CHAPTER 5: The Japanese Stems (π_N, $9 \le N \le 31$)

1. Introduction

In 1962 Toda [60] introduced the idea of a triple bracket. Using this innovation and the EHP sequence he calcualted the first nineteen stable stems. We will compute these stable stems in Section 2 using our spectral sequence. However, we will not use Toda's notation. Instead we will use the new notation that was introduced at the end of Chapter 1 as well as the notation from Chapter 4 for elements in the image of J. The stable stems in degrees 20 through 31 were computed by Mahowald, May, Mimura , Mori, Oda, Tangora and Toda [10], [37], [39], [44], [45], [46], [50]. The Japanese authors on this list gave very careful expositions, along the lines of Toda, of the computations including unstable calculations while the American authors used the Adams spectral sequence. We compute these stems from our spectral sequence in Section 3. We name these stems after Oda who computed the last seven of them. In Section 4, we collect the tables of tentative differentials which are calculated by computer from the leading differentials of this chapter. The results of this chapter are summarized in Appendices 1 - 4.

2. The Toda Stems (π^S_N, $9 \le N \le 19$)

We begin by recalling, from Chapter 3, the leaders of degree at least 10 in rows one through eight which are not determining elements of ImJ. (These leaders are of the form $\alpha_N M_1$, $\eta\alpha_N M_1$, $\eta^2\alpha_N M_1$, $\gamma_N M_1$, $\eta\gamma_N M_1$, $\eta^2\gamma_N M_1$. In Chapter 4 we showed that the third and sixth of these leaders always bounds from the 0 row, and that the others never bound.) Moreover, if a leader arises of degree greater than 21 we will list it only once with an asterisk in our tables of leaders. However, in the last table at the end of this section the leaders of all degrees will be listed.

Row	Degree	Leader	Row	Degree	Leader
3	15	$4\nu M_1^3\bar{M}_2$	7	15	σM_1^4
6	12	$\nu^2 M_1^3$	8	10	$A[8]M_1$

FIGURE 5.2.1: Leaders in Rows 1 to 8 of Degree at Least 10

There is no possibility for $A[8]M_1$ to bound, and thus $\eta A[8] = d^2(A[8]M_1)$ is a nonero element of order two in π_9^S. In Theorem 3.3.15(a) we showed that $\eta A[8] = \nu^3$. This is the only possible nonzero element of CokJ_9. Thus, we have proved the following theorem.

THEOREM 5.2.1 $\pi_9^S = Z_2\eta^2\sigma \oplus Z_2\alpha_1 \oplus Z_2\eta A[8]$

The computations from Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
3	15	$4\nu M_1^3\bar{M}_2$	*9	27	$\eta^2\sigma M_1^9$
7	15	σM_1^4	9	21	$\nu^3 M_1^3\bar{M}_2$
8	18	$A[8]M_1^2\bar{M}_2$			

FIGURE 5.2.2: Leaders in Rows 1 to 9 of Degree at Least 11

It follows from the table in Figure 5.2.2 that $\pi_N^S = \mathrm{ImJ}_N$ for $10 \le N \le 13$. Moreover, $d^8(\sigma M_1^4) = \sigma^2$ determines a nonzero element of order two in π_{14}^S. By Theorem 3.3.7(c), $d^{12}(4\nu M_1^3\bar{M}_2)$ has a nonzero representative $A[14]$ in π_{14}^S of order two. We have thus proved the following theorem.

THEOREM 5.2.2 (a) $\pi_{10}^S = Z_2\eta\alpha_1$

(b) $\pi_{11}^S = Z_8\beta_1$

(c) $\pi_{12}^S = \pi_{13}^S = 0$

(d) $\pi_{14}^S = Z_2\sigma^2 \oplus Z_2 A[14]$

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
7	17	$2\sigma M_1^5$	*11	31	$\beta_1 M_1^{10}$
8	18	$A[8]M_1^2\overline{M}_2$	14	20	$\sigma^2 M_1^3$
9	21	$\nu^3 M_1^3 \overline{M}_2$	14	16	$A[14]M_1$

FIGURE 5.2.3: Leaders in Rows 1 to 14 of Degree at Least 16

By Theorem 3.4.6(a), $2\sigma M_1^5$ transgresses. Thus $\eta A[14] = d^2(A[14]M_1)$ is the only nonzero element of CokJ_{15}. We have thus proved the following theorem.

THEOREM 5.2.3 $\pi_{15}^S = Z_2\eta A[14] \oplus Z_{32}\gamma_1$

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
7	17	$2\sigma M_1^5$	14	18	$A[14]M_1^2$
8	18	$A[8]M_1^2\overline{M}_2$	15	17	$\eta A[14]M_1$
9	21	$\nu^3 M_1^3 \overline{M}_2$	*15	37	$\gamma_1 M_1^8 \overline{M}_2$
14	20	$\sigma^2 M_1^3$			

FIGURE 5.2.4: Leaders in Rows 1 to 15 of Degree at Least 17

There are two leaders of degree 17 and two leaders of degree 18. Clearly $A[14]M_1^2$ transgresses. Now $A[14]\eta^2 = A[14]\langle 2,\eta,2\rangle = \langle A[14],2,\eta\rangle 2$ by Theorem 2.3.3(b). However, $\mathrm{ImJ}_{16} = Z_2(\eta\gamma_1)$, and A[16] has order two. Thus $\eta^2 A[14] = 0$ and $\eta A[14]M_1$ must be a boundary. The only possiblity is $d^{10}(A[8]M_1^2\overline{M}_2) = \eta A[14]M_1$. By Theorem 3.4.6(a), $d^{10}(2\sigma M_1^5) = A[16]$ is a nonzero element of π_{16}^S of order two. We have thus proved the following theorem.

THEOREM 5.2.4 $\pi^S_{16} = Z_2 A[16] \oplus Z_2 \eta\gamma_1$ and $\eta^2 A[14] = 0$.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
7	19	$2\sigma M_1^6$	14	18	$A[14]M_1^2$
9	21	$\nu^3 M_1^3 \overline{M}_2$	15	21	$\eta A[14]M_1^3$
14	20	$\sigma^2 M_1^3$	16	18	$A[16]M_1$

FIGURE 5.2.5: Leaders in Rows 1 to 16 of Degree at Least 18

By Theorem 3.4.7(a), the only leader of degree 19, $2\sigma M_1^6$, transgresses . Thus, both leaders of degree 18 transgress to nonzero elements, $\eta A[16]$ and $\nu A[14]$, both clearly of order two. We have thus proved the following theorem.

THEOREM 5.2.5 $\pi^S_{17} = Z_2 \eta A[16] \oplus Z_2 \nu A[14] \oplus Z_2 \alpha_2 \oplus Z_2 \eta^2 \gamma_1$

Observe that $\sigma A[8] \in \sigma\langle\nu, 2\nu, \eta\rangle = \langle\sigma, \nu, 2\nu\rangle\eta$ and $\nu\langle\sigma, \nu, 2\nu\rangle = \langle\nu, \sigma, \nu\rangle 2\nu = \sigma^2(2\nu) = 0$. Thus, $\langle\sigma, \nu, 2\nu\rangle$ can not be $A[14]$. Since $\pi^S_{14} = Z_2 A[14] \oplus Z_2 \sigma^2$ and $\eta\sigma^2 = 0$, we have $\langle\sigma, \nu, 2\nu\rangle\eta = 0$ and

$$\sigma A[8] = 0. \qquad [5.1]$$

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
7	19	$2\sigma M_1^6$	16	34	$A[16]M_1^6 \overline{M}_2$
9	21	$\nu^3 M_1^3 \overline{M}_2$	17	19	$\eta A[16]M_1$
14	20	$\sigma^2 M_1^3$	17	21	$\nu A[14]M_1^2$
15	21	$\eta A[14]M_1^3$	*17	51	$\alpha_2 M_1^{14} \overline{M}_2$

FIGURE 5.2.6: Leaders in Rows 1 to 17 of Degree at Least 19

Note that $\sigma^2 M_1^3$ clearly survives to E^6 since $\eta\sigma^2 = 0$ and $\nu\sigma = 0$. Thus, both leaders of degree 19 transgress to nonzero elements of π_{18}^S. Thus we have the following composition series for CokJ_{18}:

$$0 \longrightarrow Z_2\eta^2 A[16] \longrightarrow \pi_{18}^S \longrightarrow Z_4 d^{12}(2\sigma M_1^6) \longrightarrow 0$$

By Lemma 3.3.14, $\eta^2 A[16]$ must be divisible by two. The only possiblity is that $d^{12}(2\sigma M_1^6)$ is represented by an element $C[18]$ of order eight such that $4C[18] = \eta^2 A[16]$. Thus, we have proved the following theorem.

THEOREM 5.2.6 $\pi_{18}^S = Z_8 C[18] \oplus Z_2\eta\alpha_2$ and $\eta^2 A[16] = 4C[18]$.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
*7	31	$2\sigma M_1^{12}$	15	21	$\eta A[14]M_1^3$
9	21	$\nu^3 M_1^3 \overline{M}_2$	17	21	$\nu A[14]M_1^2$
14	20	$\sigma^2 M_1^3$	18	22	$C[18]M_1^2$

FIGURE 5.2.7: Leaders in Rows 1 to 18 of Degree at Least 20

There is one leader of degree 20 and three leaders of degree 21. Clearly $\eta A[14]M_1^3$ is a d^4-cycle and thus transgresses. Also, $\nu A[14]M_1^2$ clearly transgresses. Thus, the only possibility for the leader $\sigma^2 M_1^3$ to bound is from $\nu^3 M_1^3 \overline{M}_2$, which is in the 9 row. If that were the case then σ^2 would be $d^6(\nu^3 \overline{M}_2)$. However, σ^2 bounds from the 8 row. Thus, $\sigma^2 M_1^3$ does not bound and transgresses to a nonzero element $A[19]$. By Theorem 2.4.4(a),(b),

$$A[19] \in \langle \eta, \sigma^2, \nu \rangle = \langle \nu, \eta, \sigma^2 \rangle \qquad [5.2]$$

We have thus proved the following theorem.

THEOREM 5.2.7 $\pi_{19}^S = Z_2 A[19] \oplus Z_8\beta_2$ and $\eta C[18] = \nu A[16] = 0$.

The computations of Section 4 show that we have the following table of leaders. Those leaders of degree greater than 21 which were omitted from the previous tables are included in the table below.

Row	Degree	Leader	Row	Degree	Leader
7	31	$2\sigma M_1^{12}$	16	34	$A[16]M_1^6\overline{M}_2$
9	27	$\eta^2\sigma M_1^9$	17	21	$\nu A[14]M_1^2$
9	21	$\nu^3 M_1^3\overline{M}_2$	17	51	$\alpha_2 M_1^{14}\overline{M}_2$
11	31	$\beta_1 M_1^{10}$	18	22	$C[18]M_1^2$
14	28	$\sigma^2 M_1^4\overline{M}_2$	19	23	$A[19]M_1^2$
15	21	$\eta A[14]M_1^3$	19	55	$\beta_2 M_1^{18}$
15	37	$\gamma_1 M_1^8\overline{M}_2$			

FIGURE 5.2.8: Leaders in Rows 1 to 19 of Degree at Least 21

3. The Oda Stems (π_N^S, $20 \le N \le 31$)

We continue the computations of the preceeding section. In the tables of leaders of this section we only list leaders of degree greater than 33 once with an asterisk. The final table of leaders at the end of this section will list the leaders of all degrees.

Note that the only leader of degree 22 in the table of Figure 5.2.8 is $C[18]M_1^2$ which transgresses to $\nu C[18]$. Thus, the three leaders of degree 21 all transgress to nonzero elements. Therefore, π_{20}^S has a composition series consisting of three Z_2s. Let $C[20] = d^{12}(\nu^3 M_1^3\overline{M}_2)$. Note that there is only one leader of degree 23, 24 or 25, $A[19]M_1^2$, which transgresses to $\nu A[19]$. Therefore $0 \ne \eta^3 C[20] = 4\nu C[20]$ and $4C[20] \ne 0$. Thus, $C[20]$ has order eight. Observe that $\nu C[18]$ has order two because $2\nu C[18]$ can not be $\eta C[20]$ as $\eta^2 C[20] \ne 0$. We have thus proved the following theorem.

THEOREM 5.3.1 (a) $\pi^S_{20} = Z_8 C[20]$, $4C[20] = \nu^2 A[14]$ and $\eta A[19] = 0$.

