MTH 518
Topics in Random Matrix Theory: Theory of Unitary, Orthogonal and Symplectic Ensembles
Spring 2007

Time: TR 3:25–4:40pm
Place: Hylan 1101

Instructor: Dimitri Gioev

Random Matrix Theory (RMT) is an area of mathematics which is currently highly active. One of the main features of RMT is that it provides accurate models for correlated quantities that describe various complex systems arising in a broad variety of problems in physics, pure and applied mathematics, and in other branches of knowledge. The classical examples of such quantities are peaks in the diagram for neutron scattering off heavy nuclei, zeros of the Riemann zeta function, and distances between the parked cars in London. The common property of these quantities (which are modelled by the eigenvalues of a large random matrix) is that they try to "repel" each other.

Foundations of the modern RMT were laid down in the works of Wigner, Dyson, Gaudin, Mehta and others starting in the 1950's. The theory of the three so-called *invariant* classes of random matrix ensembles is well-developed at the present time. In this course we will study these three classes of random matrices: the Unitary, Orthogonal and Symplectic Ensembles (UE's, OE's and SE's, respectively). The names of the ensembles come from the assumption that the probability measure on the set of (finite) matrices in question is invariant under conjugation by unitary, orthogonal and symplectic matrices, respectively.

All interesting statistical quantities such as the expected density of the eigenvalues of a random matrix and the probability that a given interval does not contain any eigenvalues, can be expressed in terms of the so-called *correlation functions* of the eigenvalues. The main goal in this course is to derive formulae for these correlation functions for each of the three random matrix ensembles. We follow the unified approach suggested by Tracy and Widom in 1998.

The course is based on a textbook that Percy Deift and the instructor are currently working on. The material from that book and any other material will be distributed as necessary.

The course prerequisites include some basic knowledge of linear algebra, probability and analysis. The course is essentially self-contained.