Math 568 Problem Set #6 Due 10/27/14

(The first three problems are from the book. You may assume that L is separable over K in all of them.)

1. p. 29, Ex. 4

2. p. 30, Ex. 5

3. p. 32 Ex. 7

4. Let M be an $n \times n$ matrix over a field K. Suppose that $M^n = 0$ for some positive integer n. Show that the trace of M must be 0.

5. Let K be a field and let $B \supseteq K$ be an integral ring extension of K that is finitely generated as a K-module (B is not necessarily an integral domain). Then every element $x \in B$ gives rise to a multiplication map $r_x : b \mapsto xb$ from B to B. The map r_x is clearly a K-linear map and we can define its trace $\mathrm{T}_{B/K}(x)$ in the usual way. Show that the bilinear form $(x, y) = \mathrm{T}_{B/K}(xy)$ is degenerate whenever there exists an $x \in B$ such that $x \neq 0$ and for which $x^n = 0$ for some positive integer n.

6. Let $F(x) = x^n - a$. Find a formula for $\Delta(F)$ in terms of a and n.

7. Let ξ_p be a p-th primitive root of unity. Show that $\mathbb{Z}[\xi_p]$ is integrally closed.

8. Let $B = \mathbb{Z}[\xi_3]$. Factor pB for p equal to each of the following: 2, 3, 5, 7, 11, 13.

9. Let α satisfy $\alpha^3 - \alpha^2 + 5$ and let $B = \mathbb{Z}[\alpha]$. Find all the primes $\mathcal{Q} \subset B$ lying above the following primes in $\mathbb{Z}$: 2, 3, 5, 7, 11.