(b) $\pi^S_{21} = Z_2 \eta C[20] \oplus Z_2 \nu C[18]$

(c) $\pi^S_{22} = Z_2 \eta^2 C[20] \oplus Z_2 \nu A[19]$

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
7	31	$2\sigma M_1^{12}$	18	24	$2C[18]\bar{M}_2$
9	27	$\eta^2 \sigma M_1^9$	*19	55	$\beta_2 M_1^{18}$
11	31	$\beta_1 M_1^{10}$	20	24	$C[20]M_1^2$
14	28	$\sigma^2 M_1^4 \bar{M}_2$	21	26	$\nu C[18]M_1^3$
*15	37	$\gamma_1 M_1^8 \bar{M}_2$	22	24	$\eta^2 C[20]M_1$
*16	34	$A[16]M_1^6 \bar{M}_2$	22	28	$\nu A[19]M_1^3$
*17	51	$\alpha_2 M_1^{14} \bar{M}_2$			

FIGURE 5.3.1: Leaders in Rows 1 to 22 of Degree at Least 24

Note that there are no leaders of degree 25. Therefore, the leaders of degree 24 must transgress to nonzero elements. We already observed that $\nu C[20]$ has order eight and $4\nu C[20] = \eta^3 C[20]$. It remains to check that $A[23] = d^6(2C[18]\bar{M}_2)$ has order two. By Theorem 2.4.4(c),

$$A[23] \in \langle \eta, \nu, 2C[18] \rangle. \qquad [5.3]$$

By Theorem 2.3.3(b), $2A[23] \in 2\langle \eta, \nu, 2C[18] \rangle = \langle 2, \eta, \nu \rangle 2C[18] = 0$ because $\langle 2, \eta, \nu \rangle \in \pi^S_5 = 0$. We have thus proved the following theorem.

THEOREM 5.3.2 $\pi^S_{23} = Z_8 \nu C[20] \oplus Z_2 A[23] \oplus Z_{16} \gamma_2$ and $4\nu C[20] = \eta^3 C[20]$.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
7	31	$2\sigma M_1^{12}$	21	27	$\nu C[18]M_1^3$
9	27	$\eta^2\sigma M_1^9$	22	28	$\nu A[19]M_1^3$
11	31	$\beta_1 M_1^{10}$	23	27	$\nu C[20]M_1^2$
14	28	$\sigma^2 M_1^4\bar{M}_2$	23	25	$A[23]M_1$
*18	38	$C[18](M_1^4\bar{M}_2^2+2M_1^7\bar{M}_2)$	*23	63	$\gamma_1 M_1^{20}$

FIGURE 5.3.2: Leaders in Rows 1 to 23 of Degree at Least 25

There are no leaders of degree 26. Therefore, $d^2(A[23]M_1) = \eta A[23]$ is nonzero. We have thus proved the following theorem.

THEOREM 5.3.3 $\pi_{24}^S = \eta A[23] \oplus \eta\gamma_2$

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
7	31	$2\sigma M_1^{12}$	22	28	$\nu A[19]M_1^3$
9	27	$\eta^2\sigma M_1^9$	23	27	$\nu C[20]M_1^2$
11	31	$\beta_1 M_1^{10}$	23	33	$A[23]M_1^2\bar{M}_2$
14	28	$\sigma^2 M_1^4\bar{M}_2$	24	26	$\eta A[23]M_1$
21	27	$\nu C[18]M_1^3$			

FIGURE 5.3.3: Leaders in Rows 1 to 24 of Degree at Least 26

There is one leader of degree 26 and three leaders of degree 27. By Lemma 3.3.14, if $\eta^2 A[23]$ is nonzero then it must be divisible by two. However, there are no other elements of CokJ_{25}. Thus $\eta^2 A[23] = 0$ and $\eta A[23]M_1$ must be a boundary. Note that if $d^4(\nu C[18]M_1^3) = \eta A[23]M_1$ then $\nu^2 C[18] = \eta A[23]$. Since $\nu^2 C[18] \in \langle\eta,\nu,\eta\rangle C[18] = \eta\langle\nu,\eta,C[18]\rangle$ by Theorem 2.3.3(b), it would follow that $A[23] \in \langle\nu,\eta,C[18]\rangle$ and $d^6(C[18]M_2) = A[23]$ by

Theorem 2.4.4(b). However, $C[18]M_2 = d^{12}(2\sigma(M_1^6M_2+M_1^3M_2^2))$. Thus, $\nu C[18]M_1^3$ transgresses and

$$\nu^2 C[18] = 0 \qquad [5.4]$$

Clearly, $\nu C[20]M_1^2$ transgresses. The only remaining possibility for $\eta A[23]M_1$ to bound is $d^{16}(\eta^2\sigma M_1^9) = \eta A[23]M_1$. Thus we have proved the following theorem.

THEORM 5.3.4 $\pi_{25}^S = Z_2\alpha_3 \oplus Z_2\eta^2\gamma_2$.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
7	31	$2\sigma M_1^{12}$	22	28	$\nu A[19]M_1^3$
9	29	$\eta^2\sigma M_1^7\overline{M}_2$	23	27	$\nu C[20]M_1^2$
11	31	$\beta_1 M_1^{10}$	23	33	$A[23]M_1^2\overline{M}_2$
14	28	$\sigma^2 M_1^4\overline{M}_2$	*24	60	$\eta A[23]M_1^{15}\overline{M}_2$
21	27	$\nu C[18]M_1^3$			

FIGURE 5.3.4: Leaders in Rows 1 to 25 of Degree at Least 27

LEMMA 5.3.5 (a) $\nu C[18] = \sigma^3$.

(b) $\sigma A[14] = 0$.

PROOF. (a) We showed in [3.9] that $C[18] \in \langle\nu,\sigma,2\sigma\rangle$. By Theorem 2.3.7(a),

$$C[18] \in \langle\sigma,\nu,\sigma\rangle \qquad [5.5]$$

Therefore, $\nu C[18] \in \nu\langle\sigma,\nu,\sigma\rangle = \langle\nu,\sigma,\nu\rangle\sigma = \sigma^3$ by Theorems 2.3.3(b) and 2.4.2.

(b) We showed in [3.6] that $A[14] \in \langle\nu, A[8], 2, \eta\rangle$. Thus,

$\sigma A[14] \in \sigma\langle\nu, A[8], 2, \eta\rangle \subset \langle\langle\sigma,\nu,A[8]\rangle, 2, \eta\rangle = \langle 0, 2, \eta\rangle$ because $\nu\langle\sigma,\nu,A[8]\rangle$ $= \langle\nu,\sigma,\nu\rangle A[8] = \sigma^2 A[8] = 0$ and $\mathrm{Cok}_{19} = Z_2 A[19]$, $\nu A[19] \neq 0$. Therefore, $\sigma A[14] \in \eta\cdot\pi_{20}^S = Z_2\eta C[20]$. Now $\eta^2\sigma A[14] = 0$ while $\eta^2(\eta C[20]) = 4\nu C[20] \neq 0$. Thus, $\sigma A[14] = 0$. ∎

It follows from the lemma that $d^8(\sigma^2 M_1^4 \bar{M}_2) = \nu C[18]\bar{M}_2 = \nu C[18]M_1^3$. Since there are no other leaders of degree 28, the other leader, $\nu C[20]M_1^2$, of degree 27 must transgress to a nonzero element. Observe that $\sigma A[19] \in \sigma\langle\nu,\eta,\sigma^2\rangle$ $= \langle\sigma,\nu,\eta\rangle\sigma^2 = 0$. We have thus proved the following theorem.

THEOREM 5.3.6 $\pi_{26}^S = Z_2\nu^2 C[20] \oplus Z_2\eta\alpha_3$ and $\sigma A[19] = 0$.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
7	31	$2\sigma M_1^{12}$	22	28	$\nu A[19]M_1^3$
9	29	$\eta^2\sigma M_1^7\bar{M}_2$	23	29	$2\nu C[20]M_1^3$
11	31	$\beta_1 M_1^{10}$	23	33	$A[23]M_1^2\bar{M}_2$
*14	34	$\sigma^2 M_1^4\bar{M}_2^2$	26	32	$\nu^2 C[20]M_1^3$
21	33	$\nu C[18]M_1^3\bar{M}_2$			

FIGURE 5.3.5: Leaders in Rows 1 to 26 of Degree at Least 28

There is one leader of degree 28 and two leaders of degree 29. Let $X = d^6(\nu A[19]M_1^3) = d^6(\nu A[19]\bar{M}_2)$. Assume that X is not zero. By Theorems 2.4.2 and 2.4.4(c), $X \in \langle\eta,\nu,A[19]\rangle = \langle\eta,\nu,\langle\eta\sigma,\sigma,\nu\rangle\rangle$. Note that $\langle\eta,\nu,\eta\sigma\rangle = 0$ because $\pi_{13}^S = 0$. Also by Theorem 2.4.2, $0 = d^{12}(\eta\sigma M_1^6) \in \langle\nu,\eta\sigma,\sigma\rangle$ because $\eta\sigma M_1^6$ $= d^2(\sigma M_1^7)$. By Theorem 2.2.7(a), $\langle\eta,\nu,\eta\sigma,\sigma\rangle$ is defined. By Theorem 2.3.6(a), $X \in \langle\eta,\nu,\langle\eta\sigma,\sigma,\nu\rangle\rangle \supset \langle\eta,\nu,\eta\sigma,\sigma\rangle\nu$. Since X is not a d^4-boundary, $X \in \text{Indet}\,\langle\eta,\nu,\langle\eta\sigma,\sigma,\nu\rangle\rangle = (\eta)$ as $\pi_5^S = 0$. Thus, X is a d^2-boundary, a contradiction. Therefore $X = 0$ and $\nu A[19]M_1^3$ must be a boundary. Since $2\nu C[20]M_1^3$ transgresses, the only other leader of degree 29, $\eta^2\sigma M_1^7\bar{M}_2$, must hit $\nu A[19]M_1^3$. Also observe that there are no leaders of degree 30. Thus, $A[8]C[20] =$ $d^6(2\nu C[20]M_1^3)$ must be nonzero. We have thus proved the following theorem.

THEOREM 5.3.7 (a) $\pi^S_{27} = Z_8\beta_3$

(b) $\pi^S_{28} = Z_2 A[8]C[20]$

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
7	31	$2\sigma M_1^{12}$	*23	35	$2\nu C[20]M_1^3\bar{M}_2$
9	33	$\eta^2\sigma M_1^5\bar{M}_3$	23	33	$A[23]M_1^2\bar{M}_2$
11	31	$\beta_1 M_1^{10}$	26	32	$\nu^2 C[20]M_1^3$
21	33	$\nu C[18]M_1^3\bar{M}_2$	28	30	$A[8]C[20]M_1$
22	32	$\nu A[19]M_1^2\bar{M}_2$			

FIGURE 5.3.6: Leaders in Rows 1 to 27 of Degree at Least 29

To continue our analysis we will need the following relations and Toda brackets in π^S_*.

LEMMA 5.3.8 (a) $\eta A[8] \in \langle 2, A[8], 2\rangle$

(b) $A[8]A[14] = \eta^2 C[20]$

(c) $\eta C[20] \in \langle \nu^2, 2, A[14]\rangle$

(d) $\eta A[14] \in \langle 2, A[14], 2\rangle$

(e) $C[20] \in \langle \nu, \eta, 2, A[14]\rangle$ and $2C[20] \in \langle \eta A[14], \eta, \nu\rangle$

(f) $\eta A[8]C[20] = 0$

PROOF. (a) By Theorem 2.4.2, $\eta A[8] \in \eta\langle \eta, \nu, 2\nu\rangle \subset \eta\langle \eta, \nu^2, 2\rangle \subset \langle \eta^2, \nu^2, 2\rangle$ $= \langle\langle 2, \eta, 2\rangle, \nu^2, 2\rangle = \langle 2, \langle \eta, 2, \nu^2\rangle, 2\rangle + \langle 2, \eta, \langle 2, \nu^2, 2\rangle\rangle$. By Theorem 2.4.2, $A[8] \in \langle \eta, 2, \nu^2\rangle$. Since $\pi^S_8 = Z_2\eta\sigma \oplus Z_2 A[8]$, $\eta\langle 2, \nu^2, 2\rangle = \langle \eta, 2, \nu^2\rangle 2 = 0$ and $\langle 2, \nu^2, 2\rangle \in (2\sigma)$. Thus, for some integer k, $\eta A[8] \in \langle 2, A[8], 2\rangle + \langle 2, \eta, 2k\sigma\rangle$ $= \langle 2, A[8], 2\rangle + k\sigma\langle 2, \eta, 2\rangle = \langle 2, A[8], 2\rangle + k\eta^2\sigma$. By Theorem 2.3.7(b), $\eta A[8] \in \langle 2, A[8], 2\rangle$.

(b), (c) $\eta A[8]A[14] = \nu^3 A[14] = 4\nu C[20] = \eta^3 C[20]$. Since multiplication by η on π^S_{22} has kernel $Z_2\nu A[19]$, $A[8]A[14] = \eta^2 C[20]$ modulo $(\nu A[19])$. By Theorem 2.4.2, $A[8] \in \langle \nu^2, 2.\eta\rangle$ so $\eta A[8] \in \eta\langle \nu^2, 2, \eta\rangle = \langle \eta, \nu^2, 2\rangle\eta$, and hence $A[8] \in \langle \eta, \nu^2, 2\rangle$. Thus, $A[8]A[14] \in A[14]\langle 2, \nu^2, \eta\rangle = \langle A[14], 2, \nu^2\rangle\eta$. Therefore, $A[8]A[14] = \eta^2 C[20]$ and $\eta C[20] \in \langle \nu^2, 2, A[14]\rangle$.

