

CALCULATING THE RIGHT UNIT, COPRODUCT, AND CONJUGATION IN THE HOPF ALGEBROID ASSOCIATED TO BROWN-PETERSON THEORY

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ABSTRACT. We describe how to calculate the right unit, coproduct, and conjugation in the Hopf algebroid associated to Brown-Peterson theory, give examples of calculations in terms of the Araki and Hazewinkel generators, and supply Maple source code for making these calculations.

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1. INTRODUCTION

Brown-Peterson theory is extremely useful for explicit calculations in homotopy theory via the p -component of the Adams-Novikov Spectral Sequence (ANSS)

$$E_2^{s,t} = \text{Ext}_{BP_*(BP)}^{s,t}(BP_*, BP_*) \implies \pi_{t-s}^S S^0 \otimes_{\mathbb{Z}} \mathbb{Z}_{(p)}$$

of [Rav02, p.21, Theorem 1.4.2]. The E_2 -term of this spectral sequence is the p -component of the ANSS E_2 -term, and it converges to the p -component of the stable homotopy groups $\pi_*^S S^0$ of the sphere spectrum.

The purpose of this note is to explicitly calculate the structure homomorphisms in the BP -theory Hopf algebroid. These calculations are necessary to determine the p -component of the ANSS E_2 -term as the cohomology of the cobar complex of the BP -theory Hopf algebroid.

2. THE BROWN-PETERSON THEORY HOPF ALGEBROID

2.1. The rings and generating sets. Fix a prime p , and let BP be the Brown-Peterson ring spectrum at p . The **BP-theory Hopf algebroid** over $\mathbb{Z}_{(p)}$ consists of two graded-commutative $\mathbb{Z}_{(p)}$ -algebras $\pi_*^S(BP) = BP_*$ and $\pi_*^S(BP \wedge BP) = BP_*(BP)$ together with five structure homomorphisms satisfying relations that make $(BP_*, BP_*(BP))$ represent a covariant groupoid-valued functor on the category of $\mathbb{Z}_{(p)}$ -algebras. The purpose of this section is to explicitly describe this Hopf algebroid and the formulas for its five structure maps.

The $\mathbb{Z}_{(p)}$ -algebras BP_* and $BP_*(BP)$ are both polynomial algebras. Among all of the generating sets for BP_* as a polynomial algebra, the Araki and Hazewinkel generating sets are most important for topological applications. We let $\{w_n\}_{n \geq 0}$ denote the set of Araki generators for BP_* , and $\{v_n\}_{n \geq 0}$ denote the set of Hazewinkel generators.

For the Araki generators, there are canonical isomorphisms

$$BP_* \cong \mathbb{Z}_{(p)}[w_1, w_2, w_3, \dots], \quad \text{and} \quad BP_*(BP) \cong \mathbb{Z}_{(p)}[w_1, w_2, w_3, \dots][t_1, t_2, t_3, \dots]$$

where the grading on the elements w_n and t_n is $|w_n| = |t_n| = 2p^n - 2$, and we follow the conventions that $w_0 = p$ and $t_0 = 1$. Also, for the Hazewinkel generators, there are canonical isomorphisms

$$BP_* \cong \mathbb{Z}_{(p)}[v_1, v_2, v_3, \dots], \quad \text{and} \quad BP_*(BP) \cong \mathbb{Z}_{(p)}[v_1, v_2, v_3, \dots][t_1, t_2, t_3, \dots]$$

where the grading on the elements v_n and t_n is $|v_n| = |t_n| = 2p^n - 2$, and we follow the conventions that $v_0 = p$ and $t_0 = 1$. We remark that since the generators for BP_* and $BP_*(BP)$ are in even degrees, graded-commutativity is trivial (that is, there is no sign change associated with interchange of elements).

The five structure homomorphisms for the rational BP -theory Hopf algebroid were determined by Quillen and Adams in terms of a set of generators $\{\lambda_n\}_{n \geq 0}$ for $\mathbb{Q} \otimes BP_*$, where $\lambda_0 = 1$ and $\lambda_n = [\mathbb{C}P^{p^n-1}]/p^n$ for $n \geq 1$. Consequently, the five structure homomorphisms for the BP -theory Hopf algebroid can be determined by relating the generators $\{\lambda_n\}_{n \geq 0}$ to the generators $\{w_n\}_{n \geq 0}$ and $\{v_n\}_{n \geq 0}$. We will calculate the structure homomorphisms eventually, but first we relate these generating sets.

The relationship between generating sets that defines the **Araki generators** is the recursion formula (see [Rav02, p.316,A2.2.2])

$$(1) \quad \lambda_j = \frac{\left(\sum_{0 \leq k \leq j-1} \lambda_k w_{j-k}^{(p^k)} \right)}{p - p^{(p^j)}}$$

where $w_0 = p$. For example,

$$(2) \quad \lambda_1 = \frac{w_1}{(p - p^p)}, \quad \lambda_2 = \frac{w_2}{(p - p^{(p^2)})} + \frac{w_1^{(1+p)}}{(p - p^p)(p - p^{(p^2)})},$$

and

$$(3) \quad \lambda_3 = \frac{w_3}{(p - p^{(p^3)})} + \frac{w_1 w_2^p}{(p - p^p)(p - p^{(p^3)})} + \frac{w_2 w_1^{(p^2)}}{(p - p^{(p^2)})(p - p^{(p^3)})} + \frac{w_1^{(1+p+p^2)}}{(p - p^p)(p - p^{(p^2)})(p - p^{(p^3)})}.$$

By their definition, the Araki generators are closely related to the p -series of the BP -theory formal group law via

$$(4) \quad [p]_{F_{BP}}(x) = \sum_{i \geq 0}^{F_{BP}} w_i x^{(p^i)}$$

in [Rav02, p.316,A2.2.4], where $\log_{F_{BP}}(x) = \sum_{i \geq 0} \lambda_i x^{(p^i)}$.

The relationship between generating sets that defines the **Hazewinkel generators** is the recursion formula (see [Rav02, p.315,A2.2.1])

$$(5) \quad \lambda_j = \frac{\left(\sum_{0 \leq k \leq j-1} \lambda_k v_{j-k}^{(p^k)} \right)}{p}$$

where $v_0 = p$. For example,

$$(6) \quad \lambda_1 = \frac{v_1}{p}, \quad \lambda_2 = \frac{v_2}{p} + \frac{v_1^{(1+p)}}{p^2},$$

and

$$(7) \quad \lambda_3 = \frac{v_3}{p} + \frac{v_1 v_2^p}{p^2} + \frac{v_2 v_1^{(p^2)}}{p^2} + \frac{v_1^{(1+p+p^2)}}{p^3}.$$

It can be shown [Rav02, p.316,A2.2.3] that the Araki and Hazewinkel generators are, in fact, generating sets, and that Araki and Hazewinkel generators are equivalent modulo p . Thus, whenever we are making calculations in $BP_*/(p, \dots)$ we may use either the Araki or Hazewinkel generators.

2.2. The structure homomorphisms and their relationships. The five structure homomorphisms for the BP -theory Hopf algebroid are

$$\begin{aligned} \eta_L &: BP_* \rightarrow BP_*(BP) && \text{(Left unit)} \\ \eta_R &: BP_* \rightarrow BP_*(BP) && \text{(Right unit)} \\ \epsilon &: BP_*(BP) \rightarrow BP_* && \text{(Augmentation)} \\ \Delta &: BP_*(BP) \rightarrow BP_*(BP) \otimes_{BP_*}^{ \eta_R, \eta_L } BP_*(BP) && \text{(Coproduct)} \\ c &: BP_*(BP) \rightarrow BP_*(BP) && \text{(Conjugation)} \end{aligned}$$

The left unit η_L and right unit η_R are $\mathbb{Z}_{(p)}$ -algebra homomorphisms which make $BP_*(BP)$ into a BP_* -bimodule. The bimodule structure is defined by η_L and η_R acting via their images: for any $u \in BP_*$ and any $x \in BP_*(BP)$ the left action is defined by $u \cdot x = \eta_L(u)x$, and the right action is defined by $x \cdot u = x\eta_R(u)$.

The augmentation ϵ and coproduct Δ are $\mathbb{Z}_{(p)}$ -algebra homomorphisms and also BP_* -bimodule homomorphisms. The tensor product $BP_*(BP) \otimes_{BP_*} BP_*(BP)$ is the usual tensor product of bimodules: for any $u \in BP_*$ and $x, y \in BP_*(BP)$ we have $x \otimes (\eta_L(u)y) = (x\eta_R(u)) \otimes y$. The BP_* -bimodule structure on the tensor product $BP_*(BP) \otimes_{BP_*} BP_*(BP)$ is defined by $\Delta \circ \eta_L$ and $\Delta \circ \eta_R$ acting through their images: for any $u \in BP_*$ and any $x \in BP_*(BP) \otimes_{BP_*} BP_*(BP)$ the left action is defined by $u \cdot x = \Delta(\eta_L(u))x$, and the right action is defined by $x \cdot u = x(\Delta(\eta_R(u)))$.

The conjugation c is a $\mathbb{Z}_{(p)}$ -algebra homomorphism.

The relationships among these five maps are:

- (1) $\epsilon \circ \eta_L = \epsilon \circ \eta_R = 1_{BP_*}$,
- (2) $(1_{BP_*(BP)} \otimes \epsilon) \circ \Delta = (\epsilon \otimes 1_{BP_*(BP)}) \circ \Delta = 1_{BP_*(BP)}$,
- (3) $(1_{BP_*(BP)} \otimes \Delta) \circ \Delta = (\Delta \otimes 1_{BP_*(BP)}) \circ \Delta$,
- (4) $c \circ c = 1_{BP_*(BP)}$,
- (5) $c \circ \eta_R = \eta_L$, and thus $c \circ \eta_L = \eta_R$,
- (6) $(x \cdot \eta_L(u))y = x(\eta_R(u) \cdot y)$ for all $u \in BP_*$ and $x, y \in BP_*(BP)$,
- (7) The multiplication ϕ in the ring $BP_*(BP)$ induces a multiplication

$$\Phi : BP_*(BP) \otimes_{BP_*}^{ \eta_R, \eta_L } BP_*(BP) \rightarrow BP_*(BP),$$

and both $\Phi \circ (c \otimes 1)$ and $\Phi \circ (1 \otimes c)$ are well-defined and satisfy

$$[\Phi \circ (c \otimes 1)]\Delta = \eta_R \circ \epsilon \quad \text{and} \quad [\Phi \circ (1 \otimes c)]\Delta = \eta_L \circ \epsilon.$$

In the list above, (2) is called the counit condition, (3) the coassociative condition, (4) the involution condition, and (7) the conjugation condition. For a more detailed description, see [Rav02, p.301, A1.1.1] or [Koc96, p.159].

We now present explicit formulas for the structure homomorphisms in the BP -theory Hopf algebroid. These formulas are in [MRW77, p.473-474]. They are also in [Rav02, p.110-111, Theorem 4.1.18], [Rav02, p.312, Theorem A2.1.27], and [Rav92, p.166-172], but some of these formulas may be stated incorrectly.

2.3. The left unit. If $i : S^0 \rightarrow BP$ denotes the unit map for the ring spectrum BP , then

$$1 \wedge i : BP \simeq BP \wedge S^0 \rightarrow BP \wedge BP$$

induces the left unit $\eta_L = \pi_*(1 \wedge i)$. It is called the left unit because it induces left multiplication: for any $u \in BP_*$ and any $x \in BP_*(BP)$ the left action is defined by $u \cdot x = \eta_L(u)x$.

For both the Araki and Hazewinkel generators, the left unit $\mathbb{Z}_{(p)}$ -algebra homomorphism is the standard inclusion

$$\mathbb{Z}_{(p)}[w_1, w_2, \dots] \hookrightarrow \mathbb{Z}_{(p)}[w_1, w_2, \dots][t_1, t_2, \dots]$$

or

$$\mathbb{Z}_{(p)}[v_1, v_2, \dots] \hookrightarrow \mathbb{Z}_{(p)}[v_1, v_2, \dots][t_1, t_2, \dots].$$

2.4. The right unit. If $i : S^0 \rightarrow BP$ denotes the unit map for the ring spectrum BP , then

$$i \wedge 1 : BP \simeq S^0 \wedge BP \rightarrow BP \wedge BP$$

induces the right unit $\eta_R = \pi_*(i \wedge 1)$. It is called the right unit because it induces right multiplication: for any $u \in BP_*$ and any $x \in BP_*(BP)$ the right action is defined by $x \cdot u = x\eta_R(u)$.

The right unit is considerably more complicated than the left unit. An outline of the calculation of the $\mathbb{Z}_{(p)}$ -algebra homomorphism η_R is as follows: to calculate $\eta_R(w_n)$ in terms of the Araki generators $\{w_1, \dots, w_n\}$ and the generators $\{t_1, \dots, t_n\}$, apply the $\mathbb{Z}_{(p)}$ -algebra homomorphism η_R to equation (8) to obtain equation (9). Then, use equations (10) and (11) to express the terms on the right-hand side of equation (9) in the generators $\{w_1, \dots, w_n\}$ and $\{t_1, \dots, t_n\}$.

2.4.1. The right unit on Araki generators. We begin by expressing w_n in terms of λ_* by the equation

$$(8) \quad w_n = (p - p^{(p^n)})\lambda_n - \sum_{1 \leq i \leq n-1} \lambda_i w_{n-i}^{(p^i)}$$

in [Rav02, p.316, A2.2.2] that defines the Araki generators. We apply the $\mathbb{Z}_{(p)}$ -algebra homomorphism η_R to equation (8) to obtain

$$(9) \quad \eta_R(w_n) = (p - p^{(p^n)})\eta_R(\lambda_n) - \sum_{1 \leq i \leq n-1} \eta_R(\lambda_i)\eta_R(w_{n-i})^{(p^i)}.$$

Each $\eta_R(\lambda_i)$ on the right-hand side of equation (9) may be expressed in terms of the generators λ_* and t_* via the recursion formula

$$(10) \quad \eta_R(\lambda_i) = \sum_{0 \leq j \leq i} \lambda_j t_{i-j}^{(p^j)}$$

from [Rav02, p.111, Theorem 4.1.8(d)]. Further, each λ_j on the right-hand side of equation (10) may be expressed in terms of the w_* by the recursion formula

$$(11) \quad \lambda_j = \frac{\left(\sum_{0 \leq k \leq j-1} \lambda_k w_{j-k}^{(p^k)} \right)}{p - p^{(p^j)}}.$$

in [Rav02, p.316, A2.2.2] that defines the Araki generators. In fact, we will often need to express $\eta_R(\lambda_i)$ in terms of the Araki generators w_* and t_* , so we define μ_i by

$$(12) \quad \mu_i = \eta_R(\lambda_i) = \sum_{0 \leq j \leq i} \left(\frac{\left(\sum_{0 \leq k \leq j-1} \lambda_k w_{j-k}^{(p^k)} \right)}{p - p^{(p^j)}} \right) t_{i-j}^{(p^j)}.$$

Digressive remark: The λ_i are coefficients for the logarithm $\log_{F_{BP}}(x) = \sum_{i \geq 0} \lambda_i x^{(p^i)}$ for the formal group law $F_{BP}(x, y)$ over BP_* , and the μ_i are coefficients for the modified logarithm

$$\text{mog}_{F_{BP}}(x) = \eta_R^*(\log_{F_{BP}}(x)) = \sum_{i \geq 0} \eta_R(\lambda_i) x^{(p^i)} = \sum_{i \geq 0} \mu_i x^{(p^i)}$$

for the formal group law $\eta_R^* F_{BP}(x, y)$ over $BP_*(BP)$. The importance of η_R is that it classifies the universal isomorphism between any two p -typical formal group laws over a $\mathbb{Z}_{(p)}$ -algebra.

