

# Math 391 – Mixing Times for Markov Chains

## Homework Set #2

Assigned: Tuesday, Feb. 26

Due: Tuesday, Mar. 11 (by 5pm in my office 1017 Hylan)

Do the following Exercises from Levin, Peres and Wilmer:

**Chapter 3:** #3.9 and #3.11 on pg. 19, #3.17 and #3.18 on pg. 22, and #3.21 on pg. 26.

In addition, do the following problem(s).

**A1.** For each  $N$ , let  $\Omega_N$  be the set of all  $N$ -tuples  $(x_1, \dots, x_N)$  where each  $x_i$  is 0 or 1 and satisfying that  $x_i x_{i+1} = 0$  for each  $i = 1, \dots, N - 1$ . This is the state space for the hard-core lattice gas defined in Example 2.2 on page 6 (in Chapter 2) of Levin, Peres and Wilmer's text. Let  $p_N$  be the probability mass function for the uniform probability measure on  $\Omega_N$ :

$$p_N(x_1, \dots, x_N) = \begin{cases} 1/|\Omega_N| & \text{if } (x_1, \dots, x_N) \text{ is in } \Omega_N, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

Let  $X_1, \dots, X_N$  be random variables, such that

$$P\{X_1 = x_1, \dots, X_N = x_N\} = p_N(x_1, \dots, x_N).$$

(a) Show that  $X_1, \dots, X_N$  do satisfy the following condition:

$$P\{X_n = x_n \mid X_1 = x_1, \dots, X_{n-1} = x_{n-1}\} = P\{X_n = x_n \mid X_{n-1} = x_{n-1}\},$$

for  $n = 2, \dots, N$  and any  $x_1, \dots, x_{n-1}$  such that  $P\{X_1 = x_1, \dots, X_{n-1} = x_{n-1}\} > 0$ .

(b) This is almost the same as the definition of Markov chain on page 11, but not quite. What is different about  $X_1, \dots, X_N$ ?