(d), (e) Note that by Theorem 2.2.7(a), $\langle \nu, \eta, 2, A[14]\rangle$ is defined because $\langle \nu, \eta, 2\rangle \in \pi^S_5 = 0$ and $\langle \eta, 2, A[14]\rangle$ contains 0. (It can not contain $A[16]$ because as we shall see in Chapter 6, $\sigma A[16] = A[23]$ and $\sigma\langle \eta, 2, A[14]\rangle = \eta\langle 2, A[14], \sigma\rangle$ which can not be $A[23]$.) Now $\eta\langle \nu, \eta, 2, A[14]\rangle \subset \langle\langle \eta, \nu, \eta\rangle, 2, A[14]\rangle = \langle \nu^2, 2, A[14]\rangle$ which contains $\eta C[20]$. This proves (e). Now $2C[20] \in 2\langle A[14], 2, \eta, \nu\rangle \subset \langle\langle 2, A[14], 2\rangle, \eta, \nu\rangle$, and by Theorem 2.4.4(b), $2C[20] \in \langle \eta A[14], \eta, \nu\rangle$. By Theorem 2.3.7(b), $\langle 2, A[14], 2\rangle$ contains $\eta A[14]$.

(f) $\eta A[8]C[20] \in C[20]\langle 2, A[8], 2\rangle \subset \langle 2C[20], A[8], 2\rangle \subset \langle\langle \eta A[14], \eta, \nu\rangle, A[8], 2\rangle \supset \eta A[14]\langle \eta, \nu, A[8], 2\rangle$. This four fold Toda bracket is defined by Theorem 2.2.7(a) because $\langle \eta, \nu, A[8]\rangle \subset \pi^S_{13} = 0$ and $\langle \nu, A[8], 2\rangle \subset \pi^S_{12} = 0$. Thus, $\eta A[8]C[20] \in \eta A[14]\cdot\pi^S_{14}$ modulo Indet $\langle\langle \eta A[14], \eta, \nu\rangle, A[8], 2\rangle = 2\pi^S_{29}$ and $2\pi^S_{29}$ will be zero whether (f) holds or not. (If (f) holds then $\pi^S_{29} = 0$ and if (f) is false then $\pi^S_{29} = Z_2\eta A[8]C[20]$.) Note that $\eta A[14]\cdot\pi^S_{14} = (\eta A[14]^2)$. Thus, $\eta A[8]C[20] \in (\eta A[14]^2)$. By Barratt's argument [25], $\eta A[14]^2 \in A[14]\langle 2, A[14], 2\rangle = \langle A[14], 2, A[14]\rangle 2 \subset 2\cdot\pi^S_{29} = 0$, as we remarked above.∎

THEOREM 5.3.9 (a) $\pi^S_{29} = 0$

(b) $d^{18}(\beta_2 M_1^{10}) = A[8]C[20]M_1$

PROOF. By Lemma 5.3.8(f), $A[8]C[20]M_1$ must bound and $\pi^S_{29} = 0$. From the table in Figure 5.3.6, we see that there are two possibilities: either $\beta_2 M_1^{10}$ or $2\sigma M_1^{12}$ hits $A[8]C[20]M_1$. We will show that $2\sigma M_1^{12}$ transgresses. D. Kahn [25] defined a cup one product $\cup_1$ such that if B is a ring spectrum, $F: DU \longrightarrow B$ and $G: DV \longrightarrow B$ then $F\cup_1 G: I \ltimes DU\wedge DV \longrightarrow B$ with the following properties:

(a) $(F\cup_1 G)|\{0\}\times DU\wedge DV = F\wedge G$;

(b) $(F\cup_1 G)|\{1\}\times DU\wedge DV = G\wedge F$;

(c) $(F\cup_1 G)|I \ltimes SU\wedge DV = (\partial F)\cup_1 G$;

(d) $(F\cup_1 G)|I \ltimes DU\wedge SV = F\cup_1 (\partial G)$;

(e) if Image $F \subset B^{(h)}$ and Image $G \subset B^{(k)}$ then Image $(F\cup_1 G) \subset B^{(h+k)}$.

It follows that there is an induced map $\cup_1 : E^1_{M,s} \otimes E^1_{N,t} \longrightarrow E^1_{M+N,s+t}$ on our spectral sequence such that

$$d^1(X\cup_1 Y) = X\cdot Y - (-1)^{(\deg X)(\deg Y)} Y\cdot X - \partial(X)\cup_1 Y - (-1)^{\deg X} X\cup_1 Y.$$

Using this cup-one product we can construct a representative F of $2\sigma M_1^{12}$. For $1 \le i \le 5$, let $G_i : (DU_i, SU_i) \longrightarrow (BP^{(8)}, S)$ be a representative of $\langle M_1^4\rangle$, i.e. $G'_i = G_i|SU_i$ represents σ. For $1 \le i \le 4$, let $H_i(2\sigma^2): DV \longrightarrow SV'$ such that $H_i(2\sigma^2)|SV = 2G_i \wedge G_{i+1}$. Since $\pi^S_{29} = 0$, $0 = \langle\sigma^2, 2, \sigma^2\rangle$. Thus, let $K_1 : DW_1 \longrightarrow SW'_1$ such that $K_1|SW_1 = [H_1(2\sigma^2) \wedge G'_3 \wedge G'_4] \cup [G'_1 \wedge G'_2 \wedge H_3(2\sigma^2)]$ and let $K_2 : DW_2 \longrightarrow SW'_2$ such that $K_2|SW_2 = [G'_2 \wedge G'_3 \wedge H_4(2\sigma^2)] \cup [H_2(2\sigma^2) \wedge G'_4 \wedge G'_5]$. Note that $(2\sigma^2)\cup_1\sigma \in 2\pi^S_{22} = 0$. Thus, let $L : DW \longrightarrow SW'$ such that $L|\partial W = (2G'_2 \wedge G'_3)\cup_1 G'_4$. To save on long and hideous notation we supress the domains and ranges of the maps in the following definition of F. Define

$$\begin{aligned} F = {} & 2G_1 \wedge G'_2 \wedge G_3 \wedge G'_4 \wedge G_5 \cup H_1(2\sigma^2) \wedge G_3 \wedge G'_4 \wedge G_5 \cup G_1 \wedge H_2(2\sigma^2) \wedge G'_4 \wedge G_5 \\ & \cup G_1 \wedge G'_2 \wedge G_3 \wedge H_4(2\sigma^2) \cup G_1 \wedge K_2 \cup H_1(2\sigma^2) \wedge [(G'_3 \wedge G'_4)\cup_1 G_5] \\ & \cup G'_1 \wedge [H_2(2\sigma^2)\cup_1 G'_4] \wedge G_5 \cup G'_1 \wedge G'_2 \wedge [H_4(2\sigma^2)\cup_1 G_3] \cup K_1 \wedge G_5 \cup G'_1 \wedge L \wedge G_5. \end{aligned}$$

Clearly all the unionands except the first one have range in $BP^{(8)}$. Therefore, F represents $2\sigma M_1^{12}$. A straightforward calculation shows that ∂F maps to $BP^{(0)}$. Thus $2\sigma M_1^{12}$ transgresses and $d^{18}(\beta_2 M_1^{10}) = A[8]C[20]M_1$. ∎

We are about to stumble over the first hidden differential! Therefore instead of updating our table of leaders incorrectly, we compute π^S_{30} first and note this hidden differential. There is only one leader of degree 31, $2\sigma M_1^{12}$ which clearly can not bound and therefore transgresses to define the unique nonzero element $A[30] = d^{24}(2\sigma M_1^{12})$ of π_{30}^S. Observe that $\beta_1 M_1^{11}$ can not survive to E^{18} because if it did then $r_{\Delta_1} \circ d^{18}(\beta_1 M_1^{11}) = d^{18}(\beta_1 M_1^{10}) = A[8]C[20]M_1$ and the latter element is not in Image r_{Δ_1}. There are two leaders, $\nu A[19]M_1^2\bar{M}_2$ and $\nu^2 C[20]M_1^3$, of degree 32 below the 28 row. However, $2^e\beta_1 M_1^{11}$, $e = 0,1$, are in Image $r_{2\Delta_1}$ and $\nu A[19]M_1^2\bar{M}_2$ is not in Image $r_{2\Delta_1}$. Therefore $d^{12}(2^e\beta_1 M_1^{11})$ can not be $\nu A[19]M_1^2\bar{M}_2$ for $e = 0,1$. Thus, $d^{16}(\beta_1 M_1^{11}) = \nu^2 C[20]M_1^3$ and $2\beta_1 M_1^{11}$ transgresses. Let $A[31] = d^{22}(2\beta_1 M_1^{11})$. Since $A[30]$ is not a d^8-boundary, $\sigma A[23] = 0$. We have proved the first part of the following theorem.

THEOREM 5.3.10 (a) $\pi_{30}^S = Z_2 A[30]$ and $\sigma A[23] = 0$.

(b) $\pi_{31}^S = Z_2\eta A[30] \oplus Z_2 A[31] \oplus Z_{64}\gamma_3$.

PROOF. (b) We observed above that $\nu A[19]M_1^2\bar{M}_2$ transgresses to a nonzero element $A[31]$ of π_{31}^S. The only other nonzero leader of degree 32 is $A[30]M_1$ which could only bound from an element below the 7 row, and there are no leaders of degree 33 there. Thus, $\eta A[30] = d^2(A[30]M_1) \neq 0$. Therefore π_{31}^S has a composition series of two Z_2s. By Theorem 2.2.7(b), $\langle\eta,\nu,\nu A[19],\nu\rangle$ is defined because $\langle\nu,\nu A[19],\nu\rangle \in \pi_{29}^S = 0$ and $\langle\eta,\nu,\nu A[19]\rangle \in \pi_{27}^S = Z_8\beta_3$. Thus, $\langle\eta,\nu,\nu A[19]\rangle$ contains 0 because $4\beta_3 = \eta^2\alpha_3 \in \mathrm{Indet}\langle\eta,\nu,\nu A[19]\rangle$ and $2\langle\eta,\nu,\nu A[19]\rangle = \langle 2,\eta,\nu\rangle\nu A[19] = 0$. By Theorem 2.4.5(b),

$$A[31] \in \langle\eta,\nu,\nu A[19],\nu\rangle. \qquad [5.6]$$

Thus, $2A[31] \in 2\langle\eta,\nu,\nu A[19],\nu\rangle \subset \langle\langle 2,\eta,\nu\rangle,\nu A[19],\nu\rangle = \langle 0,\nu A[19],\nu\rangle = (\nu)$. However, $\nu\cdot\pi_{28}^S = \nu A[8]C[20] = 0$. Hence $2A[31] = 0$. ∎

The computations of Section 4 show that we have the following leaders. Since this is the last table of leaders of this chapter, we include the leaders of all degrees.

Row	Degree	Leader	Row	Degree	Leader
9	33	$\eta^2\sigma M_1^5\overline{M}_3$	22	62	$\nu A[19]M_1^7M_2^2\langle M_3\rangle$
11	33	$2\beta_1M_1^{11}$	23	33	$A[23]M_1^2\overline{M}_2$
14	34	$\sigma^2M_1^4\overline{M}_2^2$	23	35	$4\nu C[20]M_1^3\overline{M}_2$
15	37	$\gamma_1M_1^8\overline{M}_2$	23	63	$\gamma_2M_1^{20}$
16	34	$A[16]M_1^6\overline{M}_2$	24	60	$\eta A[23]M_1^{15}\overline{M}_2$
17	51	$\alpha_2M_1^{14}\overline{M}_2$	28	36	$A[8]C[20]M_1\overline{M}_2$
18	38	$C[18](M_1^4\overline{M}_2^2+2M_1^7\overline{M}_2)$	30	34	$A[30]M_1^2$
19	55	$\beta_2M_1^{18}$	31	33	$\eta A[30]M_1$
21	33	$\nu C[18]M_1^3\overline{M}_2$	31	35	$A[31]M_1^2$

FIGURE 5.3.7: Leaders in Rows 1 to 31 of Degree at Least 33

4. Tentative Differentials

In this section we give the tentative differentials determined by the differentials on leaders of degrees less than or equal to 32 which were determined in this chapter. We omit the differentials originating on the 7 row since they were determined in Chapter 3. Recall that these differentials are tentative in the sense that they are only valid under the assumption that there are no hidden differentials interfering with the computation.

We order the differentials by row for easy reference. We use the same notation as in Section 4.4 to display the bases in the various bidegrees. In a Z_2-vector space we omit the group in front of each basis element, and monomials which are to be added are bunched together. For example, see the first basis element in degree (28,9) below.