Therefore, we may recursively solve for $\eta_R(w_n)$ in terms of the generators $\{w_1, \dots, w_n\}$ and $\{t_1, \dots, t_n\}$ by working backwards from equation (11) to equation (9).

For example, using equations (2) and (10) we calculate

$$\eta_R(\lambda_1) = t_1 + \lambda_1 = t_1 + \frac{w_1}{(p - p^p)}$$

(recall that $\lambda_0 = 1$, $t_0 = 1$ and $w_0 = p$) and thus

$$\eta_R(w_1) = (p - p^p)\eta_R(\lambda_1) = (p - p^p)t_1 + w_1.$$

2.4.2. The right unit on Hazewinkel generators. The procedure for calculating $\eta_R(v_n)$ for a Hazewinkel generator v_n is analogous to the one above for the Araki generators. To calculate $\eta_R(v_n)$ use the equation

$$(13) \quad v_n = p\lambda_n - \sum_{1 \leq i \leq n-1} \lambda_i v_{n-i}^{(p^i)}$$

that defines the Hazewinkel generators to get

$$(14) \quad \eta_R(v_n) = p\eta_R(\lambda_n) - \sum_{1 \leq i \leq n-1} \eta_R(\lambda_i)\eta_R(v_{n-i})^{(p^i)},$$

where

$$(15) \quad \mu_i = \eta_R(\lambda_i) = \sum_{0 \leq j \leq i} \left(\frac{\left(\sum_{0 \leq k \leq j-1} \lambda_k v_{j-k}^{(p^k)} \right)}{p} \right) t_{i-j}^{(p^j)}.$$

For example, using equations (6) and (15) we calculate

$$\eta_R(\lambda_1) = t_1 + \lambda_1 = t_1 + \frac{v_1}{p}$$

(recall that $\lambda_0 = 1$, $t_0 = 1$ and $v_0 = p$) and thus

$$\eta_R(v_1) = p\eta_R(\lambda_1) = pt_1 + v_1.$$

2.5. The augmentation. The multiplication $m : BP \wedge BP \rightarrow BP$ in the ring spectrum BP induces the augmentation $\epsilon = \pi_*(m)$.

The augmentation $\epsilon : BP_*(BP) \rightarrow BP_*$ is a $\mathbb{Z}_{(p)}$ -algebra homomorphism and BP_* -bimodule homomorphism given by $\epsilon(w_n) = w_n$ on an Araki generator, $\epsilon(v_n) = v_n$ on a Hazewinkel generator, and $\epsilon(t_n) = 0$ for all $n \geq 1$.

2.6. The coproduct. If $i : S^0 \rightarrow BP$ denotes the unit map for the ring spectrum BP , then the map

$$1 \wedge i \wedge 1 : BP \wedge S^0 \wedge BP \rightarrow BP \wedge BP \wedge BP$$

induces the coproduct $\Delta = \pi_*(1 \wedge i \wedge 1)$. We have implicitly used here that

$$\pi_*^S(BP \wedge BP \wedge BP) \leftarrow BP_*(BP) \otimes_{BP_*}^{\eta_R, \eta_L} BP_*(BP)$$

is an isomorphism since BP_* is bounded below and η_R is flat (see [Koc96, p.158, Prop. 4.5.1]).

The BP -theory coproduct $\Delta : BP_*(BP) \rightarrow BP_*(BP) \otimes_{BP_*}^{\eta_R, \eta_L} BP_*(BP)$ is a $\mathbb{Z}_{(p)}$ -algebra homomorphism and BP_* -bimodule homomorphism. On the Araki generators it is given by $\Delta(w_n) = w_n \otimes 1$, and on the Hazewinkel generators it is given by $\Delta(v_n) = v_n \otimes 1$. The formulas $\Delta(w_n) = w_n \otimes 1$ and $\Delta(v_n) = v_n \otimes 1$ follow from the relation $(1_{BP_*(BP)} \otimes \epsilon)\Delta = 1_{BP_*(BP)}$ in the Hopf algebroid. On a generator t_n , $\Delta(t_n)$ is determined by [MRW77, p.473]

$$(16) \quad \sum_{i+j=n} \lambda_i (\Delta(t_j))^{(p^i)} = \sum_{i+j+k=n} \lambda_i t_j^{(p^i)} \otimes t_k^{(p^{i+j})}.$$

We may rewrite equation (16) as

$$(17) \quad \sum_{0 \leq i \leq n} (\lambda_i \otimes 1) (\Delta(t_{n-i}))^{(p^i)} = \sum_{0 \leq k \leq n} \left(\sum_{0 \leq i \leq n-k} (\lambda_i t_{n-i-k}^{(p^i)}) \otimes t_k^{(p^{n-k})} \right).$$

We have included another tensor product symbol on the left-hand side of equation (17) to clarify that the λ_i are in the left factor of the tensor product. We rearrange equation (17) to solve for

$$(18) \quad \Delta(t_n) = \sum_{0 \leq k \leq n} \left(\sum_{0 \leq i \leq n-k} (\lambda_i t_{n-i-k}^{(p^i)}) \otimes t_k^{(p^{n-k})} \right) - \left(\sum_{1 \leq i \leq n} (\lambda_i \otimes 1) (\Delta(t_{n-i}))^{(p^i)} \right)$$

in terms of $\{\lambda_1, \dots, \lambda_n\}$ and $\{t_1, \dots, t_n\}$. Each λ_i in equation (18) may be expressed in terms of the Araki or Hazewinkel generators by equation (1) or equation (5), respectively. Thus, it is possible to write down explicit recursion formulas for the BP -theory coproduct in terms of the Araki or Hazewinkel generators and the generators $\{t_n\}_{n \geq 1}$.

2.7. The conjugation. The map $\tau : BP \wedge BP \rightarrow BP \wedge BP$ that interchanges the two factors of the smash product induces the conjugation homomorphism $c = \pi_*(\tau)$.

The BP -theory conjugation $c : BP_*(BP) \rightarrow BP_*(BP)$ is a $\mathbb{Z}_{(p)}$ -algebra homomorphism. From the relation $c \circ \eta_L = \eta_R$ in the Hopf algebroid, we calculate $c(w_n) = c(\eta_L(w_n)) = \eta_R(w_n)$ for an Araki generator w_n , and $c(v_n) = c(\eta_L(v_n)) = \eta_R(v_n)$. The value of c on a generator t_n is determined by [MRW77, p.474]

$$(19) \quad \sum_{i+j+k=n} \lambda_i t_j^{(p^i)} (c(t_k))^{(p^{i+j})} = \lambda_n.$$

Rearrange equation (19) to obtain

$$(20) \quad c(t_n) = \lambda_n - \left(\sum_{0 \leq k \leq n-1} \left(\sum_{0 \leq i \leq n-k} \lambda_i t_{n-i-k}^{(p^i)} (c(t_k))^{(p^{n-k})} \right) \right).$$

As usual, each λ_i in equation (20) may be expressed in terms of the Araki or Hazewinkel generators by equation (1) or equation (5), respectively. Thus, it is possible to write down explicit recursion formulas for the BP -theory conjugation in terms of the Araki or Hazewinkel generators and the generators $\{t_n\}_{n \geq 1}$.

3. CALCULATIONS FOR ARAKI GENERATORS

3.1. Araki generators at $p=2$: Right unit calculations. These results for $\eta_R(w_1)$ and $\eta_R(w_2)$ at $p = 2$ agree with those of [Rav02, p.157, 5.2.4].

$$\eta_R(w_1) = w_1 - 2t_1$$

$$\eta_R(w_1) \pmod{2} = w_1$$

$$\eta_R(w_2) = w_2 + 13w_1t_1^2 - 14t_2 - 3w_1^2t_1 - 4t_1^3$$

$$\eta_R(w_2) \pmod{2} = w_2 + w_1t_1^2 + w_1^2t_1$$

$$\eta_R(w_2) \pmod{(2, w_1)} = w_2$$

$$\begin{aligned} \eta_R(w_3) = & -16t_1^7 - 14w_1w_2t_2 + 28t_1w_2t_2 - 128t_2t_1^4 - t_1w_2^2 - 196t_1t_2^2 + 225w_1t_2^2 + \frac{191}{7}t_1^4w_2 + \\ & \frac{1238}{7}t_1^4w_1^3 + 2/7w_1^6t_1 + \frac{29}{7}w_1^5t_1^2 - \frac{356}{7}w_1^4t_1^3 - 261w_1^2t_1^5 + 120w_1t_1^6 - t_2w_1^4 - \frac{25}{7}w_2w_1^3t_1 + \\ & \frac{145}{7}w_2w_1^2t_1^2 - \frac{226}{7}w_2w_1t_1^3 + 50t_2w_1^3t_1 - 290t_2w_1^2t_1^2 + 452t_2w_1t_1^3 + w_3 - 254t_3 \end{aligned}$$

$$\eta_R(w_3) \pmod{2} = t_1w_2^2 + w_1t_2^2 + t_1^4w_2 + w_1^5t_1^2 + w_1^2t_1^5 + t_2w_1^4 + w_2w_1^3t_1 + w_2w_1^2t_1^2 + w_3$$

$$\eta_R(w_3) \pmod{(2, w_1)} = t_1w_2^2 + t_1^4w_2 + w_3$$

$$\eta_R(w_3) \pmod{(2, w_1, w_2)} = w_3$$

$$\begin{aligned}
\eta_R(w_4) = & w_4 - 65534 t_4 - 256 t_1^{15} - \frac{9153}{7} w_1^5 w_2 t_2^2 t_1 - \frac{1361}{7} w_1^7 w_2 t_2 t_1^2 + 33 w_1^5 w_2^2 t_2 t_1 - \\
& 1821 w_2^2 t_2^2 w_1^2 t_1 - 626 w_2^2 t_2 w_1^4 t_1^2 + 6576 w_2^2 w_1 t_1^5 t_2 + 6778 w_2^2 w_1 t_1^2 t_2^2 - 8948 w_2^2 w_1^2 t_1^4 t_2 + \\
& 3834 w_2^2 w_1^3 t_1^3 t_2 + 62 w_2^3 t_2 w_1^2 t_1 - 292 w_2^3 w_1 t_1^2 t_2 + \frac{22248}{7} w_2 t_2 w_1^6 t_1^3 + \frac{105419}{7} w_2 t_2^2 w_1^4 t_1^2 + \\
& 24752 w_2 t_2^3 w_1^2 t_1 + \frac{521074}{7} w_2 w_1^4 t_1^5 t_2 - \frac{935422}{7} w_2 w_1^3 t_1^6 t_2 + \frac{996687}{7} w_2 w_1^2 t_1^4 t_2^2 - \frac{655722}{7} w_2 w_1 t_1^5 t_2^2 - \\
& 50376 w_2 w_1 t_1^8 t_2 - 69832 w_2 w_1 t_1^2 t_2^3 + \frac{801168}{7} w_2 w_1^2 t_1^7 t_2 - \frac{150027}{7} w_2 w_1^5 t_1^4 t_2 - \frac{491832}{7} w_2 w_1^3 t_1^3 t_2^2 - \\
& \frac{25}{7} w_2 w_1^4 t_1 w_3 + \frac{6350}{7} w_2 w_1^4 t_1 t_3 + \frac{195}{7} w_2 w_1^3 t_1^2 w_3 - \frac{49530}{7} w_2 w_1^3 t_1^2 t_3 - \frac{516}{7} w_2 w_1^2 t_1^3 w_3 + \frac{131064}{7} w_2 w_1^2 t_1^3 t_3 + \\
& 52 t_2 w_1^4 t_1 w_3 - 13208 t_2 w_1^4 t_1 t_3 - 390 t_2 w_1^3 t_1^2 w_3 + 99060 t_2 w_1^3 t_1^2 t_3 + 1032 t_2 w_1^2 t_1^3 w_3 - \\
& 262128 t_2 w_1^2 t_1^3 t_3 - 14 w_1^2 w_2 t_2 w_3 + 3556 w_1^2 w_2 t_2 t_3 - 3/7 w_1^8 w_2 t_2 t_1 - \frac{163322}{7} w_1 t_1^4 w_2 t_3 + \frac{643}{7} w_1 t_1^4 w_2 w_3 + \\
& 164084 w_1 t_1 t_2^2 t_3 - 646 w_1 t_1 t_2^2 w_3 + 254 w_1 t_1 w_2^2 t_3 - w_1 t_1 w_2^2 w_3 + 262128 w_1 t_2 t_1^4 t_3 - 1032 w_1 t_2 t_1^4 w_3 - \\
& 56 t_1^2 w_2 t_2 w_3 + 14224 t_1^2 w_2 t_2 t_3 + 65025 w_1 t_3^2 + \frac{36959}{127} t_1^8 w_3 + \frac{325566345}{6223} t_1^8 w_1^7 + 18401 t_2^4 w_2 + \\
& 21600 t_2^4 w_1^3 + \frac{15476044}{6223} t_1^8 w_1 w_2^2 + \frac{321970520}{6223} t_1^8 w_1^4 w_2 + 3/7 w_1^5 w_2^3 t_1 - \frac{24649}{7} w_1^8 t_1^4 t_2 + \frac{2921490}{7} w_1^5 t_1^7 t_2 - \\
& 103430 w_1^4 t_1^2 t_2^3 - 652757 w_1^4 t_1^8 t_2 - \frac{1067312}{7} w_1^6 t_1^6 t_2 + \frac{225221}{7} w_1^7 t_1^5 t_2 + 10720 w_1^5 t_2^3 t_1 + \frac{706}{7} w_1^9 t_2 t_1^3 - \\
& 2758 w_1^3 w_2 t_2^3 + 56 w_1^3 w_2^2 t_2^2 + 2 w_1^3 w_2^3 t_2 + 415264 w_1^3 t_2^3 t_1^3 + \frac{3836248}{7} w_1^3 t_2 t_1^9 + 269304 w_1 t_1^2 t_2^4 - \\
& 732784 w_1^2 t_1^4 t_2^3 - 241632 w_1^2 t_1^{10} t_2 + 404128 w_1 t_1^5 t_2^3 + 52352 w_1 t_1^{11} t_2 - 127653 t_2^4 w_1^2 t_1 + \\
& 21896 w_2 t_2^2 t_1^6 + 23520 w_2 t_2^3 t_1^3 + \frac{58752}{7} w_2 t_2 t_1^9 + \frac{1277}{7} w_2^3 w_1^2 t_1^4 - \frac{467}{7} w_2^3 w_1^3 t_1^3 - \frac{1011}{7} w_2^3 w_1 t_1^5 + \\
& 120 w_2^3 t_2 t_1^3 + \frac{39}{7} w_2^3 w_1^4 t_1^2 + \frac{33}{7} w_2^4 w_1 t_1^2 - 6/7 w_2^4 w_1^2 t_1 - 2072 w_2^2 t_2 t_1^6 - 2520 w_2^2 t_2^2 t_1^3 - \\
& 1960 w_2^2 t_2^3 + 140 w_2^3 t_2^2 + \frac{430}{7} w_2^3 t_1^6 - 5 w_2^4 t_2 - \frac{15}{7} w_2^4 t_1^3 - \frac{38945}{49} w_2^2 t_1^9 + 4096 w_1 t_1^{14} - 26336 w_1^2 t_1^{13} - \\
& 4352 t_2 t_1^{12} - 38416 t_2^5 - 68992 t_2^3 t_1^6 - 82320 t_2^4 t_1^3 - 26240 t_2^2 t_1^9 + \frac{24536}{6223} w_1^7 w_2^2 t_1^2 - \frac{500480}{6223} w_1^6 w_2^2 t_1^3 + \\
& \frac{3862447}{6223} w_1^5 w_2^2 t_1^4 - \frac{14798948}{6223} w_1^4 w_2^2 t_1^5 + \frac{29252691}{6223} w_1^3 w_2^2 t_1^6 - \frac{4112754}{889} w_1^2 w_2^2 t_1^7 + \frac{4}{889} w_1^{11} w_2 t_1 - \\
& \frac{6546}{6223} w_1^{10} w_2 t_1^2 - \frac{17758}{6223} w_1^9 w_2 t_1^3 + \frac{1439871}{6223} w_1^8 w_2 t_1^4 - \frac{14000011}{6223} w_1^7 w_2 t_1^5 + \frac{67938816}{6223} w_1^6 w_2 t_1^6 - \\
& \frac{27467939}{889} w_1^5 w_2 t_1^7 - \frac{2470877}{49} t_1^9 w_2 w_1^3 + \frac{203246}{7} t_1^{10} w_2 w_1^2 - \frac{59200}{7} t_1^{11} w_2 w_1 + \frac{37}{7} w_1^8 t_2^2 t_1 + \frac{14459}{7} w_1^7 t_2^2 t_1^2 - \\
& \frac{213494}{7} w_1^6 t_2^2 t_1^3 + \frac{1304482}{7} w_1^5 t_2^2 t_1^4 - \frac{4154939}{7} w_1^4 t_2^2 t_1^5 + 968322 w_1^3 t_2^2 t_1^6 - 703128 w_1^2 t_2^2 t_1^7 + \\
& 225760 w_1 t_2^2 t_1^8 + \frac{198}{889} w_3 w_1^7 t_1 + \frac{3567}{889} w_3 w_1^6 t_1^2 - \frac{54146}{889} w_3 w_1^5 t_1^3 + \frac{251570}{889} w_3 t_1^4 w_1^4 - \frac{552753}{889} w_3 w_1^3 t_1^5 + \\
& \frac{82430}{127} w_3 w_1^2 t_1^6 - \frac{33024}{127} w_3 w_1 t_1^7 - \frac{396}{7} t_3 w_1^7 t_1 - \frac{7134}{7} t_3 w_1^6 t_1^2 + \frac{108292}{7} t_3 w_1^5 t_1^3 - \frac{503140}{7} t_3 t_1^4 w_1^4 + \\
& \frac{1105506}{7} t_3 w_1^3 t_1^5 - 164860 t_3 w_1^2 t_1^6 + 66048 t_3 w_1 t_1^7 + \frac{4}{127} w_1^8 w_2^2 t_1 - \frac{2}{889} w_1^{14} t_1 + \frac{352}{6223} w_1^{13} t_1^2 + \\
& \frac{6466}{6223} w_1^{12} t_1^3 - \frac{68997}{6223} w_1^{11} t_1^4 - \frac{608628}{6223} w_1^{10} t_1^5 + \frac{12241066}{6223} w_1^9 t_1^6 - \frac{11990960}{889} w_1^8 t_1^7 + \frac{6368}{7} t_1^{12} w_2 - \frac{6151608}{49} t_1^9 w_1^6 + \\
& \frac{1308751}{7} t_1^{10} w_1^5 - \frac{1172559}{7} t_1^{11} w_1^4 + \frac{617072}{7} t_1^{12} w_1^3 + w_1^9 t_2^2 - t_3 w_1^8 - 8384 t_3 t_1^8 - 14224 w_1 t_1 w_2 t_2 t_3 + \\
& 56 w_1 t_1 w_2 t_2 w_3 + 256 t_2 t_1^5 w_3 - 65024 t_2 t_1^5 t_3 + 2 t_1^2 w_2^2 w_3 - 508 t_1^2 w_2^2 t_3 + 392 t_1^2 t_2^2 w_3 - 99568 t_1^2 t_2^2 t_3 - \\
& \frac{382}{7} t_1^5 w_2 w_3 + \frac{97028}{7} t_1^5 w_2 t_3 - 254 w_1 w_3 t_3 + 508 t_1 w_3 t_3 - 225 w_1^6 t_2^3 - 64516 t_1 t_3^2 - t_1 w_3^2 - \\
& 2/7 w_1^{11} t_1 t_2 + \frac{75}{7} w_1^{10} t_1^2 t_2 + 14 w_1^6 w_2 t_2^2 + 225 w_1^2 t_2^2 w_3 - 57150 w_1^2 t_2^2 t_3 - t_2 w_1^5 w_3 + 254 t_2 w_1^5 t_3
\end{aligned}$$

