

Anna Haensch. "The representation problem for inhomogeneous quadratic polynomials."
Abstract: The representation problem for quadratic polynomials, asks for an effective determination of all integers represented by a given quadratic polynomial. This problem has a rich history and has been widely studied. One related problem asks, can we determine when a quadratic polynomial represents all natural numbers? What about all but finitely many? Polynomials satisfying such conditions are called universal, or almost universal, respectively. Imposing some mild arithmetic conditions, I will give a complete characterization of inhomogeneous quadratic polynomials, which are almost universal. This generalizes the recent work by Chan and Oh on almost universal ternary sums of triangular numbers.

C. Douglas Haessig. "Unit roots everywhere! (aka. Variation of the unit root along a family of unit root L-functions)."
Abstract: The theme of the talk will focus on describing the variation of the unit root of a family of unit root L-functions via a p-adic analytic function. We will discuss why such a function was expected and unexpected. (This is work in progress with Steven Sperber.)

Phong Le. "On the state of Wan's Conjecture."
Abstract: In this talk we will state and provide background on Wan's Conjecture. We will also highlight areas where progress has been made as well as areas that remain open.

Aaron Levin. "Linear forms in logarithms and integral points on varieties."
Abstract: One of the few effective methods in number theory comes from the theory of linear forms in logarithms, primarily due to Alan Baker. This theory has been applied, with notable success, largely in the context of curves. We will discuss an application to integral points on higher-dimensional varieties, generalizing an effective result of Vojta on the four-term (homogeneous) S-unit equation , $|S| < 4$.

Marcin Mazur. "Representations of analytic functions as infinite products and some arithmetic applications."
Abstract: I will discuss certain infinite product decomposition of functions analytic in a disc around 0. Then I will show some applications to arithmetic and to numerical computations of products of the form $\prod_p f(1/p)$, where the product is taken over all (sufficiently large) prime numbers and f is a function analytic in a neighborhood of 0 and such that $f(0) = 1$ and $f'(0) = 0$. This is a joint work with B. Petrenko.

Jay Pottharst. "Applications of triangulation of the eigencurve."
Abstract: We explain how the triangulation of the eigencurve and related finiteness results, proved in joint work with K.S. Kedlaya and L. Xiao, allows one to p-adically interpolate the Selmer groups of the modular forms in the family. This allows one to move questions about the ranks of Selmer groups around in the family, which motivates a conjecture on the geometry of the eigencurve.

Robert Rumely. "The Fekete-Szego theorem with local rationality conditions."
Abstract: The classical Fekete-Szego theorem says that if E is a compact set in the complex plane, stable under complex conjugation, and if E has logarithmic capacity greater than 1, then each neighborhood of E contains infinitely many conjugate sets of algebraic integers. Raphael Robinson sharpened this by showing that if E is contained in the real line and has logarithmic capacity greater than 1, then every real neighborhood of E contains infinitely many conjugate sets of totally real algebraic integers. Robinson's theorem turns out to be a very general phenomenon, which generalizes to an adelic setting on algebraic curves. This talk will discuss that generalization and illustrate it with examples on the projective line, elliptic curves, Fermat curves, and Modular curves.

Adam Towsley. "A Hasse principle for periodic points."
Abstract: We prove that for a rational map f of degree at least 2 a point being periodic is equivalent to it being periodic modulo p for almost every prime p. The result is true over any number field or function field with finite field of constants. Over a number field the Hasse principle is immediate from a Theorem of Benedetto, Ghioca, Hutz, Kurlberg, Scanlon and Tucker. To prove the Hasse principle in the function field case we prove an analog of their theorem, the key to which is an integrality result proved using a method of Runge.

Dimitri Vaintrob. "Mazur's isogeny theorem and generalizations."
Abstract: In 1978 Mazur proved a remarkable theorem: that any isogeny $E \longrightarrow E'$ of elliptic curves defined over the rationals must have degree ≤ 163, as long as the degree is prime. (In fact, it suffices for the isogeny to have cyclic kernel). I'll talk about work of Merel and Momose and a joint paper with Eric Larson which gives a generalization of this theorem to arbitrary number fields. Time permitting, I'll also talk about some related results for abelian varieties.

Paul Vojta (University of California, Berkeley). "Understanding Dyson's lemma for products of arbitrary curves."
Abstract: In 1989, I proved a Dyson lemma for products of two smooth projective curves of arbitrary genus. In 1995, M. Nakamaye extended this to a result for a product of an arbitrary number of smooth projective curves of arbitrary genus, in a formulation involving an additional "perturbation divisor." In 1998, he also found an example in which a hoped-for Dyson lemma is false without such a perturbation divisor. This talk will present some recent work suggesting that it may be possible to eliminate the perturbation divisor by using a different definition of "volume" at the points under consideration.

Daqing Wan. "L-functions of p-adic characters."
Abstract: Our main question is the p-adic meromorphic continuation of the L-function attached to a p-adic character for the rational function field over a finite field of characteristic p. In this talk, I will explain a new and (hopefully) transparent approach to this problem. (This is ongoing joint work with Chris Davis).

Melanie Matchett Wood. "The probability that a curve over a finite field is smooth."
Abstract: Given a fixed surface over a finite field, we ask what proportion of curves in that surface are smooth. Poonen's work on Bertini theorems over finite fields answers this question for certain families of curves in the surface. In this case the probability of smoothness is predicted by a simple heuristic assuming smoothness is independent at different points in the surface. In joint work with Erman, we consider this question for other families of curves in $\mathbb{P}^1 \times \mathbb{P}^1$ and Hirzebruch surfaces. Here the simple heuristic of independence fails, but the answer can still be determined and follows from a richer heuristic that predicts at which points smoothness is independent and at which points it is dependent.

Liang Xiao. " Global triangulation over eigenvarieties."''
Abstract: In Coleman and Mazur's ground breaking paper, they introduced a rigid analytic curve, called the eigencurve, parametrizing the p-adic overconvergent modular forms. The associated family of Galois representations is crystalline at p for a Zariski dense subset of points on the eigencurve. It was first observed by Kisin that the crystalline periods of this family of Galois representations vary continuously; this fact is one of the crucial ingredients of his proof of Fontaine-Mazur conjecture. Following Colmez, one may interpret Kisin's construction as the existence of a triangulation for the associated family of (phi, Gamma)-modules. We will prove that such triangulation extends to the whole eigencurve, generalizing Kisin's result on local affinoid neighborhoods of classical points. We will show that the same argument generalizes for eigenvarieties. This is a joint work with Jay Pottharst and Kiran Kedlaya.