<u>DEGREE 9:</u> α_1 and $\eta\,\sigma$

The leading differential $d^{14}(\eta^2\sigma M_1^7\bar{M}_2) = \nu A[19]M_1^3$ determines tentative differentials by assigning the following values to monomials of degree 29 of $[Z_2(\eta^2\sigma M_1) \oplus Z_2\alpha_1] \otimes B<2>$: $\eta^2\sigma M_1\bar{M}_2^3$ and $\eta^2\sigma M_1^7\bar{M}_2$ are assigned 1 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below. In this table as well as in the following one, the monomials with an even factor of M_1 have coefficient α_1 while the monomials with an odd factor of M_1 have coefficient $\eta^2\sigma$. The new leader is $\alpha_1 M_1^6\bar{M}_2$.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(18,9)	6 1 0 0	(22,9)	11 0 0 0	(24,9)	5 0 1 0
	9 1 0 0	(26,9)	6 0 1 0		7 2 0 0
(28,9)	5 3 0 0 7 0 1 0		11 1 0 0	(30,9)	5 1 1 0
	6 3 0 0		15 0 0 0	(32,9)	6 1 1 0
	13 1 0 0	(34,9)	11 2 0 0		14 1 0 0
(36,9)	3 5 0 0		5 2 1 0 15 1 0 0		9 3 0 0 15 1 0 0
	11 0 1 0	(38,9)	3 3 1 0		6 2 1 0
	9 1 1 0		13 2 0 0	(40,9)	4 3 1 0
	5 5 0 0		11 3 0 0		13 0 1 0
(42,9)	7 0 2 0		6 5 0 0		11 1 1 0
	14 0 1 0		15 2 0 0	(44,9)	5 1 2 0 15 0 1 0
	7 0 0 1 7 5 0 0 15 0 1 0		6 3 1 0		9 2 1 0 15 0 1 0
	13 3 0 0 15 0 1 0	(46,9)	5 1 0 1		6 1 2 0
	5 6 0 0		11 4 0 0		13 1 1 0
	14 3 0 0	(48,9)	3 2 0 1		6 1 0 1
	3 7 0 0		5 4 1 0		9 5 0 0

	11 2 1 0	14 1 1 0	(50,9)	3 5 1 0
	5 2 2 0	6 4 1 0		7 6 0 0
	9 3 1 0 15 1 1 0	11 0 2 0		22 1 0 0
(52,9)	1 6 1 0 5 2 0 1 13 2 1 0	3 3 2 0		5 0 3 0 13 2 1 0
	5 2 0 1 5 7 0 0 13 2 1 0	5 7 0 0 7 4 1 0		9 1 2 0 13 2 1 0
	11 0 0 1	11 5 0 0		13 2 1 0 23 1 0 0
(54,9)	3 3 0 1	5 0 1 1		6 0 3 0
	5 5 1 0	7 2 2 0		9 1 0 1
	6 7 0 0	11 3 1 0		14 2 1 0
	15 4 0 0	21 2 0 0		27 0 0 0
(56,9)	3 1 1 1	6 0 1 1		3 6 1 0
	5 3 2 0 7 0 3 0 15 2 1 0	7 2 0 1 7 7 0 0 15 2 1 0		6 5 1 0
	11 1 2 0	12 3 1 0		13 5 0 0
	21 0 1 0	25 1 0 0	(58,9)	1 7 1 0 7 5 1 0
	5 1 3 0 5 3 0 1 7 0 1 1 7 5 1 0	5 3 0 1 7 0 1 1 7 5 1 0 13 3 1 0		6 3 2 0
	11 1 0 1	11 6 0 0		15 0 2 0
	14 5 0 0	22 0 1 0		23 2 0 0
(60,9)	3 2 3 0	3 4 0 1		5 1 1 1 15 0 0 1
	6 1 3 0	6 3 0 1		5 6 1 0 15 5 0 0
	9 7 0 0 15 5 0 0	11 4 1 0		13 1 2 0 15 0 0 1 15 5 0 0
	15 0 0 1 15 5 0 0 23 0 1 0	14 3 1 0		21 3 0 0 23 0 1 0

27 1 0 0

The leading differential $d^{16}(\eta^2\sigma M_1^9) = \eta A[23]M_1$ determines tentative differentials by assigning the following values to monomials of degree 27 of $[Z_2(\eta^2\sigma M_1) \oplus Z_2\alpha_1] \otimes B<2>$: $\alpha_1 M_1^2\bar{M}_3$ is assigned 0 and all other monomials are assigned 1. The kernel of these tentative differentials is given by the table below. The new leader is $\eta^2\sigma M_1^5\bar{M}_3$.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(24,9)	5 0 1 0	(26,9)	7 2 0 0	(28,9)	5 3 0 0
					7 0 1 0
					11 1 0 0
(30,9)	5 1 1 0	(36,9)	3 5 0 0	(38,9)	3 3 1 0
	15 0 0 0		5 2 1 0		9 1 1 0
			11 0 1 0		13 2 0 0
			15 1 0 0		
(40,9)	5 5 0 0	(42,9)	7 0 2 0	(44,9)	5 1 2 0
	13 0 1 0				7 0 0 1
	4 3 1 0				7 5 0 0
(46,9)	5 1 0 1	(48,9)	3 2 0 1	(50,9)	5 2 2 0
	11 4 0 0		9 5 0 0		7 6 0 0
			6 1 0 1		11 0 2 0
(52,9)	1 6 1 0		3 3 2 0		5 0 3 0
	7 4 1 0		5 0 3 0		5 2 0 1
	13 2 1 0		5 7 0 0		7 4 1 0
	23 1 0 0		7 4 1 0		11 0 0 1
			9 1 2 0		
			11 5 0 0		
(54,9)	3 3 0 1		5 0 1 1		7 2 2 0
	5 0 1 1		15 4 0 0		21 2 0 0
	9 1 0 1				
	6 7 0 0				
	14 2 1 0				
	21 2 0 0	(56,9)	3 1 1 1		5 3 2 0
	27 0 0 0		13 5 0 0		7 0 3 0
			6 0 1 1		7 2 0 1
					7 7 0 0
					11 1 2 0

	21 1 0 1		
(58,9)		5 1 3 0	23 2 0 0
		5 3 0 1	
		7 0 1 1	
		7 5 1 0	
		11 1 0 1	
		11 6 0 0	
		15 0 2 0	
		14 5 0 0	
		22 0 1 0	
(60,9)	3 2 3 0	3 4 0 1	21 3 0 0
	3 4 0 1	5 1 1 1	23 0 1 0
	5 6 1 0	11 4 1 0	27 1 0 0
	11 4 1 0	15 5 0 0	
	13 1 2 0	23 0 1 0	
	15 0 0 1		
	21 3 0 0		
	23 0 1 0		
	6 1 3 0		

<u>DEGREE 9:</u> $\eta A[8] = \nu^3$

The leading differential $d^2(A[8]M_1) = \eta A[8]$ determines tentative differentials with cokernel $Z_2\eta A[8]M_1 \otimes B\langle 2\rangle$, and the $\eta A[8]$-leader is $\eta A[8]M_1$.

The leading differential $d^4(\nu^2 M_1^3) = \eta A[8]M_1$ determines tentative differentials with image $Z_2\eta A[8]\{M_1, M_1^3, M_1\bar{M}_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2(\eta A[8]M_1^3\bar{M}_2) \otimes B\langle 4\rangle$, and the new $\eta A[8]$-leader is $\eta A[8]M_1^3\bar{M}_2$.

The leading differential $d^{12}(\eta A[8]M_1^3\bar{M}_2) = C[20]$ determines tentative differentials which are a monomorphism on $Z_2(\eta A[8]M_1^3\bar{M}_2) \otimes B\langle 4\rangle$. Thus, there are no remaining elements.

<u>DEGREE 11:</u> β_1

The leading differential $d^{16}(\beta_1 M_1^{11}) = \nu^2 C[20]M_1^3$ determines tentative differentials by assigning the following values to monomials of degree 33 of $Z_8\beta_1 \otimes H_*BP$: $\beta_1 M_1^{11}$ is assigned 1 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below. The new β_1-leader is $\beta_1 M_1^{10}$.

DEGREE	GROUP	GENERATOR
(20,11)	Z_2	10 0 0 0
(22,11)	Z_2	2/ 8 1 0 0
(24,11)	Z_2	6 2 0 0
	Z_2	9 1 0 0
(26,11)	Z_2	1/ 6 0 1 0 2/ 10 1 0 0
	Z_2	2/ 10 1 0 0
(28,11)	Z_2	2/ 11 1 0 0 1/ 4 1 1 0 3/ 14 0 0 0
	Z_2	2/ 11 1 0 0 3/ 14 0 0 0
	Z_4	14 0 0 0
(30,11)	Z_2	2/ 12 1 0 0
	Z_4	6 3 0 0
(32,11)	Z_2	7 3 0 0 6 1 1 0
	Z_2	2/ 7 3 0 0 1/ 13 1 0 0
	Z_2	5/ 13 1 0 0 1/ 10 2 0 0
	Z_4	1/ 7 3 0 0 5/ 13 1 0 0
(34,11)	Z_2	1/ 7 1 1 0 6/ 14 1 0 0
	Z_2	2/ 8 3 0 0
	Z_2	1/ 10 0 1 0 3/ 14 1 0 0
	Z_2	2/ 14 1 0 0
(36,11)	Z_2	3/ 9 3 0 0 1/ 2 3 1 0 3/ 8 1 1 0 2/ 12 2 0 0
	Z_2	2/ 9 3 0 0 2/ 8 1 1 0
	Z_2	1/ 9 3 0 0 2/ 15 1 0 0 3/ 12 2 0 0
	Z_2	6/ 15 1 0 0 1/ 12 2 0 0
	Z_4	2/ 15 1 0 0
(38,11)	Z_2	2/ 6 2 1 0 2/ 10 3 0 0
	Z_2	1/ 9 1 1 0 2/ 10 3 0 0 6/ 12 0 1 0
	Z_2	1/ 10 3 0 0 6/ 12 0 1 0
	Z_4	6 2 1 0
(40,11)	Z_2	3/ 7 2 1 0 6/ 11 3 0 0 6/ 13 0 1 0 1/ 4 3 1 0 3/ 10 1 1 0 6/ 14 2 0 0
	Z_2	6/ 7 2 1 0 2/ 11 3 0 0 4/ 13 0 1 0 1/ 6 0 2 0 6/ 10 1 1 0 7/ 14 2 0 0
	Z_2	2/ 7 2 1 0 6/ 11 3 0 0 4/ 13 0 1 0 2/ 10 1 1 0 1/ 14 2 0 0
	Z_2	2/ 11 3 0 0 1/ 14 2 0 0
	Z_2	1/ 13 0 1 0 2/ 14 2 0 0
	Z_4	14 2 0 0
	Z_4	1/ 7 2 1 0 2/ 10 1 1 0 1/ 14 2 0 0
(42,11)	Z_2	2/ 5 3 1 0 2/ 11 1 1 0 6/ 12 3 0 0
	Z_2	1/ 8 2 1 0 5/ 12 3 0 0 3/ 14 0 1 0
	Z_2	6/ 5 3 1 0 6/ 11 1 1 0 1/ 6 0 0 1 6/ 14 0 1 0
	Z_2	2/ 12 3 0 0
	Z_4	2/ 11 1 1 0
	Z_4	5 3 1 0 11 1 1 0 12 3 0 0 14 0 1 0
(44,11)	Z_2	1/ 4 1 0 1 6/ 6 3 1 0 6/ 12 1 1 0
	Z_2	1/ 4 6 0 0 3/ 10 4 0 0
	Z_2	2/ 13 3 0 0 2/ 6 3 1 0 5/ 10 4 0 0 2/ 12 1 1 0

	Z_2 2/ 13 3 0 0 1/ 10 4 0 0	Z_2 7/ 13 3 0 0 1/ 15 0 1 0 1/ 12 1 1 0	Z_4 6 3 1 0 Z_4 2/ 13 3 0 0
(46,11)	Z_2 1/ 4 4 1 0 1/ 6 1 2 0 3/ 10 2 1 0 1/ 14 3 0 0	Z_2 2/ 6 1 2 0 6/ 10 2 1 0	Z_2 2/ 8 5 0 0 2/ 10 2 1 0 2/ 14 3 0 0
	Z_2 1/ 13 1 1 0 1/ 10 2 1 0 5/ 14 3 0 0	Z_2 2/ 14 3 0 0 Z_4 6 1 2 0	Z_8 1/ 7 3 1 0 5/ 13 1 1 0
(48,11)	Z_2 1/ 4 2 2 0 6/ 6 1 0 1 3/ 6 6 0 0 6/ 8 3 1 0 1/ 10 0 2 0	Z_2 5/ 7 1 2 0 6/ 9 5 0 0 7/ 11 2 1 0 5/ 15 3 0 0 1/ 6 1 0 1 7/ 8 3 1 0 3/ 10 0 2 0 6/ 14 1 1 0	Z_2 2/ 7 1 2 0 4/ 9 5 0 0 6/ 11 2 1 0 2/ 15 3 0 0 3/ 6 6 0 0 6/ 10 0 2 0 2/ 14 1 1 0
	Z_2 1/ 6 6 0 0 2/ 8 3 1 0	Z_2 7/ 11 2 1 0 7/ 15 3 0 0 1/ 8 3 1 0 5/ 10 0 2 0 6/ 14 1 1 0	Z_2 2/ 11 2 1 0 2/ 15 3 0 0 1/ 10 0 2 0 2/ 14 1 1 0
	Z_2 2/ 11 2 1 0 2/ 15 3 0 0 6/ 14 1 1 0	Z_2 1/ 9 5 0 0 Z_4 2/ 15 3 0 0	Z_4 1/ 7 1 2 0 6/ 15 3 0 0 6/ 14 1 1 0
(50,11)	Z_2 3/ 7 1 0 1 6/ 15 1 1 0 1/ 4 2 0 1 6/ 4 7 0 0 6/ 6 4 1 0 3/ 10 0 0 1 6/ 10 5 0 0	Z_2 1/ 7 1 0 1 2/ 15 1 1 0 2/ 4 7 0 0 2/ 6 4 1 0 Z_2 2/ 6 4 1 0	Z_2 6/ 15 1 1 0 1/ 4 7 0 0 6/ 8 1 2 0 3/ 10 5 0 0 3/ 12 2 1 0
	Z_2 2/ 15 1 1 0 1/ 10 0 0 1 3/ 10 5 0 0	Z_2 1/ 9 3 1 0 4/ 15 1 1 0 2/ 10 5 0 0	Z_2 4/ 15 1 1 0 2/ 10 5 0 0 2/ 12 2 1 0
	Z_2 2/ 8 1 2 0	Z_2 2/ 12 2 1 0	Z_4 2/ 15 1 1 0
(52,11)	Z_2 3/ 5 7 0 0 1/ 11 5 0 0 3/ 13 2 1 0 1/ 2 3 0 1 1/ 4 5 1 0 1/ 8 1 0 1 3/ 26 0 0 0	Z_2 7/ 5 7 0 0 3/ 9 1 2 0 3/ 11 5 0 0 5/ 13 2 1 0 1/ 4 0 1 1 3/ 4 5 1 0 6/ 6 2 2 0 6/ 8 1 0 1 3/ 14 4 0 0	Z_2 7/ 5 7 0 0 3/ 7 4 1 0 6/ 9 1 2 0 7/ 11 5 0 0 2/ 13 2 1 0 1/ 4 5 1 0 7/ 6 2 2 0 6/ 14 4 0 0