$$\begin{aligned}
\eta_R(w_4) \bmod 2 = & w_4 + w_1^5 w_2 t_2^2 t_1 + w_1^7 w_2 t_2 t_1^2 + w_1^5 w_2^2 t_2 t_1 + w_2^2 t_2^2 w_1^2 t_1 + w_2 t_2^2 w_1^4 t_1^2 + \\
& w_2 w_1^2 t_1^4 t_2^2 + w_2 w_1^5 t_1^4 t_2 + w_2 w_1^4 t_1 w_3 + w_2 w_1^3 t_1^2 w_3 + w_1^8 w_2 t_2 t_1 + w_1 t_1^4 w_2 w_3 + w_1 t_1 w_2^2 w_3 + \\
& w_1 t_3^2 + t_1^8 w_3 + t_1^8 w_1^7 + t_2^4 w_2 + w_1^5 w_2^3 t_1 + w_1^8 t_1^4 t_2 + w_1^4 t_1^8 t_2 + w_1^7 t_1^5 t_2 + t_2^4 w_1^2 t_1 + w_2^3 w_1^2 t_1^4 + \\
& w_2^3 w_1^3 t_1^3 + w_2^3 w_1 t_1^5 + w_2^3 w_1^4 t_1^2 + w_2^4 w_1 t_1^2 + w_2^4 t_2 + w_2^4 t_1^3 + w_2^2 t_1^9 + w_1^5 w_2^2 t_1^4 + w_1^3 w_2^2 t_1^6 + \\
& w_1^8 w_2 t_1^4 + w_1^7 w_2 t_1^5 + w_1^5 w_2 t_1^7 + t_1^9 w_2 w_1^3 + w_1^8 t_2^2 t_1 + w_1^7 t_2^2 t_1^2 + w_1^4 t_2^2 t_1^5 + w_3 w_1^6 t_1^2 + w_3 w_1^3 t_1^5 + \\
& w_1^{11} t_1^4 + t_1^{10} w_1^5 + t_1^{11} w_1^4 + w_1^9 t_2^2 + t_3 w_1^8 + w_1^6 t_2^3 + t_1 w_3^2 + w_1^{10} t_1^2 t_2 + w_1^2 t_2^2 w_3 + t_2 w_1^5 w_3
\end{aligned}$$

$$\eta_R(w_4) \bmod (2, w_1) = w_4 + t_1^8 w_3 + t_2^4 w_2 + w_2^4 t_2 + w_2^4 t_1^3 + w_2^2 t_1^9 + t_1 w_3^2$$

$$\eta_R(w_4) \bmod (2, w_1, w_2) = w_4 + t_1^8 w_3 + t_1 w_3^2$$

$$\eta_R(w_4) \bmod (2, w_1, w_2, w_3) = w_4$$

3.2. Araki generators at p=2: Coproduct calculations. These results for $\Delta(w_1)$ and $\Delta(w_2)$ agree with those in [Rav92, p.168-169].

$$\Delta(1) = 1 \otimes 1$$

$$\Delta(t_1) = t_1 \otimes 1 + 1 \otimes t_1$$

$$\Delta(t_2) = t_2 \otimes 1 + t_1 \otimes t_1^2 + 1 \otimes t_2 + w_1 t_1 \otimes t_1$$

$$\Delta(t_2) \bmod (2, w_1) = t_2 \otimes 1 + t_1 \otimes t_1^2 + 1 \otimes t_2$$

$$\begin{aligned} \Delta(t_3) = & w_1 t_2 t_1 \otimes t_1^2 + w_1 t_2 \otimes t_2 + w_1^2 t_2 t_1 \otimes t_1 + w_1 t_1 \otimes t_1^2 t_2 + w_1^2 t_1^2 \otimes t_1^3 + w_1^2 t_1 \otimes t_2 t_1 + \\ & \frac{2}{7} w_1^3 t_1^2 \otimes t_1^2 + \frac{2}{7} w_2 t_1^3 \otimes t_1 + \frac{3}{7} w_2 t_1^2 \otimes t_1^2 + \frac{2}{7} w_2 t_1 \otimes t_1^3 - \frac{1}{7} w_1^3 t_1^3 \otimes t_1 - \frac{1}{7} w_1^3 t_1 \otimes t_1^3 + t_3 \otimes \\ & 1 + t_2 \otimes t_1^4 + t_1 \otimes t_2^2 + 1 \otimes t_3 \end{aligned}$$

$$\Delta(t_3) \bmod 2 = w_1 t_2 t_1 \otimes t_1^2 + w_1 t_2 \otimes t_2 + w_1^2 t_2 t_1 \otimes t_1 + w_1 t_1 \otimes t_1^2 t_2 + w_1^2 t_1^2 \otimes t_1^3 + w_1^2 t_1 \otimes t_2 t_1 + w_2 t_1^2 \otimes t_1^2 + w_1^3 t_1^3 \otimes t_1 + w_1^3 t_1 \otimes t_1^3 + t_3 \otimes 1 + t_2 \otimes t_1^4 + t_1 \otimes t_2^2 + 1 \otimes t_3$$

$$\Delta(t_3) \bmod (2, w_1) = w_2 t_1^2 \otimes t_1^2 + t_3 \otimes 1 + t_2 \otimes t_1^4 + t_1 \otimes t_2^2 + 1 \otimes t_3$$

$$\Delta(t_3) \bmod (2, w_1, w_2) = t_3 \otimes 1 + t_2 \otimes t_1^4 + t_1 \otimes t_2^2 + 1 \otimes t_3$$

$$\begin{aligned} \Delta(t_4) = & \frac{3}{7} w_2 t_1^2 \otimes t_1^4 t_2^2 - \frac{1}{7} w_1^3 t_2^3 \otimes t_2 - \frac{1}{7} w_1^3 t_1^3 \otimes t_1^6 t_2 - \frac{1}{7} w_1^3 t_2 t_1^3 \otimes t_1^6 - \frac{1}{7} w_1^3 t_1 \otimes t_2^3 t_1^2 - \\ & \frac{1}{7} w_1^4 t_1^4 \otimes t_1^7 - \frac{1}{7} w_1^4 t_2^3 t_1 \otimes t_1 - \frac{1}{7} w_1^4 t_1 \otimes t_2^3 t_1 + \frac{4}{127} w_3 t_1^7 \otimes t_1 + \frac{14}{127} w_3 t_1^6 \otimes t_1^2 + \frac{28}{127} w_3 t_1^5 \otimes \\ & t_1^3 + \frac{35}{127} w_3 t_1^4 \otimes t_1^4 + \frac{28}{127} w_3 t_1^3 \otimes t_1^5 + \frac{14}{127} w_3 t_1^2 \otimes t_1^6 + \frac{1}{889} w_1^7 t_1^7 \otimes t_1 + \frac{1}{889} w_1^7 t_1 \otimes t_1^7 + \frac{2}{7} w_1^3 t_2^2 \otimes \\ & t_2^2 + \frac{2}{7} w_1^3 t_2^2 t_1^2 \otimes t_1^4 + \frac{2}{7} w_1^5 t_1^2 \otimes t_2^2 t_1^2 + \frac{2}{7} w_1^5 t_2^2 t_1^2 \otimes t_1^2 + \frac{2}{7} w_1^3 t_1^2 \otimes t_1^4 t_2^2 + \frac{2}{7} w_1^5 t_1^4 \otimes t_1^6 + \\ & w_1 t_3 t_2 \otimes t_1^4 + w_1 t_3 t_1 \otimes t_2^2 + w_1 t_3 \otimes t_3 + w_1 t_2 t_1 \otimes t_1^4 t_2^2 + w_1 t_1 \otimes t_2^2 t_3 + w_1 t_2 \otimes t_1^4 t_3 + w_1^2 t_2^2 t_1 \otimes \\ & t_1^6 + \frac{4}{7} w_1^3 t_2^2 t_1 \otimes t_1^2 t_2 + \frac{4}{7} w_1^3 t_2 t_1 \otimes t_2^2 t_1^2 + \frac{4}{7} w_1^3 t_1^2 t_2 \otimes t_1^4 t_2 + \frac{3}{7} w_1^4 t_2 t_1^3 \otimes t_1^5 + w_1^3 t_2 t_1 t_3 \otimes t_1 + \\ & w_1^2 t_3 t_1 \otimes t_1^2 t_2 + w_1^3 t_1^2 t_3 \otimes t_1^3 + w_1^3 t_3 t_1 \otimes t_2 t_1 + w_1^2 t_2^2 \otimes t_1^4 t_2 + w_1^3 t_1^2 t_2 \otimes t_1^7 + w_1^3 t_2 t_1 \otimes t_1^5 t_2 + \\ & \frac{2}{7} w_1^4 t_1^2 t_3 \otimes t_1^2 - \frac{2}{889} w_1^4 w_2 t_1^7 \otimes t_1 - \frac{2}{889} w_1^4 w_2 t_1 \otimes t_1^7 + \frac{5}{7} w_1^3 w_2 t_2 t_1^3 \otimes t_1^3 + \frac{2}{7} w_1^2 w_2 t_1^4 \otimes t_2 t_1^3 + \\ & \frac{9}{7} w_1^2 w_2 t_1^3 \otimes t_1^4 t_2 + \frac{2}{7} w_1^2 w_2 t_1^2 \otimes t_1^5 t_2 + \frac{5}{7} w_1^3 w_2 t_1^4 \otimes t_1^5 - \frac{303}{6223} w_2 w_1^4 t_1^2 \otimes t_1^6 + \frac{2}{7} w_1^3 w_2 t_1^4 t_2 \otimes \\ & t_1^2 + \frac{2}{7} w_1^3 w_2 t_1^4 \otimes t_1^2 t_2 + \frac{29}{6223} w_1^4 w_2 t_1^3 \otimes t_1^5 + \frac{29}{6223} w_1^4 w_2 t_1^5 \otimes t_1^3 + \frac{2}{7} w_1^2 w_2 t_1^4 t_2 \otimes t_1^3 + \\ & \frac{2}{7} w_1^2 w_2 t_2 t_1 \otimes t_2 t_1^3 + \frac{2}{7} w_1^3 w_2 t_1^5 \otimes t_1^4 + \frac{5}{7} w_1^3 w_2 t_1^3 \otimes t_2 t_1^3 - \frac{303}{6223} w_2 w_1^4 t_1^6 \otimes t_1^2 + \frac{2}{7} w_1^3 w_2 t_1^3 \otimes \\ & t_1^6 + \frac{76}{6223} w_1 w_2^2 t_1^3 \otimes t_1^5 + \frac{2}{7} w_1 w_2 t_3 t_1^3 \otimes t_1 + \frac{3}{7} w_1 w_2 t_1^2 t_3 \otimes t_1^2 + \frac{2}{7} w_1 w_2 t_3 t_1 \otimes t_1^3 + \frac{2}{7} w_1 w_2 t_2 t_1 \otimes \\ & t_1^7 + \frac{2}{7} w_1 w_2 t_1^4 \otimes t_2^2 t_1 + \frac{3}{7} w_1 w_2 t_1^3 \otimes t_2^2 t_1^2 + \frac{8}{7} w_1 w_2 t_2 t_1^3 \otimes t_1^5 + \frac{3}{7} w_1 w_2 t_1^2 t_2 \otimes t_1^6 + \\ & \frac{8}{7} w_1 w_2 t_1^2 \otimes t_2^2 t_1^3 + \frac{2}{7} w_1 w_2 t_1^3 \otimes t_3 t_1 + \frac{3}{7} w_1 w_2 t_1^2 \otimes t_1^2 t_3 + \frac{2}{7} w_1 w_2 t_1 \otimes t_3 t_1^3 - \frac{89}{6223} w_1 w_2^2 t_1^6 \otimes \\ & t_1^2 + \frac{76}{6223} w_1 w_2^2 t_1^5 \otimes t_1^3 + \frac{222}{6223} w_1 w_2^2 t_1^4 \otimes t_1^4 - \frac{89}{6223} w_1 w_2^2 t_1^2 \otimes t_1^6 + \frac{6}{7} w_2 w_1 t_2^2 t_1 \otimes t_2 t_1 + \\ & \frac{12}{7} w_2 w_1 t_1^2 t_2 \otimes t_2 t_1^3 + \frac{2}{7} w_2 w_1 t_1^4 \otimes t_1^7 + \frac{6}{7} w_2 w_1 t_1^3 \otimes t_1^5 t_2 + \frac{3}{7} w_2 w_1^2 t_1^4 \otimes t_1^6 + \frac{2}{7} w_2 w_1 t_2^3 t_1 \otimes \\ & t_1 + \frac{6}{7} w_2 w_1 t_2^2 t_1^2 \otimes t_1^3 + \frac{6}{7} w_2 w_1 t_2 t_1 \otimes t_2^2 t_1 + \frac{3}{7} w_2 w_1^2 t_2^2 t_1^2 \otimes t_1^2 + \frac{2}{7} w_2 w_1 t_1 \otimes t_2^3 t_1 + \\ & \frac{3}{7} w_2 w_1^2 t_1^2 \otimes t_2^2 t_1^2 - \frac{2}{127} w_1 w_2^2 t_1^7 \otimes t_1 - \frac{2}{127} w_1 w_2^2 t_1 \otimes t_1^7 + \frac{2}{7} w_1^3 w_2 t_1^2 t_2 \otimes t_1^4 + \frac{9}{7} w_1^2 w_2 t_2 t_1^3 \otimes \\ & t_1^4 + \frac{2}{7} w_1^2 w_2 t_1^2 t_2 \otimes t_1^5 + \frac{2}{7} w_1^3 w_2 t_1^2 \otimes t_1^4 t_2 + \frac{576}{6223} w_1^4 w_2 t_1^4 \otimes t_1^4 + \frac{2}{7} w_1^2 w_2 t_2 t_1^3 \otimes t_2 t_1 + \\ & \frac{9}{7} w_1^2 w_2 t_1^2 t_2 \otimes t_1^2 t_2 + \frac{220}{6223} w_1^7 t_1^4 \otimes t_1^4 + \frac{6}{7} w_1^5 t_1^2 t_2 \otimes t_1^2 t_2 - \frac{1}{7} w_1^5 t_2 t_1^3 \otimes t_2 t_1 - \frac{1}{7} w_1^5 t_2 t_1 \otimes \\ & t_2 t_1^3 - \frac{1}{7} w_1^6 t_1^4 t_2 \otimes t_1^2 - \frac{1}{7} w_1^6 t_1^2 t_2 \otimes t_1^4 + \frac{4}{7} w_1^4 t_1^3 \otimes t_1^5 t_2 + \frac{1}{7} w_1^6 t_2 t_1^3 \otimes t_1^3 + \frac{1}{7} w_1^6 t_1^4 \otimes t_1^5 - \\ & \frac{1}{7} w_1^6 t_1^5 \otimes t_1^4 - \frac{1}{7} w_1^6 t_1^3 \otimes t_1^6 + \frac{6}{7} w_1^5 t_1^3 \otimes t_1^4 t_2 - \frac{1}{7} w_1^5 t_1^4 \otimes t_2 t_1^3 - \frac{1}{7} w_1^5 t_1^2 \otimes t_1^5 t_2 + \frac{88}{6223} w_1^7 t_1^2 \otimes \\ & t_1^6 + w_1^2 t_1^2 \otimes t_2^3 t_1^2 + w_1^2 t_2 t_1 t_3 \otimes t_1^2 + w_1^3 t_1^2 \otimes t_2^3 t_1 + \frac{4}{7} w_1^4 t_2^2 t_1 \otimes t_2 t_1 + w_1^3 t_1 \otimes t_2 t_1 t_3 - \\ & \frac{205}{6223} w_1^7 t_1^5 \otimes t_1^3 + \frac{4}{7} w_1^4 t_2^2 t_1^2 \otimes t_1^3 + w_1^3 t_1^2 t_2 \otimes t_2^2 t_1 + w_1^2 t_2 t_1 \otimes t_1^2 t_3 + w_1^3 t_2^2 t_1 \otimes t_1^5 + w_1^2 t_1 \otimes \end{aligned}$$

