Math 568 Problem Set #10 "Due" 12/15/14

1. Let $d < -1$ be a squarefree integer that is congruent to 1 (mod 4) and let p be an odd prime less than $|d|/4$. Show that there is a prime $\mathcal{Q}$ in $\mathbb{Z}[\frac{1+\sqrt{d}}{2}]$ with $\mathcal{Q} \cap \mathbb{Z} = p$ and $[\mathcal{Q}] \neq 1$ if and only if d is square modulo p.

2. Let $d < -\frac{8}{\pi}^2$ be a squarefree integer congruent to 5 (mod 8) and let $L = \mathbb{Q}(\sqrt{d})$. Show that $\mathrm{Cl}(\mathcal{O}_L) = 1$ if and only if d is not a square modulo p for every odd prime $p < \frac{1}{2}\frac{4}{\pi}\sqrt{\mathcal{O}_L/\mathbb{Z}}$.

3. Let d be any squarefree integer that is congruent to 3 (mod 4). Show that $\mathbb{Q}(\sqrt{d})$ admits a nontrivial extension that is unramified at all primes in $\mathcal{O}_{\mathbb{Q}(\sqrt{d})}$.

4. Let d be any composite squarefree integer. Show that $\mathbb{Q}(\sqrt{d})$ admits a nontrivial extension that is unramified at all primes in $\mathcal{O}_{\mathbb{Q}(\sqrt{d})}$. [Hint: Let p be an odd prime factor of d. Try adjoining $\sqrt{\pm p}$ to $\mathbb{Q}(\sqrt{d})$].

5. Compute the class group of $\mathbb{Z}[\frac{1+\sqrt{-31}}{2}]$.

6. p. 81, Ex. 3

7. p. 81, Ex. 4: The entry under $\mathbb{Z}[\sqrt{10}]$ is wrong, it should be $3 + \sqrt{10}$.

8. p. 81, Ex. 5