Z_2 2/ 7 4 1 0
4/ 9 1 2 0
2/ 11 5 0 0
1/ 6 2 2 0
2/ 8 1 0 1
3/ 14 4 0 0
5/ 26 0 0 0

Z_2 6/ 5 7 0 0
4/ 13 2 1 0
2/ 8 1 0 1
7/ 14 4 0 0
5/ 26 0 0 0

Z_2 2/ 5 7 0 0
2/ 7 4 1 0
4/ 9 1 2 0
2/ 13 2 1 0
3/ 14 4 0 0
7/ 26 0 0 0

Z_2 1/ 7 4 1 0
4/ 9 1 2 0
4/ 11 5 0 0
7/ 13 2 1 0
1/ 14 4 0 0

Z_2 1/ 9 1 2 0
3/ 26 0 0 0

Z_2 2/ 10 3 1 0

Z_2 2/ 11 5 0 0
2/ 13 2 1 0
5/ 14 4 0 0
5/ 26 0 0 0

Z_2 26 0 0 0

Z_4 2/ 13 2 1 0
1/ 14 4 0 0

Z_4 14 4 0 0

(54,11) Z_2 1/ 5 5 1 0
1/ 2 6 1 0
1/ 6 7 0 0
2/ 12 5 0 0
2/ 14 2 1 0
2/ 24 1 0 0

Z_2 3/ 5 5 1 0
1/ 9 1 0 1
1/ 4 3 2 0
2/ 6 0 3 0
2/ 6 2 0 1
3/ 6 7 0 0
1/ 10 1 2 0
3/ 12 5 0 0
1/ 20 0 1 0
1/ 24 1 0 0

Z_2 2/ 5 5 1 0
7/ 9 1 0 1
1/ 6 2 0 1
5/ 6 7 0 0
2/ 10 1 2 0
5/ 14 2 1 0
2/ 24 1 0 0

Z_2 2/ 6 0 3 0
2/ 6 7 0 0

Z_2 2/ 5 5 1 0
2/ 9 1 0 1
6/ 14 2 1 0
6/ 20 0 1 0

Z_2 1/ 9 1 0 1
2/ 11 3 1 0
2/ 6 7 0 0
6/ 10 1 2 0
6/ 12 5 0 0
6/ 24 1 0 0

Z_2 2/ 10 1 2 0
2/ 12 5 0 0
3/ 20 0 1 0
2/ 24 1 0 0

Z_2 2/ 10 1 2 0
2/ 12 5 0 0
2/ 24 1 0 0

Z_2 2/ 14 2 1 0
2/ 24 1 0 0

Z_4 1/ 6 0 3 0
1/ 6 7 0 0
1/ 14 2 1 0
2/ 24 1 0 0

Z_4 1/ 5 5 1 0
6/ 11 3 1 0
1/ 6 7 0 0
3/ 12 5 0 0
1/ 14 2 1 0
7/ 24 1 0 0

Z_4 1/ 6 7 0 0
1/ 14 2 1 0
2/ 24 1 0 0

Z_4 2/ 11 3 1 0

(56,11) Z_2 2/ 7 7 0 0
1/ 0 7 1 0
2/ 4 1 3 0
1/ 4 3 0 1
1/ 6 5 1 0
1/ 10 1 0 1
2/ 10 6 0 0
3/ 12 3 1 0
2/ 14 0 2 0
2/ 22 2 0 0

Z_2 4/ 7 7 0 0
6/ 11 1 2 0
1/ 4 1 3 0
7/ 4 3 0 1
3/ 6 0 1 1
7/ 6 5 1 0
5/ 10 1 0 1
1/ 14 0 2 0
7/ 22 2 0 0

Z_2 3/ 5 3 2 0
1/ 7 0 3 0
4/ 7 7 0 0
3/ 11 1 2 0
2/ 13 5 0 0
3/ 15 2 1 0
1/ 25 1 0 0
1/ 4 3 0 1
5/ 6 0 1 1
1/ 6 5 1 0
3/ 10 1 0 1
1/ 12 3 1 0
7/ 14 0 2 0
3/ 22 2 0 0

Z_2 5/ 5 3 2 0
2/ 7 0 3 0
1/ 7 7 0 0
7/ 11 1 2 0
6/ 13 5 0 0
1/ 6 0 1 1
2/ 6 5 1 0
6/ 10 1 0 1
6/ 10 6 0 0
6/ 12 3 1 0
7/ 14 0 2 0
6/ 22 2 0 0

Z_2 2/ 5 3 2 0
4/ 7 0 3 0
2/ 7 7 0 0
6/ 11 1 2 0
4/ 13 5 0 0
2/ 12 3 1 0
6/ 14 0 2 0

Z_2 4/ 7 0 3 0
2/ 7 7 0 0
4/ 15 2 1 0
4/ 25 1 0 0
6/ 6 5 1 0
6/ 10 1 0 1
5/ 10 6 0 0
2/ 14 0 2 0

Z_2 3/ 7 7 0 0
3/ 15 2 1 0
3/ 25 1 0 0
1/ 6 5 1 0
6/ 10 1 0 1
2/ 10 6 0 0
1/ 12 3 1 0
6/ 14 0 2 0
7/ 22 2 0 0

Z_2 6/ 7 7 0 0
2/ 10 1 0 1
7/ 10 6 0 0
6/ 14 0 2 0
3/ 22 2 0 0

Z_2 2/ 11 1 2 0
1/ 14 0 2 0

Z_2 2/ 7 7 0 0
1/ 10 6 0 0
6/ 12 3 1 0
2/ 14 0 2 0
5/ 22 2 0 0

Z_2 7/ 13 5 0 0
4/ 15 2 1 0
4/ 25 1 0 0
1/ 10 6 0 0
2/ 12 3 1 0
2/ 22 2 0 0

Z_2 7/ 13 5 0 0
2/ 15 2 1 0
5/ 25 1 0 0
2/ 12 3 1 0
1/ 22 2 0 0

Z_2 2/ 13 5 0 0
4/ 15 2 1 0
6/ 25 1 0 0
3/ 22 2 0 0

Z_4 1/ 5 3 2 0
1/ 7 0 3 0
1/ 7 7 0 0
1/ 11 1 2 0
6/ 15 2 1 0
2/ 25 1 0 0
1/ 14 0 2 0
2/ 22 2 0 0

Z_4 1/ 7 0 3 0
1/ 13 5 0 0
4/ 25 1 0 0
3/ 14 0 2 0
5/ 22 2 0 0

Z_4 1/ 13 5 0 0
2/ 25 1 0 0
3/ 22 2 0 0

Z_4 14 0 2 0

(58, 11) Z_2 1/ 5 3 0 1
3/ 7 5 1 0
1/ 11 1 0 1
1/ 13 3 1 0
1/ 4 1 1 1
1/ 4 6 1 0
1/ 8 7 0 0
2/ 14 0 0 1
2/ 14 5 0 0
3/ 22 0 1 0
1/ 26 1 0 0

Z_2 2/ 5 3 0 1
3/ 7 0 1 1
6/ 7 5 1 0
2/ 11 1 0 1
2/ 4 6 1 0
2/ 8 7 0 0
6/ 12 1 2 0
7/ 14 0 0 1
7/ 14 5 0 0
6/ 22 0 1 0
2/ 26 1 0 0

Z_2 1/ 7 0 1 1
6/ 7 5 1 0
6/ 11 1 0 1
2/ 13 3 1 0
6/ 8 7 0 0
6/ 14 5 0 0
5/ 20 3 0 0
6/ 22 0 1 0
5/ 26 1 0 0

Z_2 2/ 4 6 1 0 2/ 6 3 2 0 2/ 14 5 0 0 2/ 26 1 0 0 Z_2 2/ 6 3 2 0	Z_2 2/ 7 5 1 0 6/ 13 3 1 0 6/ 14 5 0 0 2/ 22 0 1 0 6/ 26 1 0 0	Z_2 2/ 11 1 0 1 2/ 13 3 1 0 2/ 8 7 0 0 1/ 14 0 0 1 1/ 14 5 0 0 7/ 20 3 0 0 1/ 22 0 1 0 3/ 26 1 0 0
Z_2 6/ 13 3 1 0 1/ 10 4 1 0 2/ 12 1 2 0 1/ 14 5 0 0 3/ 20 3 0 0 2/ 22 0 1 0 1/ 26 1 0 0	Z_2 2/ 8 7 0 0 2/ 12 1 2 0 Z_2 2/ 12 1 2 0 6/ 26 1 0 0	Z_2 2/ 14 5 0 0 3/ 20 3 0 0 6/ 22 0 1 0 5/ 26 1 0 0
Z_4 1/ 5 3 0 1 1/ 11 1 0 1 1/ 8 7 0 0 5/ 22 0 1 0	Z_4 2/ 13 3 1 0 1/ 14 0 0 1 7/ 14 5 0 0 3/ 22 0 1 0	Z_4 2/ 13 3 1 0 5/ 20 3 0 0 5/ 26 1 0 0
Z_4 1/ 7 5 1 0 3/ 13 3 1 0 1/ 4 6 1 0 1/ 6 3 2 0 6/ 14 0 0 1 1/ 14 5 0 0 2/ 20 3 0 0 5/ 22 0 1 0 2/ 26 1 0 0	Z_4 6 3 2 0 26 1 0 0	

The leading differential $d^{18}(\beta_1 M_1^{10}) = A[8]C[20]M_1$ determines tentative differentials by assigning the following values to monomials of degree 31 of $Z_8\beta_1 \otimes H_*BP$: $\beta_1 M_1^{10}$ is assigned 1 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below. The new β_1-leader ia $2\beta_1 M_1^8 \bar{M}_2$.

DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR
(22,11)	Z_2	2/ 8 1 0 0	(24,11)	Z_2	6 2 0 0	(26,11)	Z_2	2/ 10 1 0 0
	Z_8	1/ 6 0 1 0 2/ 10 1 0 0	(28,11)	Z_2	2/ 11 1 0 0 1/ 4 1 1 0 3/ 14 0 0 0		Z_4	2/ 11 1 0 0 2/ 14 0 0 0
(30,11)	Z_2	2/ 12 1 0 0		Z_4	6 3 0 0	(32,11)	Z_2	3/ 7 3 0 0 4/ 13 1 0 0 1/ 6 1 1 0 7/ 10 2 0 0

Z_4 1/ 7 3 0 0
7/ 10 2 0 0

(34,11) Z_2 1/ 7 1 1 0
7/ 10 0 1 0
3/ 14 1 0 0

Z_2 2/ 8 3 0 0

Z_2 2/ 14 1 0 0

(36,11) Z_2 2/ 9 3 0 0
6/ 15 1 0 0
1/ 2 3 1 0
3/ 8 1 1 0
7/ 12 2 0 0

Z_2 2/ 9 3 0 0
2/ 8 1 1 0

Z_4 2/ 15 1 0 0

(38,11) Z_2 2/ 6 2 1 0
2/ 10 3 0 0

Z_2 2/ 10 3 0 0
6/ 12 0 1 0

Z_4 6 2 1 0

(40,11) Z_2 3/ 7 2 1 0
6/ 11 3 0 0
6/ 13 0 1 0
1/ 4 3 1 0
3/ 10 1 1 0
6/ 14 2 0 0

Z_2 6/ 7 2 1 0
4/ 13 0 1 0
1/ 6 0 2 0
6/ 10 1 1 0
6/ 14 2 0 0

Z_2 2/ 7 2 1 0
4/ 11 3 0 0
4/ 13 0 1 0
2/ 10 1 1 0

Z_4 1/ 7 2 1 0
2/ 10 1 1 0
1/ 14 2 0 0

Z_4 2/ 11 3 0 0

(42,11) Z_2 2/ 5 3 1 0
2/ 11 1 1 0
6/ 12 3 0 0

Z_2 6/ 5 3 1 0
6/ 11 1 1 0
1/ 6 0 0 1
6/ 14 0 1 0

Z_2 2/ 12 3 0 0

Z_4 2/ 11 1 1 0

Z_4 5 3 1 0
11 1 1 0
12 3 0 0
14 0 1 0

(44,11) Z_2 6/ 13 3 0 0
1/ 4 1 0 1
6/ 6 3 1 0
7/ 10 4 0 0
6/ 12 1 1 0

Z_2 2/ 6 3 1 0
2/ 12 1 1 0

Z_4 1/ 4 6 0 0
7/ 6 3 1 0
3/ 10 4 0 0

Z_4 2/ 13 3 0 0

(46,11) Z_2 2/ 8 5 0 0
2/ 10 2 1 0
2/ 14 3 0 0

Z_2 2/ 6 1 2 0
6/ 10 2 1 0

Z_2 2/ 14 3 0 0

Z_4 6 1 2 0

Z_8 1/ 7 3 1 0
4/ 13 1 1 0
7/ 10 2 1 0
1/ 14 3 0 0

(48,11) Z_2 1/ 4 2 2 0
6/ 6 1 0 1
3/ 6 6 0 0
6/ 8 3 1 0

Z_2 5/ 7 1 2 0
6/ 9 5 0 0
4/ 11 2 1 0
2/ 15 3 0 0
1/ 6 1 0 1
7/ 10 0 2 0
2/ 14 1 1 0

Z_2 2/ 7 1 2 0
4/ 9 5 0 0
6/ 11 2 1 0
2/ 15 3 0 0
2/ 6 6 0 0
2/ 8 3 1 0
6/ 10 0 2 0
6/ 14 1 1 0

Z_2 2/ 11 2 1 0
2/ 15 3 0 0
6/ 14 1 1 0

Z_4 1/ 7 1 2 0
6/ 15 3 0 0
7/ 10 0 2 0
2/ 14 1 1 0

Z_4 2/ 15 3 0 0

(50,11) Z_2 3/ 7 1 0 1
4/ 15 1 1 0
1/ 4 2 0 1
6/ 4 7 0 0
6/ 6 4 1 0
2/ 10 0 0 1
3/ 10 5 0 0

Z_2 1/ 7 1 0 1
2/ 4 7 0 0
2/ 6 4 1 0
7/ 10 0 0 1
5/ 10 5 0 0

Z_2 4/ 15 1 1 0
2/ 10 5 0 0
2/ 12 2 1 0

Z_2 2/ 6 4 1 0

Z_2 2/ 8 1 2 0

Z_2 2/ 12 2 1 0

Z_4 2/ 15 1 1 0

(52,11) Z_2 3/ 5 7 0 0
6/ 7 4 1 0
4/ 9 1 2 0
7/ 11 5 0 0
3/ 13 2 1 0
1/ 2 3 0 1
1/ 4 5 1 0
7/ 6 2 2 0
7/ 8 1 0 1
5/ 14 4 0 0
6/ 26 0 0 0

Z_2 7/ 5 7 0 0
2/ 9 1 2 0
3/ 11 5 0 0
5/ 13 2 1 0
1/ 4 0 1 1
3/ 4 5 1 0
6/ 6 2 2 0
6/ 8 1 0 1
7/ 14 4 0 0
5/ 26 0 0 0

Z_2 7/ 5 7 0 0
6/ 9 1 2 0
1/ 11 5 0 0
3/ 13 2 1 0
1/ 4 5 1 0
6/ 6 2 2 0
2/ 8 1 0 1
2/ 14 4 0 0
3/ 26 0 0 0

Z_2 2/ 5 7 0 0
2/ 7 4 1 0
4/ 9 1 2 0
2/ 11 5 0 0
4/ 13 2 1 0
1/ 6 2 2 0
1/ 26 0 0 0

Z_2 6/ 5 7 0 0
6/ 11 5 0 0
2/ 13 2 1 0
2/ 8 1 0 1
2/ 14 4 0 0

Z_2 2/ 5 7 0 0
2/ 7 4 1 0
4/ 9 1 2 0
6/ 11 5 0 0
6/ 14 4 0 0
2/ 26 0 0 0

Z_2 2/ 10 3 1 0

Z_4 2/ 11 5 0 0

Z_4 2/ 13 2 1 0

(54,11) Z_2 2/ 5 5 1 0
2/ 9 1 0 1
1/ 2 6 1 0
1/ 4 3 2 0
2/ 6 0 3 0
1/ 6 2 0 1
7/ 6 7 0 0
7/ 10 1 2 0
5/ 12 5 0 0
5/ 14 2 1 0
1/ 20 0 1 0
1/ 24 1 0 0

Z_2 2/ 5 5 1 0
2/ 9 1 0 1
6/ 14 2 1 0
6/ 20 0 1 0

Z_2 2/ 10 1 2 0
2/ 12 5 0 0
2/ 24 1 0 0

Z_2 2/ 10 1 2 0
2/ 12 5 0 0
3/ 20 0 1 0
2/ 24 1 0 0

Z_2 2/ 6 0 3 0
2/ 6 7 0 0

Z_2 2/ 14 2 1 0
2/ 24 1 0 0

Z_4 6/ 11 3 1 0
1/ 4 3 2 0
1/ 6 2 0 1
1/ 6 7 0 0
1/ 10 1 2 0
2/ 24 1 0 0

Z_4 2/ 11 3 1 0

Z_4 6/ 9 1 0 1
2/ 11 3 1 0
1/ 6 0 3 0
1/ 6 2 0 1
6/ 6 7 0 0
6/ 10 1 2 0
6/ 12 5 0 0
3/ 20 0 1 0
6/ 24 1 0 0

Z_4 6/ 9 1 0 1
2/ 11 3 1 0
1/ 6 2 0 1
6/ 6 7 0 0
6/ 10 1 2 0
6/ 12 5 0 0
3/ 20 0 1 0
6/ 24 1 0 0

(56,11)

Z_2	1/	5	3	2	0
	2/	7	0	3	0
	6/	7	7	0	0
	3/	11	1	2	0
	6/	13	5	0	0
	1/	15	2	1	0
	2/	25	1	0	0
	1/	0	7	1	0
	2/	4	1	3	0
	1/	4	3	0	1
	5/	6	0	1	1
	5/	10	1	0	1
	7/	10	6	0	0
	6/	12	3	1	0
	7/	14	0	2	0

Z_2	3/	5	3	2	0
	6/	7	0	3	0
	3/	7	7	0	0
	7/	11	1	2	0
	2/	13	5	0	0
	1/	4	1	3	0
	7/	4	3	0	1
	2/	6	0	1	1
	5/	6	5	1	0
	7/	10	1	0	1
	6/	10	6	0	0
	2/	12	3	1	0
	2/	14	0	2	0
	1/	22	2	0	0

Z_2	5/	5	3	2	0
	5/	7	0	3	0
	7/	7	7	0	0
	7/	11	1	2	0
	6/	13	5	0	0
	1/	4	3	0	1
	7/	6	0	1	1
	1/	10	1	0	1
	6/	14	0	2	0
	6/	22	2	0	0

Z_2	2/	13	5	0	0
	4/	15	2	1	0
	6/	25	1	0	0
	3/	22	2	0	0

Z_2	5/	5	3	2	0
	2/	7	0	3	0
	7/	7	7	0	0
	7/	11	1	2	0
	6/	13	5	0	0
	1/	6	0	1	1
	2/	6	5	1	0
	6/	10	1	0	1
	5/	10	6	0	0
	5/	14	0	2	0
	1/	22	2	0	0

Z_2	2/	5	3	2	0
	4/	7	0	3	0
	2/	7	7	0	0
	6/	11	1	2	0
	4/	13	5	0	0
	2/	12	3	1	0
	6/	14	0	2	0

Z_2	4/	7	0	3	0
	4/	15	2	1	0
	4/	25	1	0	0
	6/	6	5	1	0
	6/	10	1	0	1
	6/	12	3	1	0
	7/	22	2	0	0

Z_2	4/	7	7	0	0
	2/	10	1	0	1
	6/	10	6	0	0
	2/	12	3	1	0
	6/	22	2	0	0

Z_4	2/	11	1	2	0

Z_4	1/	5	3	2	0
	1/	7	0	3	0
	7/	7	7	0	0
	7/	11	1	2	0
	6/	15	2	1	0
	2/	25	1	0	0
	7/	10	6	0	0
	2/	12	3	1	0
	6/	14	0	2	0
	5/	22	2	0	0

Z_4	1/	7	0	3	0
	4/	7	7	0	0
	6/	15	2	1	0
	6/	25	1	0	0
	6/	6	5	1	0
	2/	10	6	0	0
	6/	12	3	1	0
	5/	14	0	2	0
	5/	22	2	0	0

Z_4	4/	7	7	0	0
	2/	15	2	1	0
	4/	25	1	0	0
	2/	6	5	1	0
	6/	10	6	0	0
	2/	12	3	1	0
	6/	14	0	2	0
	1/	22	2	0	0

(58,11)

Z_2	4/	7	0	1	1
	2/	7	5	1	0
	4/	11	1	0	1
	2/	13	3	1	0
	2/	4	1	1	1
	6/	12	1	2	0
	6/	14	0	0	1
	2/	10	3	0	0
	3/	22	0	1	0
	6/	26	1	0	0

Z_2	2/	5	3	0	1
	7/	7	0	1	1
	6/	7	5	1	0
	2/	11	1	0	1
	2/	13	3	1	0
	2/	4	6	1	0
	2/	8	7	0	0
	3/	10	4	1	0
	7/	14	0	0	1
	6/	14	5	0	0
	1/	20	3	0	0
	5/	26	1	0	0

Z_2 2/ 7 5 1 0
6/ 13 3 1 0
6/ 14 5 0 0
2/ 22 0 1 0
6/ 26 1 0 0

Z_2 2/ 8 7 0 0
2/ 12 1 2 0

Z_4 1/ 5 3 0 1
1/ 11 1 0 1
2/ 13 3 1 0
1/ 4 1 1 1
1/ 6 3 2 0
1/ 8 7 0 0
7/ 10 4 1 0
1/ 20 3 0 0
6/ 22 0 1 0

Z_2 2/ 4 6 1 0
2/ 6 3 2 0
2/ 14 5 0 0
2/ 26 1 0 0

Z_2 2/ 12 1 2 0
6/ 26 1 0 0

Z_4 1/ 5 3 0 1
7/ 11 1 0 1
6/ 13 3 1 0
1/ 8 7 0 0
7/ 14 0 0 1
1/ 14 5 0 0
2/ 22 0 1 0

Z_4 2/ 11 1 0 1

Z_2 2/ 14 5 0 0
3/ 20 3 0 0
6/ 22 0 1 0
5/ 26 1 0 0

Z_2 2/ 6 3 2 0

Z_4 2/ 13 3 1 0
7/ 20 3 0 0
3/ 26 1 0 0

Z_4 6 3 2 0
26 1 0 0

DEGREE 14: A[14]

The leading differential $d^{12}(4\nu M_1^3\overline{M}_2) = A[14]$ determines tentative differentials with image $Z_2A[14] \otimes B\langle 4\rangle$. Since $\eta A[14] \neq 0$, the remaining elements are $Z_2A[14]\{M_1^2, \overline{M}_2, M_1^2\overline{M}_2\} \otimes B\langle 4\rangle$, and the A[14]-leader is $A[14]M_1^2$.

The leading differential $d^4(A[14]M_1^2) = \nu A[14]$ determines tentative differentials which are a monomorphism on $Z_2A[14]\{M_1^2, \overline{M}_2, M_1^2\overline{M}_2\} \otimes B\langle 4\rangle$. Thus, there are no remaining elements.

DEGREE 14: σ^2

The leading differential $d^8(\sigma^2\overline{M}_2\langle M_1^4\rangle) = \sigma^3\overline{M}_2$ determines tentative differentials by assigning the following values to monomials of degree 28 of $Z_2\sigma^2 \otimes H_*BP$: $\sigma^2M_1M_2^2$, σ^2M_3 and $\sigma^2M_1^4M_2$ are assigned 1 and $\sigma^2M_1^7$ is assigned 0. The elements of $E^{10}_{*,14}$ in degrees less than 69 with a representative in $Z_2\sigma^2 \otimes H_*BP$ are given by the table below. The new σ^2-leader is $\sigma^2M_1^4\overline{M}_2^2$.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(20,14)	4 2 0 0	(24,14)	2 1 1 0	(36,14)	12 2 0 0
(40,14)	14 2 0 0	(44,14)	4 6 0 0		
(48,14)	2 5 1 0		4 2 2 0	(50,14)	4 2 0 1 4 7 0 0 12 2 1 0
(52,14)	2 1 3 0 2 3 0 1 4 0 1 1 4 5 1 0		20 2 0 0	(54,14)	6 2 0 1 6 7 0 0 14 2 1 0

DEGREE 15: $\eta A[14]$

The leading differential $d^2(A[14]M_1) = \eta A[14]$ determines tentative differentials with cokernel $Z_2\eta A[14]M_1 \otimes B\langle 2\rangle$. The $\eta A[14]$-leader is $\eta A[14]M_1$.

