Math 391 : Markov Chains and Mixing Times
Homework #4
Assigned Thursday, June 24. Due Friday, July 3.

1. Exercise 1.9 from Levin, Peres, Wilmer.

2. Exercise 1.11 from Levin, Peres and Wilmer.
Hint: You may want to use the Bolzano-Weierstrass theorem from Real Analysis. It says that every bounded sequence $\vec{v}_1, \vec{v}_2, \cdots \in \mathbb{R}^d$ *(for d fixed) has a convergent subsequence* $\vec{v}_{i_j}$*. In other words for this subsequence there is a vector* $\vec{w}$ *such that* $\lim_{j\to\infty} \vec{v}_{i_j} = \vec{w}$*.*

3. Exercise 1.12 from Levin, Peres, Wilmer.

4. Exercise 1.14 from Levin, Peres, Wilmer.
Hint: Compare to Exercise 1.9. This is a special case.

5. Consider the Markov chain on $\Omega = \{1, 2, 3\}$ with the following transition matrix

$$P = \begin{bmatrix} P(1,1) & P(1,2) & P(1,3) \\ P(2,1) & P(2,2) & P(2,3) \\ P(3,1) & P(3,2) & P(3,3) \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}.$$

Find the stationary measure π, and prove that it is the unique stationary measure.