$$t_3 t_1^2 t_2 + w_1^2 t_3 t_2 \otimes t_2 + w_1^3 t_2 t_1 \otimes t_3 t_1 + \frac{15}{7} w_1^4 t_1^2 t_2 \otimes t_2 t_1^3 - \frac{205}{6223} w_1^7 t_1^3 \otimes t_1^5 + w_1^2 t_1^2 t_2 \otimes t_2^2 t_1^2 + \frac{4}{7} w_1^4 t_2 t_1 \otimes t_2^2 t_1 + w_1^2 t_2 t_1 \otimes t_1^6 t_2 + \frac{88}{6223} w_1^7 t_1^6 \otimes t_1^2 + \frac{2}{7} w_2 t_2^3 \otimes t_2 + \frac{2}{7} w_2 t_2^3 t_1 \otimes t_1^2 + \frac{2}{7} w_2 t_1^3 \otimes t_1^6 t_2 + \frac{2}{7} w_2 t_2 t_1^3 \otimes t_1^6 + \frac{2}{7} w_2 t_2 \otimes t_2^3 + \frac{2}{7} w_2 t_1 \otimes t_2^3 t_1^2 + \frac{3}{7} w_2 t_2^2 \otimes t_2^2 + \frac{3}{7} w_2 t_2^2 t_1^2 \otimes t_1^4 + \frac{6}{7} w_2 t_2^2 t_1 \otimes t_1^2 t_2 + \frac{6}{7} w_2 t_2 t_1 \otimes t_2^2 t_1^2 + \frac{6}{7} w_2 t_1^2 t_2 \otimes t_1^4 t_2 + \frac{6}{7} w_1^5 t_2 t_1^3 \otimes t_1^4 - \frac{1}{7} w_1^5 t_1^4 t_2 \otimes t_1^3 - \frac{1}{7} w_1^5 t_1^2 t_2 \otimes t_1^5 - \frac{1}{7} w_1^4 t_3 t_1^3 \otimes t_1 - \frac{1}{7} w_1^4 t_3 t_1 \otimes t_1^3 + w_1^2 t_2 t_1 \otimes t_2^3 + w_1^3 t_1^3 \otimes t_2^2 t_1^3 + w_1^2 t_2 \otimes t_3 t_2 + w_1^3 t_1^2 \otimes t_3 t_1^3 - \frac{1}{7} w_1^4 t_2 t_1 \otimes t_1^7 + \frac{2}{7} w_1^4 t_1^3 \otimes t_2^2 t_1^2 + \frac{2}{7} w_1^4 t_1^2 t_2 \otimes t_1^6 - \frac{1}{7} w_1^4 t_1^4 \otimes t_2^2 t_1 + \frac{3}{7} w_1^4 t_1^2 \otimes t_2^2 t_1^3 + \frac{2}{7} w_1^4 t_1^2 \otimes t_1^2 t_3 - \frac{1}{7} w_1^4 t_1^3 \otimes t_3 t_1 - \frac{1}{7} w_1^4 t_1 \otimes t_3 t_1^3 + \frac{1}{7} w_1^6 t_1^3 \otimes t_2 t_1^3 - \frac{1}{7} w_1^6 t_1^4 \otimes t_1^2 t_2 - \frac{1}{7} w_1^6 t_1^2 \otimes t_1^4 t_2 - \frac{1}{7} w_1^3 t_2 \otimes t_2^3 - \frac{1}{7} w_1^3 t_2^3 t_1 \otimes t_1^2 + \frac{4}{127} w_3 t_1 \otimes t_1^7 + t_2 \otimes t_2^4 + t_3 \otimes t_1^8 + t_4 \otimes 1 + 1 \otimes t_4 + t_1 \otimes t_3^2$$

$$\Delta(t_4) \bmod 2 = w_2 t_1^2 \otimes t_1^4 t_2^2 + w_1^3 t_2^3 \otimes t_2 + w_1^3 t_1^3 \otimes t_1^6 t_2 + w_1^3 t_2 t_1^3 \otimes t_1^6 + w_1^3 t_1 \otimes t_2^3 t_1^2 + w_1^4 t_1^4 \otimes t_1^7 + w_1^4 t_2^3 t_1 \otimes t_1 + w_1^4 t_1 \otimes t_2^3 t_1 + w_3 t_1^4 \otimes t_1^4 + w_1^7 t_1^7 \otimes t_1 + w_1^7 t_1 \otimes t_1^7 + w_1 t_3 t_2 \otimes t_1^4 + w_1 t_3 t_1 \otimes t_2^2 + w_1 t_3 \otimes t_3 + w_1 t_2 t_1 \otimes t_1^4 t_2^2 + w_1 t_1 \otimes t_2^2 t_3 + w_1 t_2 \otimes t_1^4 t_3 + w_1^2 t_2^2 t_1 \otimes t_1^6 + w_1^4 t_2 t_1^3 \otimes t_1^5 + w_1^3 t_2 t_1 t_3 \otimes t_1 + w_1^2 t_3 t_1 \otimes t_1^2 t_2 + w_1^3 t_1^2 t_3 \otimes t_1^3 + w_1^3 t_3 t_1 \otimes t_2 t_1 + w_1^2 t_2^2 \otimes t_1^4 t_2 + w_1^3 t_1^2 t_2 \otimes t_1^7 + w_1^3 t_2 t_1 \otimes t_1^5 t_2 + w_1^3 w_2 t_2 t_1^3 \otimes t_1^3 + w_1^2 w_2 t_1^3 \otimes t_1^4 t_2 + w_1^3 w_2 t_1^4 \otimes t_1^5 + w_2 w_1^4 t_1^2 \otimes t_1^6 + w_1^4 w_2 t_1^3 \otimes t_1^5 + w_1^4 w_2 t_1^5 \otimes t_1^3 + w_1^3 w_2 t_1^3 \otimes t_2 t_1^3 + w_2 w_1^4 t_1^6 \otimes t_1^2 + w_1 w_2 t_1^2 t_3 \otimes t_1^2 + w_1 w_2 t_1^3 \otimes t_2^2 t_1^2 + w_1 w_2 t_1^2 t_2 \otimes t_1^6 + w_1 w_2 t_1^2 \otimes t_1^2 t_3 + w_1 w_2^2 t_1^6 \otimes t_1^2 + w_1 w_2^2 t_1^2 \otimes t_1^6 + w_2 w_1^2 t_1^4 \otimes t_1^6 + w_2 w_1^2 t_2^2 t_1^2 \otimes t_1^2 + w_2 w_1^2 t_1^2 \otimes t_2^2 t_1^2 + w_1^2 w_2 t_2 t_1^3 \otimes t_1^4 + w_1^2 w_2 t_1^2 t_2 \otimes t_1^2 t_2 + w_1^5 t_2 t_1^3 \otimes t_2 t_1 + w_1^5 t_2 t_1 \otimes t_2 t_1^3 + w_1^6 t_1^4 t_2 \otimes t_1^2 + w_1^6 t_1^2 t_2 \otimes t_1^4 + w_1^6 t_2 t_1^3 \otimes t_1^3 + w_1^6 t_1^4 \otimes t_1^5 + w_1^6 t_1^5 \otimes t_1^4 + w_1^6 t_1^3 \otimes t_1^6 + w_1^5 t_1^4 \otimes t_2 t_1^3 + w_1^5 t_1^2 \otimes t_1^5 t_2 + w_1^2 t_1^2 \otimes t_2^3 t_1^2 + w_1^2 t_2 t_1 t_3 \otimes t_1^2 + w_1^3 t_1^2 \otimes t_2^3 t_1 + w_1^3 t_1 \otimes t_2 t_1 t_3 + w_1^7 t_1^5 \otimes t_1^3 + w_1^3 t_1^2 t_2 \otimes t_2^2 t_1 + w_1^2 t_2 t_1 \otimes t_1^2 t_3 + w_1^3 t_2^2 t_1 \otimes t_1^5 + w_1^2 t_1 \otimes t_3 t_1^2 t_2 + w_1^2 t_3 t_2 \otimes t_2 + w_1^3 t_2 t_1 \otimes t_3 t_1 + w_1^4 t_1^2 t_2 \otimes t_2 t_1^3 + w_1^7 t_1^3 \otimes t_1^5 + w_1^2 t_1^2 t_2 \otimes t_2^2 t_1^2 + w_1^2 t_2 t_1 \otimes t_1^6 t_2 + w_2 t_2^2 \otimes t_2^2 + w_2 t_2^2 t_1^2 \otimes t_1^4 + w_1^5 t_1^4 t_2 \otimes t_1^3 + w_1^5 t_1^2 t_2 \otimes t_1^5 + w_1^4 t_3 t_1^3 \otimes t_1 + w_1^4 t_3 t_1 \otimes t_1^3 + w_1^2 t_2 t_1 \otimes t_2^3 + w_1^3 t_1^3 \otimes t_2^2 t_1^3 + w_1^2 t_2 \otimes t_3 t_2 + w_1^3 t_1^2 \otimes t_3 t_1^3 + w_1^4 t_2 t_1 \otimes t_1^7 + w_1^4 t_1^4 \otimes t_2^2 t_1 + w_1^4 t_1^2 \otimes t_2^2 t_1^3 + w_1^4 t_1^3 \otimes t_3 t_1 + w_1^4 t_1 \otimes t_3 t_1^3 + w_1^6 t_1^3 \otimes t_2 t_1^3 + w_1^6 t_1^4 \otimes t_1^2 t_2 + w_1^6 t_1^2 \otimes t_1^4 t_2 + w_1^3 t_2 \otimes t_2^3 + w_1^3 t_2^3 t_1 \otimes t_1^2 + t_2 \otimes t_2^4 + t_3 \otimes t_1^8 + t_4 \otimes 1 + 1 \otimes t_4 + t_1 \otimes t_3^2$$

$$\Delta(t_4) \bmod (2, w_1) = w_2 t_1^2 \otimes t_1^4 t_2^2 + w_3 t_1^4 \otimes t_1^4 + w_2 t_2^2 \otimes t_2^2 + w_2 t_2^2 t_1^2 \otimes t_1^4 + t_2 \otimes t_2^4 + t_3 \otimes t_1^8 + t_4 \otimes 1 + 1 \otimes t_4 + t_1 \otimes t_3^2$$

$$\Delta(t_4) \bmod (2, w_1, w_2) = w_3 t_1^4 \otimes t_1^4 + t_2 \otimes t_2^4 + t_3 \otimes t_1^8 + t_4 \otimes 1 + 1 \otimes t_4 + t_1 \otimes t_3^2$$

$$\Delta(t_4) \bmod (2, w_1, w_2, w_3) = t_2 \otimes t_2^4 + t_3 \otimes t_1^8 + t_4 \otimes 1 + 1 \otimes t_4 + t_1 \otimes t_3^2$$