The leading differential $d^8(A[8]M_1^2\bar{M}_2) = \eta A[14]M_1$ determines tentative differentials with image $Z_2\eta A[14]M_1 \otimes B\langle 4\rangle$. The remaining elements are $Z_2\eta A[14]\{M_1^3, M_1\bar{M}_2, M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$, and the new $\eta A[14]$-leader is $\eta A[14]M_1^3$.

The leading differential $d^6(\eta A[14]M_1^3) = 2C[20]$ determines tentative differentials which are a monomorphism on $Z_2\eta A[14]\{M_1^3, M_1\bar{M}_2, M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$. Thus, there are no remaining elements.

DEGREE 17: $\eta A[16]$

Since $\eta^2 A[16] \neq 0$, the only element of $E^4_{*,17}$ with a representative in $Z_2\eta A[16] \otimes H_*BP$ is zero.

DEGREE 17: $\nu A[14]$

Since $\nu^2 A[14] \neq 0$, the only element of $E^6_{*,17}$ with a representative in $Z_2\nu A[14] \otimes H_*BP$ is zero.

DEGREE 18: C[18]

The leading differential $d^2(\eta A[16]M_1) = 4C[18]$ determines tentative differentials with cokernel $[Z_8(C[18]M_1) \otimes B\langle 2\rangle] \oplus [Z_4C[18] \otimes B\langle 2\rangle]$.

The leading differential $d^4(C[18]M_1^2) = \nu C[18]$ determines tentative differentials with kernel $[Z_8C[18]\{M_1, M_2\} \otimes B\langle 4\rangle]$

$$\oplus\ [Z_4(2C[18])\{M_1\overline{M}_2, M_1^3\overline{M}_2\} \otimes B\langle 4\rangle] \oplus [Z_2(2C[18]) \otimes B\langle 2\rangle].$$

In Section 3.4 we computed the image of $d^{12}: E^{12}_{*,7} \longrightarrow E^{12}_{*,18}$. However, that computation was done in three stages so that the global image of these d^{12}-differentials is hard to unravel. Therefore, we give the computer calculation of the cokernel of these differentials in the table below. The new C[18]-leader is $2C[18]\overline{M}_2$.

DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR
(6,18)	Z_2	2/ 0 1 0 0	(10,18)	Z_2	2/ 2 1 0 0	(12,18)	Z_2	2/ 3 1 0 0 2/ 6 0 0 0
(14,18)	Z_2	2/ 4 1 0 0	(18,18)	Z_2	2/ 0 3 0 0		Z_2	2/ 6 1 0 0
(20,18)	Z_2	2/ 0 1 1 0		Z_2	2/ 7 1 0 0 1/ 4 2 0 0		Z_2	2/ 7 1 0 0 6/ 10 0 0 0
(22,18)	Z_2	2/ 2 3 0 0		Z_2	2/ 4 0 1 0		Z_4	4 0 1 0 8 1 0 0
(24,18)	Z_2	2/ 2 1 1 0		Z_2	2/ 3 3 0 0 6/ 6 2 0 0		Z_2	5 0 1 0 9 1 0 0
(26,18)	Z_2	4/ 3 1 1 0 2/ 0 2 1 0 2/ 4 3 0 0 2/ 10 1 0 0		Z_2 Z_2	2/ 3 1 1 0 6/ 6 0 1 0 2/ 4 3 0 0		Z_4	1/ 0 2 1 0 1/ 4 3 0 0 2/ 6 0 1 0 2/ 10 1 0 0
(28,18)	Z_2	4/ 5 3 0 0 4/ 7 0 1 0 4/ 11 1 0 0 2/ 4 1 1 0		Z_2	2/ 7 0 1 0 4/ 11 1 0 0 2/ 14 0 0 0		Z_8	2/ 5 3 0 0 1/ 7 0 1 0 5/ 11 1 0 0 1/ 4 1 1 0
(30,18)	Z_2	2/ 0 5 0 0		Z_2	2/ 5 1 1 0 6/ 6 3 0 0		Z_2	2/ 6 3 0 0
	Z_2	2/ 12 1 0 0	(32,18)	Z_2	3 2 1 0 7 3 0 0 0 3 1 0		Z_2	2/ 3 2 1 0 6/ 10 2 0 0

Z_2 2/ 6 1 1 0

Z_2	2/ 7 1 1 0
	6/ 10 0 1 0
	6/ 14 1 0 0

(36,18) Z_2	2/ 3 5 0 0
	4/ 5 2 1 0
	2/ 15 1 0 0
	2/ 6 4 0 0
	6/ 18 0 0 0

Z_2 2/ 8 1 1 0

Z_8	1/ 5 2 1 0
	7/ 9 3 0 0
	2/ 15 1 0 0
	2/ 18 0 0 0

Z_2 2/ 10 3 0 0

Z_4	12 0 1 0
	16 1 0 0

Z_2	2/ 7 2 1 0
	4/ 11 3 0 0
	6/ 13 0 1 0
	2/ 17 1 0 0
	6/ 14 2 0 0

Z_8	1/ 7 2 1 0
	1/ 11 3 0 0
	1/ 13 0 1 0
	3/ 17 1 0 0
	1/ 4 3 1 0

Z_2	2/ 3 1 0 1
	6/ 5 3 1 0
	2/ 11 1 1 0
	6/ 0 7 0 0
	2/ 4 1 2 0
	2/ 6 0 0 1
	6/ 8 2 1 0
	6/ 12 3 0 0
	6/ 14 0 1 0

(44,18) Z_2	6/ 7 5 0 0
	4/ 13 3 0 0
	6/ 15 0 1 0
	2/ 0 5 1 0
	6/ 4 6 0 0
	6/ 10 4 0 0
	2/ 12 1 1 0
	2/ 22 0 0 0

Z_2	2/ 7 3 0 0
	6/ 10 2 0 0

Z_2 2/ 2 5 0 0

Z_2 2/ 4 2 1 0

Z_2	2/ 5 2 1 0
	6/ 9 3 0 0
	2/ 15 1 0 0
	2/ 18 0 0 0

Z_2 2/ 2 3 1 0

(38,18) Z_2	2/ 3 3 1 0
	2/ 6 2 1 0
	2/ 12 0 1 0
	2/ 16 1 0 0

Z_2 2/ 2 1 2 0

(40,18) Z_2 2/ 2 1 0 1

Z_2	4/ 11 3 0 0
	5/ 13 0 1 0
	7/ 17 1 0 0
	2/ 14 2 0 0

(42,18) Z_2	2/ 5 3 1 0
	2/ 11 1 1 0
	6/ 12 3 0 0
	2/ 14 0 1 0
	2/ 18 1 0 0

Z_2	2/ 5 3 1 0
	2/ 11 1 1 0
	2/ 4 1 2 0
	6/ 6 5 0 0
	6/ 12 3 0 0
	2/ 14 0 1 0
	2/ 18 1 0 0

Z_2 2/ 12 3 0 0

Z_2	4/ 7 5 0 0
	4/ 13 3 0 0
	6/ 19 1 0 0
	2/ 4 1 0 1
	6/ 6 3 1 0
	6/ 12 1 1 0
	6/ 22 0 0 0

(34,18) Z_2 2/ 0 1 2 0

Z_2 2/ 14 1 0 0

Z_4	4 2 1 0
	8 3 0 0

Z_2	2/ 15 1 0 0
	1/ 12 2 0 0
	2/ 18 0 0 0

Z_2 2/ 0 1 0 1

Z_2 2/ 4 5 0 0

Z_2 2/ 6 2 1 0

Z_2 2/ 12 0 1 0

Z_2	2/ 3 1 2 0
	2/ 6 0 2 0

Z_2 2/ 4 3 1 0

Z_2 2/ 10 1 1 0

Z_2	2/ 8 2 1 0
	2/ 12 3 0 0
	6/ 18 1 0 0

Z_2 2/ 6 5 0 0

Z_2 2/ 0 7 0 0

Z_4	1/ 8 2 1 0
	1/ 12 3 0 0

Z_4	2/ 11 1 1 0
	6/ 14 0 1 0

Z_2	2/ 7 5 0 0
	4/ 13 3 0 0
	6/ 15 0 1 0
	2/ 19 1 0 0
	6/ 10 4 0 0
	6/ 12 1 1 0

Z_2 2/ 12 1 1 0

Z_2 2/ 7 5 0 0
4/ 15 0 1 0
1/ 4 6 0 0
6/ 22 0 0 0

Z_2 2/ 19 1 0 0
6/ 22 0 0 0

Z_2 2/ 6 3 1 0

Z_8 4/ 13 3 0 0
5/ 15 0 1 0
1/ 19 1 0 0
1/ 12 1 1 0
2/ 22 0 0 0

(46,18) Z_2 6/ 13 1 1 0
6/ 14 3 0 0

Z_2 6/ 13 1 1 0
2/ 6 1 2 0

Z_2 2/ 0 3 2 0

Z_2 2/ 4 4 1 0

Z_2 2/ 14 3 0 0

Z_2 2/ 20 1 0 0

Z_2 2/ 2 7 0 0

Z_4 2/ 7 3 1 0
6/ 10 2 1 0

Z_4 4 4 1 0
8 5 0 0

(48,18) Z_2 6/ 7 1 2 0
2/ 0 1 3 0
2/ 10 0 2 0

Z_2 2/ 3 7 0 0
6/ 7 1 2 0
2/ 15 3 0 0
2/ 6 6 0 0
2/ 10 0 2 0
2/ 18 2 0 0

Z_2 1/ 5 4 1 0
6/ 7 1 2 0
5/ 9 5 0 0
6/ 15 3 0 0
6/ 8 3 1 0
2/ 14 1 1 0

Z_2 3/ 11 2 1 0
3/ 15 3 0 0
2/ 21 1 0 0
1/ 8 3 1 0
2/ 18 2 0 0

Z_2 2/ 7 1 2 0
6/ 10 0 2 0

Z_2 2/ 11 2 1 0
2/ 15 3 0 0
4/ 21 1 0 0

Z_2 2/ 0 3 0 1

Z_2 2/ 2 5 1 0

Z_2 4 2 2 0

Z_2 2/ 6 1 0 1

Z_2 2/ 14 1 1 0

Z_4 2/ 15 3 0 0
2/ 18 2 0 0

(50,18) Z_2 2/ 3 5 1 0
2/ 15 1 1 0
2/ 0 1 1 1
2/ 4 0 3 0
2/ 4 7 0 0
2/ 6 4 1 0
2/ 10 5 0 0
2/ 12 2 1 0
6/ 16 3 0 0
6/ 18 0 1 0
6/ 22 1 0 0

Z_2 2/ 3 5 1 0
2/ 15 1 1 0
2/ 4 0 3 0
2/ 4 7 0 0
2/ 6 4 1 0
2/ 10 5 0 0
2/ 12 2 1 0
6/ 16 3 0 0
6/ 18 0 1 0
6/ 22 1 0 0

Z_2 6/ 3 5 1 0
6/ 15 1 1 0
1/ 4 2 0 1
7/ 4 7 0 0
6/ 6 4 1 0
6/ 10 5 0 0
1/ 12 2 1 0
2/ 18 0 1 0
2/ 22 1 0 0

Z_2 2/ 7 1 0 1
4/ 15 1 1 0
2/ 4 7 0 0
2/ 8 1 2 0
6/ 10 0 0 1
2/ 10 5 0 0
6/ 16 3 0 0
6/ 22 1 0 0

Z_2 4/ 3 5 1 0
2/ 0 6 1 0
2/ 4 7 0 0
2/ 10 5 0 0

Z_2 2/ 8 1 2 0
6/ 12 2 1 0

Z_2 2/ 2 3 2 0

Z_2 2/ 4 7 0 0

Z_2 2/ 12 2 1 0

Z_2 2/ 22 1 0 0

GROUP	GENERATOR
Z_4	2/ 3 5 1 0 2/ 15 1 1 0 1/ 4 0 3 0 2/ 6 4 1 0 1/ 8 1 2 0 2/ 10 5 0 0 6/ 16 3 0 0 6/ 18 0 1 0
Z_4	2/ 3 5 1 0 1/ 0 6 1 0 1/ 4 7 0 0 2/ 10 5 0 0
Z_4	12 2 1 0 16 3 0 0
Z_4	2/ 15 1 1 0 6/ 18 0 1 0

The leading differential $d^6(2C[18]\overline{M}_2) = A[23]$ determines tentative differentials by assigning the following values to monomials of degree 24 of $Z_8C[18] \otimes H_*BP$: $C[18]M_1^3$ is assigned 1 and $C[18]M_2$ is assigned 2. The kernel of these tentative differentials is given by the table below. The new C[18]-leader is $C[18]M_1^4\overline{M}_2^2$.

