



# MTH150/A: Discrete Mathematics

Midterm Exam 1 (Sample)

October 23, 2009

Name (please print legibly): \_\_\_\_\_

University ID Number: \_\_\_\_\_

Andrew Ledoan MWF 11:00-11:50 am
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- Do all 10 problems in the space provided and explain your work carefully. The quality of your writeup counts. Each problem counts 10 points.
- Please show all your work. You may use the backs of pages if necessary. A correct answer with no work shown will not receive full credit. Please label and circle your final answers.
- You are responsible for checking that this exam has all 11 pages. Please tell us immediately if your exam is missing a page. Missing pages will not contribute to your total score.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

**1. (10 points)** Write out the truth table for the following propositional form using 0 and 1:

$$(\neg p \wedge q) \vee (r \wedge t).$$

**2. (10 points)** Prove by contradiction: If  $x$  is a rational number and  $y$  is an irrational number, then the sum  $x + y$  is an irrational number.

**3. (10 points)** For each natural number  $k$ , let  $A_k$  be the subset of the real numbers defined by

$$A_k = \{x : -k \leq x \leq k\}.$$

Find each of the following.

(a)  $A_4$

(b)  $A_5 \cap A_6$

(c)  $A_3 \cup A_7$

(d)  $\bigcup_{k=1}^{\infty} A_k$

(e)  $\bigcap_{k=1}^{\infty} A_k$

**4. (10 points)** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = (3x - 1)/5$ , and let  $g : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $g(x) = (x^2 + 1)/2$ . Find  $f \circ g$  and  $g \circ f$ .

**5. (10 points)** Evaluate the sum

$$\sum_{j=-1}^2 ((5j)^3 - 2j).$$

**6. (10 points)** Give a big-Oh estimate for each of the following.

(a)  $3n^2 + 2n \log n$

(b)  $2 \log n + 4n + 3n \log n$

(c)  $2 + 4 + 6 + \cdots + 2n$

(d)  $\frac{n(n+1)(n+2)}{n+3}$

(e)  $3 + \sin\left(\frac{1}{n}\right)$

**7. (10 points)** Give a big-Oh estimate for  $7n^2 \log n! + (n^3 + 11) \log n$ .

**8. (10 points)** Let  $a$  and  $b$  be positive integers. Prove: If  $a \mid b$  and  $c \mid d$ , then  $ac \mid bd$ .

**9. (10 points)** Use the Division Algorithm to find the quotient and the remainder when  $-325$  is divided by  $17$ .

**10. (10 points)** Let  $n$  be an arbitrary integer. Prove by cases that  $n^2 + n$  is an even integer.