3.3. Araki generators at p=2: Conjugation calculations.

$$c(t_1) = -t_1$$

$$c(t_2) = -t_2 + w_1 t_1^2 - t_1^3$$

$$c(t_3) = -t_3 + w_1 t_2^2 + 1/7 t_1^4 w_2 + 3/7 t_1^4 w_1^3 - 3 t_2 t_1^4 + 3 w_1 t_1^6 - t_2^2 t_1 + 3 t_2 w_1 t_1^3 - 2 w_1^2 t_1^5 - t_1^7 - t_2 w_1^2 t_1^2$$

$$c(t_4) = -t_1^{15} - 7 t_1^{12} t_2 - 15 t_1^9 t_2^2 + 2/7 t_3 t_1^5 w_2 + \frac{20}{7} t_3 t_1^5 w_1^3 - 12 t_1^6 t_2^3 + w_1 t_3^2 - t_4 - 3 t_3 t_1^8 + 47 w_1 t_2^2 t_1^8 - t_2^5 + 1/7 t_2^4 w_2 + 3/7 t_2^4 w_1^3 + \frac{1}{127} t_1^8 w_3 + \frac{351}{6223} t_1^8 w_1^7 + \frac{41}{7} w_1^3 t_2^3 t_1^3 - \frac{34}{49} t_1^9 w_2 w_1^3 + \frac{8}{7} t_1^9 w_2 t_2 - \frac{9}{7} t_1^{11} w_2 w_1 + 5/7 t_1^6 w_2 t_2^2 + \frac{10}{7} t_1^{10} w_2 w_1^2 + \frac{26}{7} t_1^7 w_1^5 t_2 - 2 t_1 w_1^2 t_2^4 - 6 t_3 t_2 t_1^5 - 2 t_3 t_2^2 t_1^2 - 7 t_3 w_1^2 t_1^6 - \frac{48}{7} w_1^4 t_2^2 t_1^5 + 7 t_3 w_1 t_1^7 - t_1 t_3^2 + \frac{37}{7} t_1^{10} w_1^5 + 7 w_1 t_1^{14} - 1/49 t_1^9 w_2^2 - \frac{44}{49} t_1^9 w_1^6 - \frac{104}{7} t_1^{11} w_1^4 - 18 w_1^2 t_1^{13} + 29 t_2^3 w_1 t_1^5 - \frac{136}{7} t_2 w_1^4 t_1^8 + \frac{325}{7} t_2 w_1^3 t_1^9 - 56 t_2 w_1^2 t_1^{10} + 33 t_2 w_1 t_1^{11} + 3/7 t_1^{12} w_2 + \frac{156}{7} t_1^{12} w_1^3 + 7 t_2^4 w_1 t_1^2 - 22 t_2^3 w_1^2 t_1^4 + \frac{211}{7} t_2^2 w_1^3 t_1^6 - 56 t_2^2 w_1^2 t_1^7 - \frac{9}{7} w_1 t_2^2 t_1^5 w_2 -$$

$$\begin{aligned} & \frac{15}{7} t_1^8 w_2 t_2 w_1 + \frac{11}{7} t_1^7 w_2 t_2 w_1^2 + 9 t_3 t_2 w_1 t_1^4 - 5 t_3 t_2 w_1^2 t_1^3 + 3 t_1 t_3 w_1 t_2^2 - 2/7 w_2 t_2^3 w_1 t_1^2 + \\ & 4/7 w_2 t_2^2 w_1^2 t_1^4 + t_3 t_2 w_1^3 t_1^2 - 1/7 w_1 t_3 t_1^4 w_2 + \frac{39}{6223} t_1^8 w_1 w_2^2 + \frac{822}{6223} t_1^8 w_1^4 w_2 - t_3 w_1^2 t_2^2 - \\ & 3/7 t_3 t_1^4 w_1^4 - 2/7 w_1^6 t_2 t_1^6 - 6/7 w_1^4 t_2^3 t_1^2 + 5/7 w_1^5 t_2^2 t_1^4 + 2/7 w_2 t_2^3 t_1^3 - 3/7 w_2 t_2 w_1^3 t_1^6 - 5 t_2^4 t_1^3 \end{aligned}$$

3.4. Araki generators at p=3: Right unit calculations.

$$\eta_R(w_1) = w_1 - 24 t_1$$

$$\eta_R(w_1) \pmod{3} = w_1$$

$$\eta_R(w_2) = w_2 - 1484 w_1 t_1^3 - 19680 t_2 - 4 w_1^3 t_1 + 144 w_1^2 t_1^2 + 13824 t_1^4$$

$$\eta_R(w_2) \pmod{3} = w_2 + w_1 t_1^3 + 2 w_1^3 t_1$$

$$\eta_R(w_2) \pmod{(3, w_1)} = w_2$$

$$\begin{aligned} \eta_R(w_3) = & -\frac{51456}{205} w_2 w_1^6 t_1^3 + \frac{1979516}{205} w_2 t_1^4 w_1^5 - \frac{44290704}{205} w_2 w_1^4 t_1^5 + \frac{656529194}{205} w_2 w_1^3 t_1^6 - \\ & \frac{6574205424}{205} w_2 w_1^2 t_1^7 + \frac{40449798144}{205} w_2 w_1 t_1^8 + 216 t_2 w_1^8 t_1 - 60096 t_2 w_1^7 t_1^2 + 4939776 t_2 w_1^6 t_1^3 - \\ & 190033536 t_2 t_1^4 w_1^5 + 4251907584 t_2 w_1^4 t_1^5 - 63026802624 t_2 w_1^3 t_1^6 + 631123720704 t_2 w_1^2 t_1^7 - \\ & 3883180621824 t_2 w_1 t_1^8 + 145260541 w_1 t_2^3 - \frac{3431723050035}{1312} t_1^9 w_1^4 - 2460 w_1 w_2^2 t_2 + \\ & 48412800 w_1 w_2 t_2^2 + 6180 w_1 t_1^4 w_2^2 + 2393528832000 w_1 t_2^2 t_1^4 + 59040 t_1 w_2^2 t_2 - \\ & 1161907200 t_1 w_2 t_2^2 + 19680 w_2 t_2 w_1^4 t_1 - 1/2 w_2^2 w_1^4 t_1 + 30 w_2^2 w_1^3 t_1^2 - \frac{1235}{2} w_2^2 w_1^2 t_1^3 - \\ & 239159232000 w_1^2 t_1^3 t_2^2 - 193651200 t_2^2 w_1^4 t_1 + 11619072000 t_2^2 w_1^3 t_1^2 + 1632337920 w_2 t_2 t_1^5 + \\ & 23651829440 w_1^3 t_1^{10} - 168059805696 w_1^2 t_1^{11} + 850790449152 w_1 t_1^{12} - t_2 w_1^9 + \\ & 13924527243264 t_2 t_1^9 + \frac{3}{6560} w_1^{12} t_1 - \frac{9}{205} w_1^{11} t_1^2 - \frac{273}{1640} w_1^{10} t_1^3 + \frac{52171}{205} w_1^9 t_1^4 - \frac{3538264}{205} w_1^8 t_1^5 + \\ & \frac{123707904}{205} w_1^7 t_1^6 - \frac{2793155624}{205} w_1^6 t_1^7 + \frac{44814121824}{205} w_1^5 t_1^8 + \frac{626}{205} w_2 w_1^7 t_1^2 - \frac{9}{820} w_2 w_1^8 t_1 - 41472 t_1^5 w_2^2 - \\ & 16062205132800 t_2^2 t_1^5 - t_1 w_2^3 + 7622111232000 t_1 t_2^3 - 2641807540224 t_1^{13} - 1180800 w_2 t_2 w_1^3 t_1^2 + \\ & 24304800 w_2 t_2 w_1^2 t_1^3 - 243244800 w_1 w_2 t_2 t_1^4 + w_3 - \frac{52491081319}{164} t_1^9 w_2 - 7625597484984 t_3 \end{aligned}$$

$$\eta_R(w_3) \pmod{3} = 2 w_2 t_1^4 w_1^5 + 2 w_2 w_1^3 t_1^6 + w_1 t_2^3 + w_2^2 w_1^4 t_1 + 2 w_2^2 w_1^2 t_1^3 + 2 w_1^3 t_1^{10} + 2 t_2 w_1^9 + w_1^9 t_1^4 + 2 w_1^8 t_1^5 + w_1^6 t_1^7 + 2 w_2 w_1^7 t_1^2 + 2 t_1 w_2^3 + w_3 + t_1^9 w_2$$

$$\eta_R(w_3) \pmod{(3, w_1)} = 2 t_1 w_2^3 + w_3 + t_1^9 w_2$$

$$\eta_R(w_3) \pmod{(3, w_1, w_2)} = w_3$$

3.5. Araki generators at p=3: Coproduct calculations. These results for $\Delta(w_1)$ and $\Delta(w_2)$ agree with those in [Rav92, p.168-169].

$$\Delta(1) = 1 \otimes 1$$

$$\Delta(t_1) = t_1 \otimes 1 + 1 \otimes t_1$$

$$\Delta(t_2) = t_2 \otimes 1 + t_1 \otimes t_1^3 + 1 \otimes t_2 + \frac{1}{8} w_1 t_1^2 \otimes t_1 + \frac{1}{8} w_1 t_1 \otimes t_1^2$$

$$\Delta(t_2) \pmod{3} = t_2 \otimes 1 + t_1 \otimes t_1^3 + 1 \otimes t_2 + 2 w_1 t_1^2 \otimes t_1 + 2 w_1 t_1 \otimes t_1^2$$

$$\Delta(t_2) \pmod{(3, w_1)} = t_2 \otimes 1 + t_1 \otimes t_1^3 + 1 \otimes t_2$$

$$\Delta(t_2) \pmod{(3, w_1, w_2)} = t_2 \otimes 1 + t_1 \otimes t_1^3 + 1 \otimes t_2$$

$$\begin{aligned} \Delta(t_3) = & t_1 \otimes t_2^3 + t_2 \otimes t_1^9 + \frac{1}{256} w_1^3 t_2 t_1^3 \otimes t_1^3 + t_3 \otimes 1 + 1 \otimes t_3 + \frac{1}{32} w_1^2 t_1^2 \otimes t_1^5 t_2 + \frac{1}{32} w_1^2 t_1^3 \otimes \\ & t_1^4 t_2 + \frac{1}{32} w_1^2 t_2 t_1^3 \otimes t_1^4 + \frac{1}{256} w_1^3 t_1^3 \otimes t_2 t_1^3 - \frac{19}{839680} w_1^4 t_1^5 \otimes t_1^4 + \frac{1}{512} w_1^3 t_1^2 \otimes t_1^4 t_2 + \frac{1}{64} w_1^2 t_1^4 \otimes \\ & t_1^7 + \frac{1}{256} w_1^3 t_1^4 \otimes t_1^6 - \frac{19}{839680} w_1^4 t_1^4 \otimes t_1^5 + \frac{1}{8} w_1 t_2 \otimes t_2^2 + \frac{1}{8} w_1 t_2^2 \otimes t_2 + \frac{1}{8} w_1 t_1^2 t_2 \otimes t_1^6 + \frac{1}{4} w_1 t_2 t_1 \otimes \\ & t_2 t_1^3 + \frac{1}{8} w_1 t_2^2 t_1 \otimes t_1^3 + \frac{1}{8} w_1 t_1 \otimes t_2^2 t_1^3 + \frac{1}{8} w_1 t_1^2 \otimes t_1^6 t_2 + \frac{1}{64} w_1^2 t_1^3 \otimes t_1^8 - \frac{81}{839680} w_1^4 t_1^3 \otimes t_1^6 + \\ & \frac{3}{6560} w_2 t_1^8 \otimes t_1 + \frac{3}{1640} w_2 t_1^7 \otimes t_1^2 + \frac{7}{1640} w_2 t_1^6 \otimes t_1^3 + \frac{21}{3280} w_2 t_1^5 \otimes t_1^4 + \frac{21}{3280} w_2 t_1^4 \otimes t_1^5 + \\ & \frac{7}{1640} w_2 t_1^3 \otimes t_1^6 + \frac{3}{1640} w_2 t_1^2 \otimes t_1^7 + \frac{3}{6560} w_2 t_1 \otimes t_1^8 - \frac{1}{52480} w_1^4 t_1^8 \otimes t_1 - \frac{1}{13120} w_1^4 t_1^7 \otimes t_1^2 - \\ & \frac{1}{13120} w_1^4 t_1^2 \otimes t_1^7 - \frac{1}{52480} w_1^4 t_1 \otimes t_1^8 + \frac{1}{64} w_1^2 t_1 \otimes t_2^2 t_1^2 + \frac{1}{512} w_1^3 t_1^4 \otimes t_1^2 t_2 + \frac{1}{512} w_1^3 t_1^5 \otimes t_1^5 - \\ & \frac{81}{839680} w_1^4 t_1^6 \otimes t_1^3 + \frac{1}{512} w_1^3 t_1^3 \otimes t_1^7 + \frac{1}{64} w_1^2 t_1^2 \otimes t_2^2 t_1 + \frac{1}{32} w_1^2 t_1^2 t_2 \otimes t_2 t_1 + \frac{1}{64} w_1^2 t_2^2 t_1 \otimes t_1^2 + \\ & \frac{1}{32} w_1^2 t_2 t_1 \otimes t_1^2 t_2 + \frac{1}{64} w_1^2 t_2^2 t_1^2 \otimes t_1 + \frac{1}{512} w_1^3 t_1^2 t_2 \otimes t_1^4 + \frac{1}{512} w_1^3 t_1^4 t_2 \otimes t_1^2 + \frac{1}{32} w_1^2 t_1^2 t_2 \otimes t_1^5 \end{aligned}$$

$$\begin{aligned} \Delta(t_3) \bmod 3 = & t_1 \otimes t_2^3 + t_2 \otimes t_1^9 + w_1^3 t_2 t_1^3 \otimes t_1^3 + t_3 \otimes 1 + 1 \otimes t_3 + 2 w_1^2 t_1^2 \otimes t_1^5 t_2 + 2 w_1^2 t_1^3 \otimes \\ & t_1^4 t_2 + 2 w_1^2 t_2 t_1^3 \otimes t_1^4 + w_1^3 t_1^3 \otimes t_2 t_1^3 + 2 w_1^4 t_1^5 \otimes t_1^4 + 2 w_1^3 t_1^2 \otimes t_1^4 t_2 + w_1^2 t_1^4 \otimes t_1^7) + \\ & w_1^3 t_1^4 \otimes t_1^6 + 2 w_1^4 t_1^4 \otimes t_1^5 + 2 w_1 t_2 \otimes t_2^2 + 2 w_1 t_2^2 \otimes t_2 + 2 w_1 t_1^2 t_2 \otimes t_1^6 + w_1 t_2 t_1 \otimes t_2 t_1^3 + \\ & 2 w_1 t_2^2 t_1 \otimes t_1^3 + 2 w_1 t_1 \otimes t_2^2 t_1^3 + 2 w_1 t_1^2 \otimes t_1^6 t_2 + w_1^2 t_1^3 \otimes t_1^8 + 2 w_2 t_1^6 \otimes t_1^3 + 2 w_2 t_1^3 \otimes \\ & t_1^6 + 2 w_1^4 t_1^8 \otimes t_1 + 2 w_1^4 t_1^7 \otimes t_1^2 + 2 w_1^4 t_1^2 \otimes t_1^7) + 2 w_1^4 t_1 \otimes t_1^8 + w_1^2 t_1 \otimes t_2^2 t_1^2 + 2 w_1^3 t_1^4 \otimes \\ & t_1^2 t_2 + 2 w_1^3 t_1^5 \otimes t_1^5) + 2 w_1^3 t_1^3 \otimes t_1^7 + w_1^2 t_1^2 \otimes t_2^2 t_1 + 2 w_1^2 t_1^2 t_2 \otimes t_2 t_1 + w_1^2 t_2^2 t_1 \otimes t_1^2 + \\ & 2 w_1^2 t_2 t_1 \otimes t_1^2 t_2 + w_1^2 t_2^2 t_1^2 \otimes t_1 + 2 w_1^3 t_1^2 t_2 \otimes t_1^4 + 2 w_1^3 t_1^4 t_2 \otimes t_1^2 + 2 w_1^2 t_1^2 t_2 \otimes t_1^5 \end{aligned}$$

$$\Delta(t_3) \bmod (3, w_1) = t_1 \otimes t_2^3 + t_2 \otimes t_1^9 + t_3 \otimes 1 + 1 \otimes t_3 + 2 w_2 t_1^6 \otimes t_1^3 + 2 w_2 t_1^3 \otimes t_1^6$$

$$\Delta(t_3) \bmod (3, w_1, w_2) = t_1 \otimes t_2^3 + t_2 \otimes t_1^9 + t_3 \otimes 1 + 1 \otimes t_3$$

