

Math 518
Topics in Random Matrix Theory
Spring 2006

Time: MW 4:50–6:05pm

Place: Hylan 1106A

Instructor: Dimitri Gioev

Random Matrix Theory (RMT) is an area of mathematics which is currently highly active. One of the main features of RMT is that it provides accurate models for correlated quantities that describe various complex systems arising in a broad variety of problems in physics, pure and applied mathematics, and in other branches of knowledge. The classical examples of such quantities are peaks in the diagram for neutron scattering off heavy nuclei, zeros of the Riemann zeta function, and distances between the parked cars in London. The common property of these quantities (which are modelled by the eigenvalues of a large random matrix) is that they try to “repel” each other.

In this course we will concentrate on the theory of the so-called Unitary Ensembles (UE’s) of random matrices. The main mathematical tool in the study of these ensembles is the theory of orthogonal polynomials (OP’s) on the real line. It turns out that the relevant asymptotic properties of a general system of OP’s can be analyzed using the solution to a certain Riemann–Hilbert problem. One of the goals in this course is to eventually prove universality for a broad class of UE’s, that is to show that (in the appropriate scaling limit) the behavior of the eigenvalues is independent of the details of the (probability distribution on the) initial matrix ensemble.

We will consider some related topics (such as Orthogonal and Symplectic Ensembles) and several further applications of RMT (in physics, combinatorics, finance) if time permits.

The course prerequisites are as follows: a basic course in complex analysis and also basic knowledge of analysis, linear algebra and some probability. The course is essentially self-contained. The textbook for the course is: “Orthogonal Polynomials and Random Matrices: A Riemann–Hilbert Approach” by Percy Deift, New York Univ., Courant Inst. of Math. Sciences–Amer. Math. Soc., 2000, 261 pp., Softcover, ISBN 0-8218-2695-6 (available for online purchase at www.ams.org (bookstore), AMS Members: \$25, list: \$31).