

ABSTRACTS

Algebraic Dynamics Workshop

June 7–11, 2010

Plenary speakers.

Matthew Baker. “Complex dynamics and adelic potential theory.”

Abstract: We will discuss the proof of the following theorem: For any fixed complex numbers a and b , and any integer d at least 2, the set of complex numbers c for which both a and b are preperiodic for $z^d + c$ is infinite if and only if $a^d = b^d$. This provides an affirmative answer to a question of Zannier. The main ingredients in the proof are a complex-analytic study of certain generalized Mandelbrot sets and an adelic equidistribution theorem for preperiodic points over number fields and function fields. Somewhat surprisingly, non-Archimedean Berkovich spaces play an essential role in the arguments even though the theorem is purely about complex dynamics.

This is joint work with Laura DeMarco.

Robert Benedetto. “Attacking the Dynamical Uniform Boundedness Conjecture.”

Abstract: Let K be a number field, and let f be a morphism from the projective line to itself, defined over K and of degree at least two. The 1994 Dynamical Uniform Boundedness Conjecture of Morton and Silverman claims, in dimension one, that the number of K -rational preperiodic points of f is bounded by a constant depending only on the degrees of f and K . In this talk, we will discuss progress towards this conjecture both when f is a polynomial and when f is not a polynomial.

Antoine Chambert-Loir. “Heights and measures on analytic spaces.”

Abstract: In this talk, I will present the construction of measures for polarized line bundles on non-archimedean analytic spaces, give examples, and review some of their properties (local character, heights and Mahler measures, equidistribution theorems).

Zoe Chatzidakis. “Some aspects of the model theory of difference field.”

Abstract: This talk will build on Scanlon’s lectures, and give additional details. I will recall some of the tools from model theory, and what their immediate application gives and doesn’t give.

Charles Favre. “Dynamics on valuation spaces.”

Abstract: I shall discuss various applications to complex dynamics on the study of the action of maps on valuation spaces.

Dragos Ghioca. “The Dynamical Mordell-Lang Conjecture.”

Abstract: Starting from a classical question regarding linear recurrence sequences, we motivate our question about the dynamics of an arbitrary endomorphism f of a quasiprojective variety X . More precisely, our problem asks for a description of the set of iterates under f of a given point of X which lie on a subvariety V of X .

Our question is closely related to the classical Mordell-Lang problem. We present several results on our conjecture in characteristic 0, and we conclude by posing a natural extension of our question in positive characteristic.

Thomas Scanlon. “Model theory, difference fields, local fields.”

Abstract: I will deliver a series of five lectures with the ambitious goal of explaining some of the ideas at the heart of model theory, such as the notion of definability, the compactness theorem, and the constructions around stability theory, as well as exposing specific results at the interface of model theory and algebraic and diophantine geometry which may bear on problems in the study of algebraic dynamics. For the latter, I shall concentrate on the theory of difference fields, especially as developed by Chatzidakis and Hrushovski, and on the model theory of real and p-adic geometry concluding with Pila’s recent proof of the André-Oort conjecture from an analysis of rational points on definable sets in o-minimal structures.

Shouwu Zhang. “Calabi-Yau Theorem and Algebraic Dynamics.”

Abstract: In this talk, I will first state a Calabi-Yau theorem for both complex and p-adic manifolds and applications to dynamical system on complex varieties in terms of preperiodic points. Then I will discuss dynamical Manin-Mumford conjectures with both supporting examples and counterexamples.

This is a report of joint works with Xinyi Yuan, and Ghioca and Tucker respectively.

Michael Zieve. “Lattès maps.”

Abstract: I will present an algebraic treatment of Lattès maps, namely the rational functions $f: \mathbb{P}^1 \rightarrow \mathbb{P}^1$ for which there exists an elliptic curve E and finite maps $\phi: E \rightarrow E$ and $\pi: E \rightarrow \mathbb{P}^1$ satisfying $\pi \circ \phi = f \circ \pi$. This approach works over an arbitrary field, and gives a recipe for determining whether a given rational function f is a Lattès map, and if so, describing all corresponding tuples (E, ϕ, π) .

Contributed talks.

Anupam Bhatnagar. “Canonical height zero over function fields.”

Abstract: We prove a result on the parametrization of points of canonical height zero of an algebraic dynamical system defined over a function field of a curve. This is joint work with Lucien Szpiro.

Manav Das. “Erdős’ result on Bernoulli Convolutions.”

Abstract: Consider the iterated function system (‘IFS’) given by

$$f_1(x) = \rho x, \quad f_2(x) = 1 - \rho x, \quad 0 < \rho < 1.$$

Let ν_ρ denote the “uniform” measure associated with this IFS. For $0 < \rho < 1/2$, ν_ρ is singular with respect to Lebesgue measure. It is well-known that for $1/2 < \rho < 1$, these measures are either purely singular or purely absolutely continuous with respect to Lebesgue measure on the line. Characterizing the parameters ρ for which this dichotomy occurs is a long-standing open problem. Erdős proved that when ρ is the reciprocal of a PV number, the measure is singular. We discuss a new proof of Erdős’ result. This is joint work with Gerald Edgar at The Ohio State University.

Paul Fili. “ L^p heights and the geometry of the algebraic numbers modulo torsion.”