3.6. Araki generators at p=3: Conjugation calculations.

$$c(t_1) = -t_1$$

$$c(t_2) = -t_2 + t_1^4$$

$$c(t_3) = -t_3 + 4 t_1^9 t_2 + t_1 t_2^3 - 3 t_2^2 t_1^5 - t_1^{13} + 1/8 w_1 t_2^2 t_1^4 - 1/8 t_2 w_1 t_1^8$$

$$\begin{aligned} c(t_4) = & 4 t_3 t_1^{27} + \frac{145}{8} w_1 t_1^{19} t_2^5 - \frac{167}{8} w_1 t_1^{23} t_2^4 - \frac{17}{4} w_1 t_1^{31} t_2^2 + 1/2 w_1 t_1^{35} t_2 - \frac{81}{839680} w_1^4 t_1^{12} t_2^6 + \\ & \frac{19}{839680} w_1^4 t_2^5 t_1^{16} - \frac{19}{839680} w_1^4 t_2^4 t_1^{20} + \frac{81}{839680} w_1^4 t_2^3 t_1^{24} - \frac{1}{13120} w_1^4 t_2^2 t_1^{28} + \frac{1}{52480} w_1^4 t_2 t_1^{32} + \\ & \frac{3}{6560} w_2 t_2^8 t_1^4 - \frac{3}{1640} w_2 t_1^8 t_2^7 + \frac{7}{1640} w_2 t_1^{12} t_2^6 - \frac{21}{3280} w_2 t_2^5 t_1^{16} + \frac{21}{3280} w_2 t_2^4 t_1^{20} - \frac{7}{1640} w_2 t_2^3 t_1^{24} + \\ & \frac{3}{1640} w_2 t_2^2 t_1^{28} - \frac{3}{6560} w_2 t_2 t_1^{32} + t_2^{10} + \frac{1}{64} t_3^2 w_1^2 t_2^2 t_1^4 - \frac{1}{64} t_3^2 t_2 w_1^2 t_1^8 - 1/32 t_3 t_1^5 t_2^5 w_1^2 - \\ & 1/32 t_3 t_1^{21} t_2 w_1^2 - \frac{1}{512} t_3 w_1^3 t_2^4 t_1^8 + \frac{1}{256} t_3 w_1^3 t_2^3 t_1^{12} - \frac{1}{512} t_3 t_2^2 w_1^3 t_1^{16} + 1/8 w_1 t_3^2 t_1 t_2^3 - \\ & 1/8 w_1 t_3 t_1^2 t_2^6 + \frac{1}{64} t_1^6 t_2^8 w_1^2 + \frac{1}{512} t_1^9 t_2^7 w_1^3 - \frac{1}{64} t_1^{34} t_2 w_1^2 - \frac{1}{512} t_1^{29} t_2^2 w_1^3 - 1/8 w_1 t_3^2 t_1^{13} - \\ & 1/8 w_1 t_3 t_1^{26} + t_1 t_3^3 - 175 t_1^{28} t_2^3 + 3 t_3^2 t_1^{14} - 300 t_1^{20} t_2^5 + 279 t_1^{24} t_2^4 + 66 t_1^{32} t_2^2 - 123 t_1^{12} t_2^7 + \\ & 228 t_1^{16} t_2^6 - 13 t_1^{36} t_2 - 10 t_1^4 t_2^9 + 45 t_1^8 t_2^8 - \frac{1}{52480} w_1^4 t_2^8 t_1^4 + \frac{1}{13120} w_1^4 t_1^8 t_2^7 - \frac{7}{8} w_1 t_1^{15} t_2^6 + \\ & 11/4 w_1 t_1^{11} t_2^7 - 3/8 w_1 t_1^7 t_2^8 - 12 t_3^2 t_1^{10} t_2 - 3 t_3^2 t_1^2 t_2^3 + 9 t_3^2 t_2^2 t_1^6 - \frac{19}{16} t_1^{18} t_2^5 w_1^2 + \frac{21}{16} t_1^{22} t_2^4 w_1^2 - \\ & 3/4 t_1^{26} t_2^3 w_1^2 - \frac{5}{32} t_1^{10} t_2^7 w_1^2 + \frac{19}{32} t_1^{14} t_2^6 w_1^2 + 3/16 t_1^{30} t_2^2 w_1^2 - \frac{3}{256} w_1^3 t_2^6 t_1^{13} + \frac{7}{256} w_1^3 t_2^5 t_1^{17} - \\ & \frac{15}{512} w_1^3 t_2^4 t_1^{21} + \frac{7}{512} t_2^3 w_1^3 t_1^{25} + 66 t_3 t_1^{19} t_2^2 + 51 t_3 t_1^{11} t_2^4 - 78 t_3 t_1^{15} t_2^3 - 24 t_3 t_1^{23} t_2 + 3 t_3 t_1^3 t_2^6 - \\ & 18 t_3 t_1^7 t_2^5 + t_1^{40} - 3/4 t_3^2 w_1 t_2^2 t_1^5 + \frac{7}{8} t_3^2 t_2 w_1 t_1^9 + 17/2 t_3 t_1^{14} t_2^3 w_1 - 13/2 t_3 t_1^{18} t_2^2 w_1 + 3/2 t_3 t_1^6 t_2^5 w_1 - \\ & \frac{41}{8} t_3 t_1^{10} t_2^4 w_1 + 7/4 t_3 t_1^{22} t_2 w_1 + \frac{11}{64} t_3 w_1^2 t_2^4 t_1^9 - \frac{5}{16} t_3 w_1^2 t_2^3 t_1^{13} + \frac{13}{64} t_3 t_2^2 w_1^2 t_1^{17} + \frac{27}{2} w_1 t_2^3 t_1^{27} - t_4 \end{aligned}$$

3.7. Araki generators at p=5: Right unit calculations.

$$\eta_R(w_1) = w_1 - 3120 t_1$$

$$\eta_R(w_1) \pmod{5} = w_1$$

$$\eta_R(w_2) = w_2 - 473030996096874 w_1 t_1^5 - 298023223876953120 t_2 - 6 w_1^5 t_1 + 46800 w_1^4 t_1^2 - 194688000 w_1^3 t_1^3 + 455569920000 w_1^2 t_1^4 + 295646655283200000 t_1^6$$

$$\eta_R(w_2) \pmod{5} = w_2 + w_1 t_1^5 + 4 w_1^5 t_1$$

$$\eta_R(w_2) \pmod{(5, w_1)} = w_2$$

$$\eta_R(w_3) = (\text{omitted due to excessive length})$$

$$\begin{aligned} \eta_R(w_3) \pmod{5} = & 3 w_1^{14} t_1^{17} + w_1^{25} t_1^6 + w_1^{10} t_1^{21} + 2 w_1^{18} t_1^{13} + 4 w_1^{22} t_1^9 + 4 t_2 w_1^{25} + 4 w_1^5 t_1^{26} \\ & + 4 w_2 t_1^8 w_1^{17} + t_1^{25} w_2 + 4 w_2^4 w_1^2 t_1^5 + w_2^4 w_1^6 t_1 + 4 w_1^{12} t_1^7 w_2^2 + 3 w_2^2 w_1^4 t_1^{15} + 2 w_1^{16} t_1^3 w_2^2 + \\ & 4 w_2^3 w_1^7 t_1^6 + w_2^2 w_1^8 t_1^{11} + 3 w_2^3 w_1^3 t_1^{10} + 3 w_2^3 w_1^{11} t_1^2 + 4 t_1 w_2^5 + w_1 t_2^5 + 4 w_2 t_1^{20} w_1^5 + \\ & 4 w_2 t_1^4 w_1^{21} + 4 w_2 t_1^{16} w_1^9 + 4 w_2 t_1^{12} w_1^{13} + w_3 \end{aligned}$$

$$\eta_R(w_3) \pmod{(5, w_1)} = t_1^{25} w_2 + 4 t_1 w_2^5 + w_3$$

$$\eta_R(w_3) \pmod{(5, w_1, w_2)} = w_3$$

3.8. Araki generators at p=5: Coproduct calculations.

$$\Delta(1) = 1 \otimes 1$$

$$\Delta(t_1) = (t_1 \otimes 1) + (1 \otimes t_1)$$

$$\Delta(t_2) = (t_2 \otimes 1) + (t_1 \otimes t_1^5) + (1 \otimes t_2) + \frac{1}{624} (t_1^4 w_1 \otimes t_1) + \frac{1}{312} (w_1 t_1^3 \otimes t_1^2) + \frac{1}{312} (w_1 t_1^2 \otimes t_1^3) + \frac{1}{624} (w_1 t_1 \otimes t_1^4)$$

$$\Delta(t_2) \pmod{5} = (t_2 \otimes 1) + (t_1 \otimes t_1^5) + (1 \otimes t_2) + 4(t_1^4 w_1 \otimes t_1) + 3(w_1 t_1^3 \otimes t_1^2) + 3(w_1 t_1^2 \otimes t_1^3) + 4(w_1 t_1 \otimes t_1^4)$$

$$\Delta(t_2) \pmod{(5, w_1)} = (t_2 \otimes 1) + (t_1 \otimes t_1^5) + (1 \otimes t_2) + 4(t_1^4 w_1 \otimes t_1) + 3(w_1 t_1^3 \otimes t_1^2) + 3(w_1 t_1^2 \otimes t_1^3) + 4(w_1 t_1 \otimes t_1^4)$$

3.9. Araki generators at p=5: Conjugation calculations.

$$c(t_1) = -t_1$$

$$c(t_2) = -t_2 + t_1^6$$

$$\begin{aligned} c(t_3) = & -t_3 + 6 t_2 t_1^{25} + t_1 t_2^5 - 5 t_2^4 t_1^7 + 10 t_1^{13} t_2^3 - 10 t_1^{19} t_2^2 - t_1^{31} + \frac{1}{624} w_1 t_2^4 t_1^6 - \frac{1}{312} w_1 t_1^{12} t_2^3 + \\ & \frac{1}{312} w_1 t_1^{18} t_2^2 - \frac{1}{624} w_1 t_2 t_1^{24} \end{aligned}$$

4. CALCULATIONS FOR HAZEWINKEL GENERATORS

4.1. Hazewinkel generators at p=2: Right unit calculations. These calculations agree with those in [Gia78].

$$\eta_R(v_1) = v_1 + 2t_1$$

$$\eta_R(v_1) \pmod{2} = v_1$$

$$\eta_R(v_2) = v_2 - 5v_1t_1^2 + 2t_2 - 3v_1^2t_1 - 4t_1^3$$

$$\eta_R(v_2) \pmod{2} = v_2 + v_1t_1^2 + v_1^2t_1$$

$$\eta_R(v_2) \pmod{(2, v_1)} = v_2$$

$$\eta_R(v_3) = v_3 - 2t_2v_1^2t_1^2 - 2v_2v_1t_1^3 - v_2v_1^3t_1 - v_2v_1^2t_1^2 - 4t_1v_2t_2 - 2t_2v_1^3t_1 - 4t_2v_1t_1^3 - 2v_1v_2t_2 - v_1t_2^2 + t_1^4v_2 - 70t_1^4v_1^3 - t_1v_2^2 - 16t_1^7 - 4t_1t_2^2 - 2v_1^6t_1 - 11v_1^5t_1^2 - 36v_1^4t_1^3 - 85v_1^2t_1^5 - 56v_1t_1^6 - t_2v_1^4 + 2t_3$$

$$\eta_R(v_3) \pmod{2} = v_3 + v_2v_1^2t_1^2 + v_2v_1^3t_1 + v_1t_2^2 + t_1^4v_2 + v_1^2t_1^5 + t_2v_1^4 + t_1v_2^2 + v_1^5t_1^2$$

$$\eta_R(v_3) \pmod{(2, v_1)} = v_3 + t_1^4v_2 + t_1v_2^2$$

$$\eta_R(v_3) \pmod{(2, v_1, v_2)} = v_3$$

4.2. Hazewinkel generators at p=2: Coproduct calculations. These calculations agree with those in [Gia78].

$$\Delta(1) = 1 \otimes 1$$

$$\Delta(t_1) = t_1 \otimes 1 + 1 \otimes t_1$$

$$\Delta(t_2) = t_2 \otimes 1 + t_1 \otimes t_1^2 + 1 \otimes t_2 - v_1t_1 \otimes t_1$$

$$\Delta(t_2) \pmod{(2, v_1)} = t_2 \otimes 1 + t_1 \otimes t_1^2 + 1 \otimes t_2$$

$$\Delta(t_3) = t_1 \otimes t_2^2 + t_2 \otimes t_1^4 + v_1^2t_1^2 \otimes t_1^3 - v_1t_1 \otimes t_1^2t_2 + v_1^2t_2t_1 \otimes t_1 - v_1t_2 \otimes t_2 - t_2v_1t_1 \otimes t_1^2 - 2v_1^3t_1^2 \otimes t_1^2 + v_1^2t_1 \otimes t_2t_1 - 2v_2t_1^3 \otimes t_1 - v_1^3t_1 \otimes t_1^3 - v_1^3t_1^3 \otimes t_1 - 2v_2t_1 \otimes t_1^3 + t_3 \otimes 1 + 1 \otimes t_3 - 3v_2t_1^2 \otimes t_1^2$$

$$\Delta(t_3) \pmod{2} = t_1 \otimes t_2^2 + t_2 \otimes t_1^4 + v_1^2t_1^2 \otimes t_1^3 + v_1t_1 \otimes t_1^2t_2 + v_1^2t_2t_1 \otimes t_1$$

$$+ v_1t_2 \otimes t_2 + t_2v_1t_1 \otimes t_1^2 + v_1^2t_1 \otimes t_2t_1 + v_1^3t_1 \otimes t_1^3 + v_1^3t_1^3 \otimes t_1 + t_3 \otimes 1 + 1 \otimes t_3 + v_2t_1^2 \otimes t_1^2$$

$$\Delta(t_3) \pmod{(2, v_1)} = t_1 \otimes t_2^2 + t_2 \otimes t_1^4 + t_3 \otimes 1 + 1 \otimes t_3 + v_2t_1^2 \otimes t_1^2$$

$$\Delta(t_3) \pmod{(2, v_1, v_2)} = t_1 \otimes t_2^2 + t_2 \otimes t_1^4 + t_3 \otimes 1 + 1 \otimes t_3$$

4.3. **Hazewinkel generators at p=2: Conjugation calculations.** These calculations agree with those in [Gia78].

