Independent Study – Reading Course
Spring 2007
Math 391

Topic: Mixing times for Markov chains

Instructor: Shannon Starr, 1017 Hylan Building, sstarr at math dot rochester dot edu

Units credit: 1 unit for doing the reading course without homework, 3 units for doing the reading course with homework

Textbook/Materials: There is a new textbook being written (completed) by Yuval Peres, Elizabeth Wilmer and David Levin which can be downloaded, one chapter at a time, in pdf format, from the following website: `http://www.oberlin.edu/markov/`
When we finish that, we will read this paper:

Persi Diaconis and David Bayer. Trailing the dovetail shuffle to its lair, *Ann. Appl. Prob.*, **2**, 294-313.

Workload: We will probably meet just once per week for 75 minutes. But in the intervening days, you will be expected to read the course material on your own. I will be available to answer questions you may have in doing the reading. If you want 3 units of credit, then you should do homework assignments which will be assigned 1 per every 2 weeks.

Description: Markov chains are a central topic in probability. These pertain to shuffling decks of cards, generating a random coloring of a graph, or performing a Monte Carlo simulation to generate a given (equilibrium) measure in physics. In a typical class on stochastic processes, you learn conditions, for finite state space Markov chains, to prove that the Markov chain eventually settles down and converges to one unique stationary state. The proposed course is independent of a course on stochastic processes, but the question it answers is more refined: how fast do you converge?

For any practical application, you want to know not only that your Markov chain converges, but how fast. E.g., in running a Monte Carlo simulation, this tells you how long you need to let your computer run. Also, the mixing times can give you valuable information about the system when the system size grows very large. E.g., how much time do you need to generate a random coloring for a very large graph, or how much time does a Monte Carlo simulation have to run to generate a good approximation to an equilibrium state for a very large physical system? (Most physical systems have on the order of $10^2 3$ atoms, so you do need to know about large systems.) These questions are also related to phase transitions.

This course is aimed at mathematicians, computer scientists, physicists and statisticians who want to know how mathematicians rigorously solve these problems. The book we will use grew out of an MSRI program several years ago about the cutting-edge of the research topic. But the authors, Levin, Peres and Wilmer found a way to make a course that is entirely accessible to undergraduate students with a basic working knowledge of mathematical probability theory.

I expect we will finish the book before the semester is over. If that happens we will start to read research papers, starting with the one listed above by Bayer and Diaconis. That paper is somewhat famous. It was reported about in the newspapers in the 1990's. It proves that 7 shuffles suffices to mix-up a standard deck of cards, using the riffle shuffle, and explains why no fewer is really enough (even though most of us only shuffle a few times when we play casual card games). It is a beautiful paper that also discusses a card-trick used by magicians (Diaconis was a professional magician before becoming a statistician) and uses the analysis of the Baker's map, a chaotic dynamical system. The topic of the article is perfectly aligned with the textbook we will read.

For more information, or if you want to join, contact Shannon Starr at the email listed above. The prerequisite is Math 201 or some equivalent, with a good grade, or at least very good understanding. The ideal number of students is between 1 and 10. We already have 1.