Abstract: The usual absolute logarithmic Weil height satisfies the axioms of a vector space norm on the multiplicative group of nonzero algebraic numbers modulo torsion. Allcock and Vaaler recently determined that when an algebraic number is viewed as a function of the algebraic places, there exists a measure on the places such that the classical Weil height of a number is essentially the L^1 norm of its associated function. In this talk, we explore the geometry of this function space associated with the natural inner product. We conclude by demonstrating the existence of canonical heights associated to iterated rational functions using the L^p norms.

Ayhan Gunaydin. “Mann pairs.”

Abstract: Let K be an ambient algebraically closed field. We first define what it means for a multiplicative subgroup G of K to have the Mann property. That diophantine property roughly states that G does not interact with the additive structure of the field too much. Then we introduce a subfield k of K and define a uniform version of Mann property with respect to k . The pair (k, G) is called a Mann pair whenever that condition holds. We shall present a large class of examples of Mann pairs; namely we shall show that (k, G) is a Mann pair when G is of finite rank and intersects k trivially. In the second part we prove that the triple (K, k, G) has good model theoretic properties whenever (k, G) is a Mann pair.

Su-Ion Ih. “Integral division points on curves.”

Abstract: We will talk about some Diophantine property of division points on curves which are integral. We will also see some related issues. (This is joint work with David Grant.)

Patrick Ingram. “Specialization and the canonical height.”

Abstract: To each rational function defined over a number field, one may associate a canonical height function, which gives certain arithmetic information about the underlying dynamics. We will explore how these height functions vary within one-parameter families.

Aaron Levin. *TBA*

Karl-Olof Lindahl. “Linearization problems in ultrametric dynamics.”

Abstract: We discuss open problems and results concerning linearization of power series near indifferent periodic points. In the positive characteristic case, the conjugacy may diverge, even for polynomials in one variable. In fact, in fields of positive characteristic, the linearizability of a power series $f(z) = \lambda z +$ (higher order terms), near the fixed point at the origin, cannot be determined solely in terms of the multiplier λ (except for some trivial cases), but depends mutually on the characteristics of the state space and the higher order terms of f .

Alice Medvedev. “What orthogonality and triviality in ACFA mean for algebraic dynamics: some hands-on examples.”

Abstract: Scanlon’s tutorial at this workshop describes a connection between algebraic dynamics and the model theory of ACFA, the theory of fields with a generic automorphism. We describe a few algebraic dynamics results that arise from the model-theoretic notions of “orthogonality” and “triviality”. (This triviality is not related to the (iso)triviality of dynamical systems over function fields.) All technical model theory will be swept under the rug, and many explicit examples will be used to illustrate and explain these notions.

Eugen Mihailescu. “Distributions of multivalued inverse iterates near repellers.”

Abstract: We introduce and study inverse SRB measures for non-invertible hyperbolic endomorphisms near folded repellers. In particular we obtain results about distributions of preimages for toral endomorphisms and perturbations of these.

Fabien Pazuki. “Dynamical Manin-Mumford conjecture.”

Abstract: We will give some generalizations to the counter-examples given by D. Ghioca and T. Tucker to the Dynamical Manin-Mumford conjecture, and try to understand where these examples come from.

Clayton Petsche. “A dynamical pairing between two rational maps.”

Abstract: I will summarize joint work with Lucien Szpiro and Thomas J. Tucker. Given two rational maps φ and ψ on \mathbb{P}^1 of degree at least two, we study a symmetric, nonnegative-real-valued pairing $\langle \varphi, \psi \rangle$ which is closely related to the canonical height functions h_φ and h_ψ associated to these maps. Our main results show a strong connection between the value of $\langle \varphi, \psi \rangle$ and the canonical heights of points which are small with respect to at least one of the two maps φ and ψ . Several necessary and sufficient conditions are given for the vanishing of $\langle \varphi, \psi \rangle$. We give an explicit upper bound on the difference between the canonical height h_ψ and the standard (naive) height h in terms of $\langle \sigma, \psi \rangle$, where $\sigma(x) = x^2$ denotes the squaring map. The pairing $\langle \sigma, \psi \rangle$ is computed or approximated for several families of rational maps ψ .

Vijay Sookdeo. “Integer points in backward orbits.”

Abstract: J. Silverman proved that a forward orbit of a rational function $f(x)$ with rational coefficients contains at most finitely many integers when $f(f(x))$ is not a polynomial. We state a similar conjecture for the backward orbits using a more general notion integrality, and prove this conjecture for the map $f(x) = x^d$.

Justin Sukiennik. “Height variations with change of variables.”

Abstract: We will look at two strict, or “best possible”, upper bounds for the differences in the height of a point and its affine linear image over a number field. From these results, we can demonstrate an asymmetry when we interchange the difference terms, which seems to run counter-intuitive to a result in a paper by C. Petsche, L. Szpiro, and T. Tucker. The proof the bounds are in fact strict follows mainly from Artin and Whaples Approximation Theorem.

Yu Yasufuku. “Integer points in orbits and Vojta’s conjecture.”

Abstract: Vojta’s conjecture is a deep inequality in Diophantine geometry, describing how the geometry of a variety controls the arithmetic. In this talk, I will describe some consequences of the conjecture to finiteness of integers in orbits, and mention some special cases which can be proved without assuming the conjecture. This is partly a joint work with A. Gregor.