$$c(t_1) = -t_1$$

$$c(t_2) = -t_2 - v_1 t_1^2 - t_1^3$$

$$c(t_3) = -t_3 - v_1 t_2^2 - t_1^4 v_2 - t_1^4 v_1^3 - 3 t_2 t_1^4 - 3 v_1 t_1^6 - t_1 t_2^2 - 3 t_2 v_1 t_1^3 - 2 v_1^2 t_1^5 - t_1^7 - t_2 v_1^2 t_1^2$$

$$\begin{aligned} c(t_4) = & -12 t_2^3 t_1^6 - 3 t_3 t_1^8 - 47 v_1 t_2^2 t_1^8 - t_1^{15} - v_1 t_3^2 - 4 v_2 t_2^2 v_1^2 t_1^4 - 9 v_2 t_2^2 v_1 t_1^5 - 3 v_2 t_2 v_1^3 t_1^6 - \\ & 11 v_2 t_2 v_1^2 t_1^7 - 15 v_2 t_2 v_1 t_1^8 - 2 v_2 t_2^3 v_1 t_1^2 - t_3 v_1^2 t_2^2 - 9 t_3 t_2 v_1 t_1^4 - 5 t_3 t_2 v_1^2 t_1^3 - 3 t_1 t_3 v_1 t_2^2 - \\ & t_3 t_2 v_1^3 t_1^2 - v_1 t_3 t_1^4 v_2 - 2 t_3 t_1^5 v_2 - 4 t_3 t_1^5 v_1^3 - 6 t_3 t_2 t_1^5 - 7 t_3 v_1 t_1^7 - 2 t_3 t_1^2 t_2^2 - 7 t_3 v_1^2 t_1^6 - \\ & 29 v_1 t_2^3 t_1^5 - 33 t_2 t_1^{11} v_1 - 56 t_2 t_1^{10} v_1^2 - 56 v_1^2 t_2^2 t_1^7 - 7 v_1 t_2^4 t_1^2 - 22 v_1^2 t_2^3 t_1^4 - 2 t_1 v_1^2 t_2^4 - \\ & 33 v_1^3 t_2^2 t_1^6 - 51 v_1^3 t_1^9 t_2 - 7 v_1^3 t_2^3 t_1^3 - 3 v_1^5 t_2^2 t_1^4 - 12 v_1^4 t_2^2 t_1^5 - 2 v_1^6 t_2 t_1^6 - 10 v_1^5 t_2 t_1^7 - \\ & 28 v_1^4 t_2 t_1^8 - 2 v_1^4 t_2^3 t_1^2 - 5 v_2 t_2^2 t_1^6 - 8 v_2 t_1^9 t_2 - 2 v_2 t_2^3 t_1^3 - 6 v_2 v_1^3 t_1^9 - 10 v_2 v_1^2 t_1^{10} - 9 v_2 v_1 t_1^{11} - \\ & t_1^8 v_1 v_2^2 - 2 t_1^8 v_1^4 v_2 - 3 v_2 t_1^{12} - 24 v_1^3 t_1^{12} - 4 v_1^6 t_1^9 - 11 v_1^5 t_1^{10} - 20 v_1^4 t_1^{11} - t_1^9 v_2^2 - 18 v_1^2 t_1^{13} - \\ & 7 v_1 t_1^{14} - 5 t_1^3 t_2^4 - 15 t_1^9 t_2^2 - t_1 t_3^2 - t_2^4 v_2 - t_2^4 v_1^3 - t_1^8 v_3 - t_1^8 v_1^7 - t_3 t_1^4 v_1^4 - 7 t_1^{12} t_2 - t_4 - t_2^5 \end{aligned}$$

4.4. **Hazewinkel generators at p=3: Right unit calculations.** These computations mostly agree with those in [Gia78]. In particular, the following misprints in [Gia78] should be noted: the term $3t_3$ in $\eta_R(v_2)$ should be $3t_2$; the term $1118v_1t_2^2t_1^4$ in $\eta_R(v_3)$ should be $1188v_1t_2^2t_1^4$; and the term $-26244t_1^2t_1^9$ in $\eta_R(v_3)$ should be $-26244t_2t_1^9$.

$$\eta_R(v_1) = v_1 + 3 t_1$$

$$\eta_R(v_1) \pmod{3} = v_1$$

$$\eta_R(v_2) = v_2 - 35 v_1 t_1^3 + 3 t_2 - 4 v_1^3 t_1 - 18 v_1^2 t_1^2 - 27 t_1^4$$

$$\eta_R(v_2) \pmod{3} = v_2 + v_1 t_1^3 + 2 v_1^3 t_1$$

$$\eta_R(v_2) \pmod{(3, v_1)} = v_2$$

$$\begin{aligned} \eta_R(v_3) = & -27 t_1 t_2^3 - t_1 v_2^3 + v_3 - 8 v_1 t_2^3 - 8747 t_1^9 v_2 + 143703 t_1^9 v_1^4 + 3 t_3 - 34725 v_2 v_1^2 t_1^7 - \\ & 948 v_2 v_1^6 t_1^3 - 26244 t_2 t_1^9 + 173636 v_1^3 t_1^{10} + 144423 v_1^2 t_1^{11} + 76545 v_1 t_1^{12} - t_2 v_1^9 - 9 v_2 v_1^8 t_1 - \\ & 231 v_1^{10} t_1^3 - 791 v_1^9 t_1^4 - 790 v_1^8 t_1^5 + 5424 v_1^7 t_1^6 + 30226 v_1^6 t_1^7 + 81975 v_1^5 t_1^8 - 124 v_2 v_1^7 t_1^2 - \\ & 3 v_1^{12} t_1 - 36 v_1^{11} t_1^2 - 4438 v_2 v_1^5 t_1^4 - 27037 v_2 v_1^3 t_1^6 - 13494 v_2 v_1^4 t_1^5 + 19683 t_1^{13} - \\ & 78246 t_2 v_1 t_1^8 - 104175 t_2 v_1^2 t_1^7 - 81111 t_2 v_1^3 t_1^6 - 40482 t_2 v_1^4 t_1^5 - 13314 t_2 v_1^5 t_1^4 + 132 v_1 v_2^2 t_1^4 - \\ & 9 v_1 v_2 t_2^2 - 9 t_1 v_2^2 t_2 - 27 t_1 v_2 t_2^2 + 1188 v_1 t_2^2 t_1^4 - 3 v_1 v_2^2 t_2 - 2844 t_2 v_1^6 t_1^3 + 729 t_2^2 t_1^5 + 81 v_2^2 t_1^5 - \\ & 372 t_2 v_1^7 t_1^2 - 27 t_2 v_1^8 t_1 + 24 v_2 t_2 v_1^4 t_1 + 534 v_2 v_1^2 t_1^3 t_2 + 180 v_2 t_2 v_1^3 t_1^2 + 792 v_1 v_2 t_2 t_1^4 + 4 v_2^2 v_1^4 t_1 - \\ & 26082 v_2 v_1 t_1^8 + 30 v_2^2 v_1^3 t_1^2 + 89 v_2^2 v_1^2 t_1^3 + 801 v_1^2 t_1^3 t_2^2 + 270 t_2^2 v_1^3 t_1^2 + 36 t_2^2 v_1^4 t_1 + 486 v_2 t_2 t_1^5 \end{aligned}$$

$$\eta_R(v_3) \pmod{3} = v_3 + v_1 t_2^3 + t_1^9 v_2 + 2 t_2 v_1^9 + 2 v_1^3 t_1^{10} + 2 v_2 v_1^7 t_1^2 + v_1^9 t_1^4 + 2 v_1^8 t_1^5 + v_1^6 t_1^7 + 2 v_2 v_1^3 t_1^6 + 2 v_2^2 v_1^2 t_1^3 + v_2^2 v_1^4 t_1 + 2 v_2 v_1^5 t_1^4 + 2 t_1 v_2^3$$

$$\eta_R(v_3) \pmod{(3, v_1)} = v_3 + 2 t_1 v_2^3 + t_1^9 v_2$$

$$\eta_R(v_3) \pmod{(3, v_1, v_2)} = v_3$$

4.5. Hazewinkel generators at p=3: Coproduct calculations. These calculations agree with those in [Gia78].

$$\Delta(1) = 1 \otimes 1$$

$$\Delta(t_1) = t_1 \otimes 1 + 1 \otimes t_1$$

$$\Delta(t_2) = t_2 \otimes 1 + t_1 \otimes t_1^3 + 1 \otimes t_2 - v_1 t_1^2 \otimes t_1 - v_1 t_1 \otimes t_1^2$$

$$\Delta(t_2) \bmod 3 = t_2 \otimes 1 + t_1 \otimes t_1^3 + 1 \otimes t_2 + 2v_1 t_1^2 \otimes t_1 + 2v_1 t_1 \otimes t_1^2$$

$$\Delta(t_2) \bmod (3, v_1) = t_2 \otimes 1 + t_1 \otimes t_1^3 + 1 \otimes t_2$$

$$\begin{aligned} \Delta(t_3) = & -v_1^4 t_1^8 \otimes t_1 - 3v_2 t_1 \otimes t_1^8 - 12v_2 t_1^2 \otimes t_1^7 - v_1^4 t_1 \otimes t_1^8 - 4v_1^4 t_1^2 \otimes t_1^7 - 4v_1^4 t_1^7 \otimes \\ & t_1^2 + t_2 \otimes t_1^9 + t_1 \otimes t_2^3 - v_1^3 t_1^2 \otimes t_1^4 t_2 - 9v_1^4 t_1^3 \otimes t_1^6 + v_1^2 t_1 \otimes t_2^2 t_1^2 - v_1^3 t_2 t_1^2 \otimes t_1^4 - \\ & 13v_1^4 t_1^5 \otimes t_1^4 + v_1^2 t_1^3 \otimes t_1^8 + 2v_1^2 t_2 t_1 \otimes t_1^2 t_2 - 13v_1^4 t_1^4 \otimes t_1^5 + v_1^2 t_1^4 \otimes t_1^7 - 9v_1^4 t_1^6 \otimes \\ & t_1^3 + t_3 \otimes 1 + 1 \otimes t_3 - v_1^3 t_1^5 \otimes t_1^5 + v_1^2 t_1^2 \otimes t_2^2 t_1 - 2t_1^4 v_1^3 \otimes t_1^6 - t_1^4 v_1^3 \otimes t_1^2 t_2 - v_1^3 t_1^3 \otimes \\ & t_1^7 - 2v_1^3 t_1^3 \otimes t_1^3 t_2 - 12v_2 t_1^7 \otimes t_1^2 - 3v_2 t_1^8 \otimes t_1 - 28v_2 t_1^6 \otimes t_1^3 + 2t_2 v_1^2 t_1^3 \otimes t_1^4 - t_2^2 v_1 t_1 \otimes \\ & t_1^3 - 42v_2 t_1^4 \otimes t_1^5 + 2v_1^2 t_1^3 \otimes t_1^4 t_2 - t_2 v_1 t_1^2 \otimes t_1^6 + 2t_2 v_1^2 t_1^2 \otimes t_1^5 - 2v_1^3 t_2 t_1^3 \otimes t_1^3 + \\ & v_1^2 t_2^2 t_1^2 \otimes t_1 + v_1^2 t_2^2 t_1 \otimes t_1^2 - 42v_2 t_1^5 \otimes t_1^4 - v_1 t_1 \otimes t_1^3 t_2^2 + 2v_1^2 t_1^2 \otimes t_1^5 t_2 + 2t_2 v_1^2 t_1^2 \otimes \\ & t_2 t_1 - v_1^3 t_2 t_1^4 \otimes t_1^2 - 2t_2 v_1 t_1 \otimes t_1^3 t_2 - v_1 t_2^2 \otimes t_2 - v_1 t_2 \otimes t_2^2 - v_1 t_1^2 \otimes t_1^6 t_2 - 28v_2 t_1^3 \otimes t_1^6 \end{aligned}$$

$$\begin{aligned} \Delta(t_3) \bmod 3 = & 2v_1^4 t_1^8 \otimes t_1 + 2v_1^4 t_1 \otimes t_1^8 + 2v_1^4 t_1^2 \otimes t_1^7 + 2v_1^4 t_1^7 \otimes t_1^2 + t_2 \otimes t_1^9 + t_1 \otimes t_2^3 \\ & + 2v_1^3 t_1^2 \otimes t_1^4 t_2 + v_1^2 t_1 \otimes t_2^2 t_1^2 + 2v_1^3 t_2 t_1^2 \otimes t_1^4 + 2v_1^4 t_1^5 \otimes t_1^4 + v_1^2 t_1^3 \otimes t_1^8 + 2v_1^2 t_2 t_1 \otimes \\ & t_1^2 t_2 + 2v_1^4 t_1^4 \otimes t_1^5 + v_1^2 t_1^4 \otimes t_1^7 + t_3 \otimes 1 + 1 \otimes t_3 + 2v_1^3 t_1^5 \otimes t_1^5 + v_1^2 t_1^2 \otimes t_2^2 t_1 + t_1^4 v_1^3 \otimes \\ & t_1^6 + 2t_1^4 v_1^3 \otimes t_1^2 t_2 + 2v_1^3 t_1^3 \otimes t_1^7 + v_1^3 t_1^3 \otimes t_1^3 t_2 + 2v_2 t_1^6 \otimes t_1^3 + 2t_2 v_1^2 t_1^3 \otimes t_1^4 + \\ & 2t_2^2 v_1 t_1 \otimes t_1^3 + 2v_1^2 t_1^3 \otimes t_1^4 t_2 + 2t_2 v_1 t_1^2 \otimes t_1^6 + 2t_2 v_1^2 t_1^2 \otimes t_1^5 + v_1^3 t_2 t_1^3 \otimes t_1^3 + \\ & v_1^2 t_2^2 t_1^2 \otimes t_1 + v_1^2 t_2^2 t_1 \otimes t_1^2 + 2v_1 t_1 \otimes t_1^3 t_2^2 + 2v_1^2 t_1^2 \otimes t_1^5 t_2 + 2t_2 v_1^2 t_1^2 \otimes t_2 t_1 + \\ & 2v_1^3 t_2 t_1^4 \otimes t_1^2 + t_2 v_1 t_1 \otimes t_1^3 t_2 + 2v_1 t_2^2 \otimes t_2 + 2v_1 t_2 \otimes t_2^2 + 2v_1 t_1^2 \otimes t_1^6 t_2 + 2v_2 t_1^3 \otimes t_1^6 \end{aligned}$$

$$\Delta(t_3) \bmod (3, v_1) = t_2 \otimes t_1^9 + t_1 \otimes t_2^3 + t_3 \otimes 1 + 1 \otimes t_3 + 2v_2 t_1^6 \otimes t_1^3 + 2v_2 t_1^3 \otimes t_1^6$$

$$\Delta(t_3) \bmod (3, v_1, v_2) = t_2 \otimes t_1^9 + t_1 \otimes t_2^3 + t_3 \otimes 1 + 1 \otimes t_3$$

