Math 531 Problem Set #8 Due 10/27/04

1. Let F be a monic polynomial over an integral domain A. Recall the definition of the resultant from Problem Set # 5. Show that

$$\Delta(F)A = \mathrm{Res}(F, F')A.$$

2. p.42, Ex. 4.
3. p. 51, Ex.2.
4. Let A be a Dedekind domain with field of fractions K. Let L and L' be finite separable extensions of K and suppose that there exist $\alpha \in L$ and $\alpha' \in L'$ such that the integral closure of A in L is $A[\alpha]$ and the integral closure of A in L' is $A[\alpha']$. Suppose furthermore that $\Delta(A[\alpha]/A) + \Delta(A[\alpha']/A) = A$. Let M be the compositum LL' over K. Is the integral closure of A in M necessarily equal to $A[\alpha, \alpha']$? Give a proof or a counterexample.
5. Let p and q be primes in $\mathbb{Z}$ with $p \neq q$. Find the integral closure of $\mathbb{Z}$ in $\mathbb{Q}(\xi_{pq})$ where ξ_{pq} a primitive pq-th root of unity. Justify your answer.
6. Let ξ_{p^2} be a p^2-th root of unity. Calculate $\mathrm{N}_{\mathbb{Q}(\xi_{p^2})/\mathbb{Q}}(1 - \xi_{p^2})$.