I like to begin the first lecture of a semester with an ice-breaker. I asked my Calculus II students at the University of Rochester this Fall 2014 to discuss a group of questions in small groups to share with the whole class later. Faced with the question “why are you taking this course?” not one student mentioned intrinsic interest. I followed up and asked this group of 25 students: “who likes mathematics here?”. Two students hesitantly raised their hands. I faced a similar audience as a lecturer at SUNY at Buffalo right before my PhD work in 2005 and as a graduate-student lecturer at University of Chicago in 2009: How do you teach mathematics to this diverse group? I could explain every aspect of integration by parts if woken up at 2 am, but this would hardly matter if I lost the attention of an anxious and possibly skeptical audience. I did a similar ice-breaker exercise two hours after the first Calculus lecture for my Real Analysis class. The audience of majors liked mathematics, but after a few difficult assignments students started wondering if the transition from “continuous curves are those drawn without lifting a pencil” to an \( \epsilon - \delta \) definition is worth the effort.

How to deal with these challenges? Years of teaching experience and the AMS Project NExT (New Experiences in Teaching) Fellowship\(^1\) have taught me to adapt to the needs of my students and be patient in developing my teaching style. Among the best lessons of the MAA conferences was a lack of a “universal teaching formula” similar to the Gromov’s thesis that “every property of all finitely generated groups is either false or trivial”. Thus, I strive to understand the difficulties the students face in a concrete classroom, try reasonable solutions and keep an eye on ways to improve. To improve, I found it helpful to maintain a network of colleagues and students, who are willing to listen to my thoughts on teaching and offer constructive feedback. This network overlaps with my research network, just as teaching, research and service interconnect in my daily professional life.

For the past two years I have kept a journal of self assessment for each class I teach. This project started from the discussion on pedagogy with my historian wife and I find that this journal adds a useful perspective to the mid-course and end-of-term student evaluations, and classroom visits by colleagues. Reviewing journals for the current semester, the classes after the first exam stand out. In an effort to move students away from rote imitation and encouraged by the MAA workshops I attended this August, I devoted a third of the Real Analysis lecture to group work and student-led discussion. On October 27, the lesson started with Korin, a student, who thrives in a discussion environment, asking for a group exercise. I obliged by asking everyone to negate the \( \epsilon - \delta \) definition of continuity. Unlike a similar exercise done a month earlier for the limit of a sequence, where I had to mix groups to insure at least one member understood quantifiers, the exercise went quickly and efficiently. Another Analysis student, Huiru, who took my probability course a year ago, commented that “discussions in class corrected my misconception on what are the limit points of the interval \([a,b]\).” However, most of the class struggled with crafting clear and rigorous proofs for more complex problems, as evidenced by the exam given on October 21. After providing a brief in-class survey to students, reflection and discussion with colleagues I pursued a three-pronged approach starting with that October class. First, I started stressing that the quality of students’ written assignments had to improve. Second, I added even more emphasis in my preparation to carefully worked out arguments for students to use and imitate, now that I was confident in their discussion skills. I asked Prof. Mueller, who taught this course in 2013, for a set of his lecture notes to contrast with the textbook presentation. Third, I kept my positive attitude and reminded the students that a gap between graphical intuition and rigorous arguments takes time to master and emphasized the

\(^{1}\text{the detailed description of my teaching and broader service experience is available in my CV}\)
rewards of this process. After a few weeks of this intervention the results on the second midterm were obvious: students’ written and analytical skills substantially improved. Despite the different challenges in my current Calculus II class, I used the students midterms and brief informal surveys to reflect on and adjust to what is most effectively helping my students’ learning. A course taught in the Spring 2014, MTH 200W – Transition to Advanced Mathematics, illustrates my experience in course design. Mark Herman, director of undergraduate studies at Rochester, asked me to redesign the course to have a substantial writing component to better align the department with a college-wide writing requirement. Mark indicated, that I had an option to treat this requirement as an administrative formality, counting routine homework as writing. Instead, I embraced this opportunity and in collaboration with Kalyani Madhu changed the format of the course: replacing midterms with two research papers, typeset in LaTeX. My department chair, Allan Greenleaf, while encouraging, expressed doubts that new math majors could successfully master LaTeX. In fact, teaching and critiquing writing was far more challenging than troubleshooting code. The skills required were rather different than mentoring 12 REU and DRP (Directed Reading Program) students at the University of Chicago. In particular, I did not anticipate the need to teach basic writing skills in addition to the mathematical guidance. If I were to teach a similar course again, I would borrow ideas from humanities colleagues: incorporating in-class discussion of writing excerpts and peer work, as well as students visiting University’s writing center. I did not have this benefit of the hindsight, but one-on-one meetings and revisions did significantly improved the students’ writing and typesetting. Discussing mathematics is complementary to writing mathematics. Thus every student gave several in-class presentation on the proofs they constructed for the homework and, at the end of the semester students presented the topic of their final paper. Finally, these presentations culminated in an oral exam, where students gave definitions and constructed proofs. According to a student, Bi Cheng, “researching a paper on the Russell’s paradox led me to re-examine what I understand in mathematics and science”. Another student, Dalyana, used this course to transition from the calculus course with a remedial component to the full math major. Dalyana presented a characterization of integer Pythagorean triplets to the class, a topic she learned by herself. She did need plenty of practice and guidance, but like every other student in this class, mastered new skills and ways of thinking. Dalyana is now leading a study group for my Calculus II class and is intending to pursue a Ph.D. in mathematics. I am certain that redesigning this course made a substantial difference in students’ mathematical abilities and broader interest in mathematics. This new format of the course, with a few tweaks based on the recommendations of Kalyani and me, is now the model for the MTH 200W.

To summarize, teaching is a task that I enjoy and work hard to do well. The main framework I developed for my teaching is careful planning, setting up a positive learning environment and adjusting to the needs of my students. I follow this framework and constantly evaluate and experiment with different strategies and techniques to improve teaching. The end result is quality teaching that fulfils and enriches my role as a teacher, scholar and member of my university.