4.6. Hazewinkel generators at p=3: Conjugation calculations. These calculations agree with those in [Gia78].

$$c(t_1) = -t_1$$

$$c(t_2) = -t_2 + t_1^4$$

$$c(t_3) = -t_3 + 4t_2 t_1^9 + t_1 t_2^3 - 3t_2^2 t_1^5 - t_1^{13} - v_1 t_2^2 t_1^4 + v_1 t_2 t_1^8$$

$$\begin{aligned} c(t_4) = & -t_4 + 4t_3 t_1^{27} + 13v_1^4 t_2^5 t_1^{16} - 9v_1^4 t_2^6 t_1^{12} - 4v_1 t_1^{35} t_2 - 13v_1^4 t_2^4 t_1^{20} + 9v_1^4 t_2^3 t_1^{24} - 4v_1^4 t_2^2 t_1^{28} + \\ & v_1^4 t_2 t_1^{32} - 3v_2 t_2^8 t_1^4 + 12v_2 t_2^7 t_1^8 + 34v_1 t_1^{31} t_2^2 - 145v_1 t_1^{19} t_2^5 + 167v_1 t_1^{23} t_2^4 + 3v_1 t_1^7 t_2^8 - \\ & 22v_1 t_1^{11} t_2^7 + 75v_1 t_1^{15} t_2^6 - 42v_2 t_2^4 t_1^{20} + 28v_2 t_2^3 t_1^{24} - 12v_2 t_2^2 t_1^{28} + 3v_2 t_2 t_1^{32} - v_1^4 t_2^8 t_1^4 + 4v_1^4 t_2^7 t_1^8 - \\ & 28v_2 t_2^6 t_1^{12} + 42v_2 t_2^5 t_1^{16} - 3t_3^2 t_1^2 t_2^3 + 9t_3^2 t_2^2 t_1^6 - 76t_2^5 t_1^{18} v_1^2 + 84t_2^4 t_1^{22} v_1^2 + t_1^{40} + 45t_1^8 t_2^8 + \\ & 3t_3^2 t_1^{14} - 13t_2 t_1^{36} + 6t_3^2 v_1 t_2^2 t_1^5 - 7t_3^2 v_1 t_2 t_1^9 + 41t_3 t_1^{10} t_2^4 v_1 - 14t_3 t_1^{22} v_1 t_2 + 11t_3 v_1^2 t_2^4 t_1^9 - \end{aligned}$$

$$\begin{aligned}
& 20 t_3 v_1^2 t_2^3 t_1^{13} - 10 t_1^{10} t_2^7 v_1^2 + 38 t_1^{14} t_2^6 v_1^2 - 12 t_3^2 t_2 t_1^{10} + t_1 t_3^3 - 123 t_2^7 t_1^{12} + 228 t_2^6 t_1^{16} + \\
& v_1 t_3 t_1^{26} + v_1 t_3^2 t_1^{13} - 2 t_3 t_1^5 t_2^5 v_1^2 - v_1 t_3^2 t_1 t_2^3 + v_1 t_3 t_1^2 t_2^6 + t_3 v_1^3 t_2^4 t_1^8 - 2 t_3 v_1^3 t_2^3 t_1^{12} + t_3 v_1^3 t_2^2 t_1^{16} + \\
& t_1^{29} v_1^3 t_2^2 - t_1^{34} v_1^2 t_2 + t_1^6 t_2^8 v_1^2 + t_3^2 v_1^2 t_2^2 t_1^4 - t_3^2 v_1^2 t_2 t_1^8 - 2 t_3 t_1^{21} v_1^2 t_2 - t_1^9 t_2^7 v_1^3 + 13 t_3 v_1^2 t_2^2 t_1^{17} - \\
& 12 t_3 t_1^6 t_2^5 v_1 - 68 t_3 t_2^3 t_1^{14} v_1 + 12 t_1^{30} v_1^2 t_2^2 + 52 t_3 t_2^2 t_1^{18} v_1 - 48 t_2^3 t_1^{26} v_1^2 + 66 t_3 t_2^2 t_1^{19} + 3 t_3 t_1^3 t_2^6 - \\
& 18 t_3 t_1^7 t_2^5 + 51 t_3 t_2^4 t_1^{11} - 78 t_3 t_2^3 t_1^{15} - 24 t_3 t_2 t_1^{23} - 7 v_1^3 t_2^3 t_1^{25} + 6 v_1^3 t_2^6 t_1^{13} - 14 v_1^3 t_2^5 t_1^{17} + \\
& 15 v_1^3 t_2^4 t_1^{21} - 175 t_2^3 t_1^{28} - 300 t_2^5 t_1^{20} + 279 t_2^4 t_1^{24} + 66 t_2^2 t_1^{32} - 10 t_1^4 t_2^9 + t_2^{10} - 108 v_1 t_2^3 t_1^{27}
\end{aligned}$$

4.7. Hazewinkel generators at p=5: Right unit calculations.

$$\eta_R(v_1) = v_1 + 5 t_1$$

$$\eta_R(v_1) \pmod{5} = v_1$$

$$\eta_R(v_2) = v_2 - 3749 v_1 t_1^5 + 5 t_2 - 6 v_1^5 t_1 - 75 v_1^4 t_1^2 - 500 v_1^3 t_1^3 - 1875 v_1^2 t_1^4 - 3125 t_1^6$$

$$\eta_R(v_2) \pmod{5} = v_2 + v_1 t_1^5 + 4 v_1^5 t_1$$

$$\eta_R(v_2) \pmod{(5, v_1)} = v_2$$

$$\eta_R(v_3) = (\text{omitted due to length})$$

$$\begin{aligned}
\eta_R(v_3) \pmod{5} &= v_1 t_2^5 + 4 v_2 t_1^{12} v_1^{13} + 4 v_2 t_1^{16} v_1^9 + 4 v_2 t_1^{20} v_1^5 + 4 v_1^5 t_1^{26} + 4 t_2 v_1^{25} + 4 v_2 t_1^8 v_1^{17} + \\
& 4 v_2 t_1^4 v_1^{21} + 4 v_2^4 v_1^2 t_1^5 + 3 v_2^3 v_1^3 t_1^{10} + 4 v_2^3 v_1^7 t_1^6 + 3 v_2^2 v_1^4 t_1^{15} + v_2^2 v_1^8 t_1^{11} + 4 v_2^2 v_1^{12} t_1^7 + v_2^4 v_1^6 t_1 + \\
& 3 v_2^3 v_1^{11} t_1^2 + 2 v_2^2 v_1^{16} t_1^3 + 4 t_1 v_2^5 + v_3 + t_1^{25} v_2 + v_1^{25} t_1^6 + 4 v_1^{22} t_1^9 + 2 v_1^{18} t_1^{13} + 3 v_1^{14} t_1^{17} + v_1^{10} t_1^{21}
\end{aligned}$$

$$\eta_R(v_3) \pmod{(5, v_1)} = t_1^{25} v_2 + 4 t_1 v_2^5 + v_3$$

$$\eta_R(v_3) \pmod{(5, v_1, v_2)} = v_3$$

4.8. Hazewinkel generators at p=5: Coproduct calculations.

$$\Delta(1) = 1 \otimes 1$$

$$\Delta(t_1) = t_1 \otimes 1 + 1 \otimes t_1$$

$$\Delta(t_2) = t_2 \otimes 1 + t_1 \otimes t_1^5 + 1 \otimes t_2 - t_1^4 v_1 \otimes t_1 - 2 v_1 t_1^3 \otimes t_1^2 - 2 v_1 t_1^2 \otimes t_1^3 - v_1 t_1 \otimes t_1^4$$

$$\Delta(t_2) \pmod{5} = t_2 \otimes 1 + t_1 \otimes t_1^5 + 1 \otimes t_2 + 4 t_1^4 v_1 \otimes t_1 + 3 v_1 t_1^3 \otimes t_1^2 + 3 v_1 t_1^2 \otimes t_1^3 + 4 v_1 t_1 \otimes t_1^4$$

$$\Delta(t_2) \pmod{(5, v_1)} = t_2 \otimes 1 + t_1 \otimes t_1^5 + 1 \otimes t_2$$

4.9. Hazewinkel generators at p=5: Conjugation calculations.

$$c(t_1) = -t_1$$

$$c(t_2) = -t_2 + t_1^6$$

$$\begin{aligned}
c(t_3) &= -t_3 + 6 t_2 t_1^{25} + t_1 t_2^5 - 5 t_2^4 t_1^7 + 10 t_2^3 t_1^{13} - 10 t_2^2 t_1^{19} - t_1^{31} - v_1 t_2^4 t_1^6 + 2 v_1 t_2^3 t_1^{12} - \\
& 2 v_1 t_2^2 t_1^{18} + v_1 t_2 t_1^{24}
\end{aligned}$$

5. MAPLE SOURCE CODE FOR CALCULATIONS

The Maple [Map] procedures and computations below are those used to generate the results in this paper. Similar procedures for calculating the right unit using Mathematica [Mat] were created by Ravenel, and are described in [NY03, p.150-155].

The author is greatly indebted to the authors of the Maple packages Bigebra [AF] and Steenrod [Mon] for making their source code available and for their helpful suggestions. Depending on what version of Maple you run, the “define” command may fail to work properly. If you install and load the Bigebra package, it contains a “define” command that works properly, no matter what version of Maple you run. Maple affords you the ability to define a neutral operator $\&o$ that is multilinear and associative just like the tensor product; however, Maple does not know how to multiply tensors. In the source code below, there are procedures PTM, $\&*$, and $\&^$ that define multiplication of “pure tensors” (tensor monomials), multiplication of tensors, and exponentiation for tensors, respectively.

5.1. Maple source code for calculating the right unit.

```
> restart: with(Bigebra):
> t[0]:=1:

> # lambdaW returns lambda_n in terms of the
  # Araki generators {w_1, ..., w_n} at the prime p.
> lambdaW:=(p,n)->
  if n=0 then 1; else
  expand(add(lambdaW(p,i)*w[n-i]^(p^i),
  i=0..(n-1)))/(p-p^(p^n));
  end if:

> # muW returns mu_n = eta_R(lambda_n) in terms of the
  # Araki generators {w_1, ..., w_n} and {t_1, ..., t_n}
  # at the prime p.
> muW:=(p,n)->if n=0 then 1; else
  expand(add(lambdaW(p,n-i)*t[i]^(p^(n-i)),i=0..n));
  end if:

> # RUW returns eta_R(w_n) in terms of the
  # Araki generators {w_1, ..., w_n} and {t_1, ..., t_n}
  # at the prime p.
> RUW:=(p,n)->expand((p-p^(p^n))*muW(p,n) -
  add(muW(p,n-i)*(RUW(p,i))^(p^(n-i)),i=1..(n-1) )):

> RUW(2,1);
> RUW(2,1) mod 2;
> RUW(2,2);
> RUW(2,2) mod 2;
> subs(w[1]=0,RUW(2,2) mod 2);
```

```

> RUW(3,1);
> RUW(3,2);

> # =====
> # lambdaV returns lambda_n in terms of the
  # Hazewinkel generators {v_1, ..., v_n} at the prime p.
> lambdaV:=(p,n)->if n=0 then 1; else
  expand(add(lambdaV(p,i)*v[n-i]^(p^i),i=0..(n-1))/p);
  end if:

> # muV returns mu_n = eta_R(lambda_n) in terms of the
  # Hazewinkel generators {v_1, ..., v_n} and
  # {t_1, ..., t_n} at the prime p.
> muV:=(p,n)->if n=0 then 1; else
  expand(add(lambdaV(p,n-i)*t[i]^(p^(n-i)),i=0..n));
  end if:

> # RUV returns eta_R(v_n) in terms of the
  # Hazewinkel generators {v_1, ..., v_n} and
  # {t_1, ..., t_n} at the prime p.
> RUV:=(p,n)->expand(p*muV(p,n) -
  add(muV(p,n-i)*(RUV(p,i))^(p^(n-i)),i=1..(n-1) )):

> RUV(2,1);
> RUV(2,2);
> RUV(3,1);
> RUV(3,2);

```

5.2. Maple source code for calculating the coproduct.

```

> # Define the tensor product.
  # Remark: to get the "define" command to work
  # correctly, you may need to download the
  # package for Maple called "Bigebra"
> define('&o',flat,multilinear);

> # Define operations for multiplication of tensors
> PTM:=proc(R,S)
  # "Pure tensor" (tensor monomial) multiplication.
  # convert R,S to lists of the form [c,[a,b]] that represent c*(a \otimes b)
  # or more generally [c,[a_1,a_2,...,a_n]]
  # represents c*(a_1 \otimes ... \otimes a_n)
  local A,B;
  if type(R,'*') then A:=[mul(op(i,R),i=1..nops(R)-1),[op([op(R)][nops(R)])]]
  else A:=[1,[op(R)]] fi;
  if type(S,'*') then B:=[mul(op(i,S),i=1..nops(S)-1),[op([op(S)][nops(S)])]]

```

```

else B:=[1,[op(S)]] fi;
RETURN(A[1]*B[1]*'&o'(seq(A[2,i]*B[2,i],i=1..nops(A[2]))));
end:

> '&*' := proc(R,S)
# Multiplies sums of "pure tensors".
local M,r,s;
if type(R,'+') then r:=nops(R) else r:=1 fi; #print(r);
if type(S,'+') then s:=nops(S) else s:=1 fi; #print(s);
if (r=1 and s=1) then
RETURN(PTM(R,S));
elif r=1 then
M:=matrix(r,s,(i,j)->PTM(R,op(j,S)));
elif s=1 then
M:=matrix(r,s,(i,j)->PTM(op(i,R),S));
else
M:=matrix(r,s,(i,j)->PTM(op(i,R),op(j,S)));
fi;
RETURN(add(add(M[i,j],i=1..r),j=1..s))
end:

> '&^' := proc(a,p)
# An exponentiation for sums of "pure tensors".
if p=0 then RETURN(1)
elif p=1 then RETURN(a)
else RETURN('&*' (a,'&^' (a,p-1))); fi;
end:

> # CPW returns the coproduct Delta(t_n) at the prime p
# in terms of the Araki generators {w_1, ..., w_n} and
# {t_1, ..., t_n}.
> CPW:=(p,n)->simplify(add(add(
&o(expand(lambdaW(p,i)*t[n-i-k]^(p^i)),t[k]^(p^(n-k))),
i=0..n-k),k=0..n) - add(&o(lambdaW(p,i),1)&*(
expand(CPW(p,n-i)&^(p^i))),i=1..n)):

> CPW(2,1);
> CPW(2,2);
> eval(subs(w[1]=0,CPW(2,2)));
> CPW(2,3);
> CPW(2,3) mod 2;
> eval(subs(w[1]=0,CPW(2,3) mod 2));
> eval(subs({w[1]=0,w[2]=0},CPW(2,3) mod 2));

> CPW(3,1);

```

```

> CPW(3,2);
> CPW(3,2) mod 3;
> eval(subs(w[1]=0,CPW(3,2) mod 3));
> CPW(3,3);
> CPW(3,3) mod 3;
> eval(subs(w[1]=0,CPW(3,3) mod 3));
> eval(subs({w[1]=0,w[2]=0},CPW(3,3) mod 3));

> # =====
> # CPV returns the coproduct Delta(t_n) at the prime p
  # in terms of the Hazewinkel generators {v_1, ..., v_n} and
  # {t_1, ..., t_n}.
> CPV:=(p,n)->simplify(add(add(
  &o(expand(lambdaV(p,i)*t[n-i-k]^(p^i)),t[k]^(p^(n-k))),
  i=0..n-k),k=0..n) - add(&o(lambdaV(p,i),1)&*
  (expand(CPV(p,n-i)&^(p^i))),i=1..n)):

> CPV(2,1);
> CPV(2,2);
> eval(subs(v[1]=0,CPV(2,2)));
> CPV(2,3);
> CPV(2,3) mod 2;
> eval(subs(v[1]=0,CPV(2,3) mod 2));
> eval(subs({v[1]=0,v[2]=0},CPV(2,3) mod 2));

> CPV(3,1);
> CPV(3,2);
> CPV(3,2) mod 3;
> eval(subs(v[1]=0,CPV(3,2) mod 3));
> CPV(3,3);
> CPV(3,3) mod 3;
> eval(subs(v[1]=0,CPV(3,3) mod 3));
> eval(subs({v[1]=0,v[2]=0},CPV(3,3) mod 3));

```

5.3. Maple source code for calculating the conjugation.

```

> # CONJW returns the conjugation c(t_n) at the prime p
  # in terms of the Araki generators {w_1, ..., w_n} and
  # {t_1, ..., t_n}.
> CONJW:=(p,n)->expand(lambdaW(p,n)-
  (add(add(lambdaW(p,i)*t[n-i-k]^(p^i)*(CONJW(p,k))^(p^(n-k)),
  i=0..n-k),k=0..n-1))):

> CONJW(2,1);
> CONJW(2,2);
> CONJW(2,3);

```

```

> CONJW(2,4);

> CONJW(3,1);
> CONJW(3,2);
> CONJW(3,3);
> CONJW(3,4);

> # =====
> # CONJV returns the conjugation c(t_n) at the prime p
  # in terms of the Hazewinkel generators {v_1, ..., v_n} and
  # {t_1, ..., t_n}.
> CONJV:=(p,n)->expand(lambdaV(p,n)-
  (add(add(lambdaV(p,i)*t[n-i-k]^(p^i)*(CONJV(p,k))^(p^(n-k)),
  i=0..n-k),k=0..n-1))):

> CONJV(2,1);
> CONJV(2,2);
> CONJV(2,3);
> CONJV(2,4);

> CONJV(3,1);
> CONJV(3,2);
> CONJV(3,3);
> CONJV(3,4);

```

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