Problem Set #8 for April 5, 2006

1.
(a) Let A and B be integral domains where $A \subseteq B$ and B is integral over A. Show that if $\mathfrak{m}$ is a maximal ideal in B, then $\mathfrak{m} \cap A$ is a maximal ideal in A.
(b) Give an example of integral domains $A \subseteq B$ and a maximal ideal $\mathfrak{m}$ in B such that $\mathfrak{m} \cap A$ is not a maximal ideal in A.
(c) Let A and B be integral domains where $A \subseteq B$ and B is integral over A. Show that if $\mathfrak{b}$ is a nonzero ideal of B, then $\mathfrak{b} \cap A$ is a nonzero ideal of A.
(d) Give an example of integral domains $A \subseteq B$ and a nonzero ideal $\mathfrak{b}$ in B such that $\mathfrak{b} \cap A = 0$.

2. Let K be a field and let v be a discrete valuation on K. Let

$$R_v = \{x \in K \mid v(x) \geq 0\} \cup \{0\}$$

and let

$$\mathfrak{m}_v = \{x \in K \mid v(x) > 0\} \cup \{0\}.$$

Show that if B is local ring with maximal ideal $\mathfrak{m}_B$ such that $R_v \subseteq B$ and $\mathfrak{m}_B \cap R_v = \mathfrak{m}_v$, then $B = R_v$.

From the book: Section 1.6: Problems 2, 4, 6, 7
(Do four problems total)