DEGREE	GROUP	GENERATOR
(20,18)	Z_2	2/ 7 1 0 0 1/ 4 2 0 0
(22,18)	Z_4	1/ 4 0 1 0 7/ 8 1 0 0
(24,18)	Z_2	3/ 5 0 1 0 3/ 9 1 0 0 2/ 2 1 1 0
(26,18)	Z_4	6/ 3 1 1 0 1/ 0 2 1 0 1/ 4 3 0 0
(28,18)	Z_8	2/ 5 3 0 0 1/ 7 0 1 0 5/ 11 1 0 0 1/ 4 1 1 0
(30,18)	Z_2	2/ 5 1 1 0
(32,18)	Z_2	1/ 3 2 1 0 7/ 7 3 0 0 1/ 0 3 1 0 2/ 10 2 0 0
	Z_8	5/ 5 2 1 0 3/ 9 3 0 0 2/ 2 3 1 0
(34,18)	Z_4	6/ 7 1 1 0 1/ 4 2 1 0 5/ 8 3 0 0 2/ 10 0 1 0 2/ 14 1 0 0
(36,18)	Z_2	2/ 15 1 0 0 1/ 12 2 0 0 2/ 18 0 0 0
	Z_4	1/ 13 0 1 0 7/ 16 1 0 0
(38,18)	Z_2	2/ 6 2 1 0 6/ 10 3 0 0
	Z_8	7/ 7 2 1 0 1/ 11 3 0 0 6/ 13 0 1 0 2/ 17 1 0 0 1/ 4 3 1 0
(40,18)	Z_2	4/ 11 3 0 0 7/ 13 0 1 0 5/ 17 1 0 0 2/ 10 1 1 0 6/ 14 2 0 0
	Z_4	2/ 11 1 1 0 1/ 8 2 1 0 1/ 12 3 0 0 6/ 14 0 1 0 6/ 18 1 0 0
(42,18)	Z_4	2/ 5 3 1 0 2/ 11 1 1 0 3/ 8 2 1 0 3/ 12 3 0 0 2/ 14 0 1 0 6/ 18 1 0 0
	Z_8	4/ 13 3 0 0 5/ 15 0 1 0 1/ 19 1 0 0 1/ 12 1 1 0 2/ 22 0 0 0
(44,18)	Z_2	2/ 7 5 0 0 4/ 15 0 1 0 1/ 4 6 0 0 6/ 22 0 0 0

(46,18)	Z_2	4/ 7 3 1 0 4/ 13 1 1 0	Z_2	2/ 13 1 1 0	Z_4	1/ 4 4 1 0 7/ 8 5 0 0
(48,18)	Z_2	7/ 5 4 1 0 2/ 7 1 2 0 3/ 9 5 0 0 2/ 15 3 0 0 2/ 2 5 1 0 1/ 4 2 2 0 2/ 8 3 1 0	Z_2	6/ 7 1 2 0 3/ 11 2 1 0 3/ 15 3 0 0 2/ 21 1 0 0 1/ 4 2 2 0 5/ 8 3 1 0 2/ 10 0 2 0 6/ 18 2 0 0	Z_4	1/ 11 2 1 0 3/ 15 3 0 0 6/ 21 1 0 0 1/ 8 3 1 0
(50,18)	Z_4	4/ 3 5 1 0 6/ 7 1 0 1 1/ 4 0 3 0 5/ 4 2 0 1 5/ 4 7 0 0 1/ 8 1 2 0 2/ 10 0 0 1 3/ 12 2 1 0 6/ 22 1 0 0	Z_4	4/ 3 5 1 0 6/ 7 1 0 1 1/ 4 2 0 1 5/ 4 7 0 0 2/ 10 0 0 1 2/ 12 2 1 0 3/ 16 3 0 0 6/ 22 1 0 0	Z_4	6/ 15 1 1 0 1/ 12 2 1 0 5/ 16 3 0 0 2/ 18 0 1 0 2/ 22 1 0 0
					Z_4	2/ 3 5 1 0 1/ 0 6 1 0 1/ 4 7 0 0

<u>DEGREE 19:</u> A[19]

The leading differential $d^6(\sigma^2\overline{M}_2) = A[19]$ determines tentative differentials with image $Z_2A[19]\{1,M_1,M_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2A[19]\{M_1^2,M_1^3,M_1M_2,M_1^2M_2,M_1^3M_2\} \otimes B\langle 4\rangle$, and the A[19]-leader is $A[19]M_1^2$.

The leading differential $d^4(A[19]M_1^2) = \nu A[19]$ determines tentative differentials which are a monomorphism on $Z_2A[19]\{M_1^2,M_1^3,M_1M_2,M_1^2M_2,M_1^3M_2\} \otimes B\langle 4\rangle$. There are no remaining elements.

<u>DEGREE 20:</u> C[20]

The leading differential $d^4(\nu A[14]M_1^2) = 4C[20]$ determines tentative differentials with image $Z_2(4C[20])\{1,M_1,M_1^2,M_2,M_1M_2\} \otimes B\langle 4\rangle$. The leading differential $d^6(\eta A[14]M_1^3) = 2C[20]$ determines tentative differentials with image $Z_2(2C[20])\{1,M_1,M_2\} \otimes B\langle 4\rangle$. The leading differential $d^{12}(\eta A[8]M_1^3\overline{M}_2) = C[20]$ determines tentative differentials with image

$Z_2C[20] \otimes B\langle 4\rangle$. The cokernel of these differentials is $[Z_8C[20]\{M_1^3, M_1^2\bar{M}_2, M_1^3\bar{M}_2\} \otimes B\langle 4\rangle] \oplus [Z_4C[20]\{M_1^2, M_1\bar{M}_2\} \otimes B\langle 4\rangle] \oplus [Z_2C[20]\{M_1, \bar{M}_2\} \otimes B\langle 4\rangle]$. The leading differential $d^2(C[20]M_1) = \eta C[20]$ determines tentative differentials which are a monomorphism on $Z_2(C[20]M_1) \otimes B\langle 2\rangle$. The leading differential $d^4(C[20]M_1^2) = \nu C[20]$ determines tentative differentials which are a monomorphism on $([Z_8C[20]\{\bar{M}_2, M_1^2\bar{M}_2] \oplus [Z_4(2C[20])M_1^3\bar{M}_2] \oplus [Z_4C[20]M_1^2] \oplus [Z_2(2C[20])M_1\bar{M}_2]) \otimes B\langle 4\rangle$. There are no remaining elements.

DEGREE 21: $\eta C[20]$

Since $\eta^2C[20] \neq 0$, the only element of $E^4_{*,21}$ with a representative in $Z_2\eta C[20] \otimes H_*BP$ is zero.

DEGREE 21: $\nu C[18] = \sigma^3$

The leading differential $d^4(C[18]M_1^2) = \nu C[18]$ determines tentative differentials with image $Z_2\nu C[18]\{1, M_1, M_1^2, M_2, M_1M_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2\nu C[18]\{M_1^3, M_1^2M_2, M_1^3M_2\} \otimes B\langle 4\rangle$, and the $\nu C[18]$-leader is $\nu C[18]M_1^3$.

The leading differential $d^8(\sigma^2M_1^4\bar{M}_2) = \sigma^3\bar{M}_2$ determines tentative differentials with cokernel given by the table below. The new $\nu C[18]$-leader is $\nu C[18]M_1^3M_2$.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(12,21)	3 1 0 0	(20,21)	7 1 0 0	(24,21)	3 3 0 0
(26,21)	3 1 1 0	(28,21)	11 1 0 0		
(32,21)	6 1 1 0		7 3 0 0	(34,21)	7 1 1 0
(36,21)	3 0 0 1		3 5 0 0	(38,21)	3 3 1 0
(40,21)	3 1 2 0		4 3 1 0		11 3 0 0
(42,21)	3 1 0 1		11 1 1 0		

(44,21)	7 0 0 1	6 3 1 0		7 5 0 0
(46,21)	0 3 2 0	7 3 1 0	(48,21)	14 1 1 0

DEGREE 22: $\eta^2 C[20]$

Since $\eta^3 C[20] = 4\nu C[20] \neq 0$, the only element of $E^4_{*,22}$ with a representative in $Z_2\eta^2 C[20] \otimes H_*BP$ is zero.

DEGREE 22: $\nu A[19]$

The leading differential $d^4(A[19]M_1^2) = \nu A[19]$ determines tentative differentials with image $Z_2\nu A[19]\{1, M_1, M_1^2, M_2, M_1M_2\} \otimes H_*BP$. The remaining elements are $Z_2\nu A[19]\{M_1^3, M_1^2M_2, M_1^3M_2\} \otimes B<4>$, and the $\nu A[19]$-leader is zero.

The leading differential $d^{14}(\eta^2\sigma M_1^7\bar{M}_2) = \nu A[19]\bar{M}_2$ determines tentative differentials with image, in degrees less than 69, of dimension given by the fourth column of the table below.

The leading differntial $d^{10}(\nu A[19]M_1^2\bar{M}_2) = A[31]$ determines tentative differentials which are a monomorphism on $A = Z_2\nu A[19]\{M_1^2\bar{M}_2, M_1^3\bar{M}_2\} \otimes B<4>$. We give the dimensions of A in the fifth column of the table below. As the reader can verify, the sum of the numbers in the last three columns equals the number in the second column except when N = 40. Thus, $E^{16}_{*,22} = E^{16}_{40,22} = Z_2\nu A[19]M_1^7M_2^2<M_3>$ in dimensions less than 69, and the new $\nu A[19]$-leader is $\nu A[19]M_1^7M_2^2<M_3>$.

N	DIM $E^4_{N,22}$	DIM $\nu E^4_{N+4,19}$	DIM $d^{14}(E^{14}_{N+14,9})$	DIM A
0	1	1	0	0
2	1	1	0	0
4	1	1	0	0
6	2	1	1	0
8	2	2	0	0
10	2	1	0	1
12	3	2	0	1
14	4	3	1	0
16	4	4	0	0
18	5	3	1	1
20	6	4	1	1
22	6	4	1	1
24	7	5	0	2
26	8	5	1	2
28	9	7	1	1
30	11	8	2	1
32	12	9	1	2
34	13	8	2	3
36	15	10	2	3
38	16	11	2	3
40	17	12	0	4
42	20	13	3	4
44	22	16	3	3
46	23	16	3	4

DEGREE 23: A[23]

The leading differrential $d^6(2C[18]M_1^3) = A[23]$ determines tentative differentials with image $Z_2A[23]\{1, M_1^2, \bar{M}_2\} \otimes B\langle 4\rangle$. Since $\eta A[23] \neq 0$, the remaining elements are $Z_2A[23]M_1^2\bar{M}_2 \otimes B\langle 4\rangle$, and the A[23]-leader is $A[23]M_1^2\bar{M}_2$.

DEGREE 23: $\nu C[20]$

The leading differential $d^2(\eta^2C[20]M_1) = 4\nu C[20]$ determines tentative differentials with image $Z_2(4\nu C[20]) \otimes B\langle 2\rangle$. The remaining elements are $[Z_8(\nu C[20]M_1) \otimes B\langle 2\rangle] \oplus [Z_4(\nu C[20]) \otimes B\langle 2\rangle]$, and the $\nu C[20]$-leader is $\nu C[20]M_1$.

The leading differential $d^4(C[20]M_1^2) = \nu C[20]$ determines tentative differentials with image

$[Z_8(\nu C[20])\{M_1, M_2\} \oplus Z_4(\nu C[20])\{1, 2M_1M_2\} \oplus Z_2(2\nu C[20])M_1] \otimes B\langle 4\rangle$. The remaining elements are

$[Z_2(\nu C[20])\{M_1^2, M_1M_2\} \oplus Z_4(\nu C[20])\{\overline{M}_2, M_1^2\overline{M}_2\} \oplus Z_8(\nu C[20]M_1^3\overline{M}_2)] \otimes B\langle 4\rangle$,

and the new $\nu C[20]$-leader is $\nu C[20]M_1^2$.

The leading differential $d^4(\nu C[20]M_1^2) = \nu^2 C[20]$ determines tentative differentials which are a monomorphism on

$Z_2(\nu C[20])\{M_1^2, M_1M_2, \overline{M}_2, M_1^2\overline{M}_2, M_1^3\overline{M}_2\} \otimes B\langle 4\rangle$. The remaining elements are $[Z_2(\nu C[20])\{2\overline{M}_2, 2M_1^2\overline{M}_2\} \oplus Z_4(2\nu C[20]M_1^3\overline{M}_2)] \otimes B\langle 4\rangle$, and the new $\nu C[20]$-leader is $2\nu C[20]\overline{M}_2$.

The leading differential $d^6(2\nu C[20]\overline{M}_2) = A[8]C[20]$ determines tentative differentials which are a monomorphism on $Z_2(\nu C[20])\{2\overline{M}_2, 2M_1^2\overline{M}_2, 2M_1^3\overline{M}_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2(4\nu C[20]M_1^3\overline{M}_2) \otimes B\langle 4\rangle$, and the new $\nu C[20]$-leader is $4\nu C[20]M_1^3\overline{M}_2$.

<u>DEGREE 24:</u> $\eta A[23]$

The leading differential $d^2(A[23]M_1) = \eta A[23]$ determines tentative differentials with image $Z_2\eta A[23] \otimes B\langle 2\rangle$. The remaining elements are $Z_2(\eta A[23]M_1) \otimes B\langle 2\rangle$, and the $\eta A[23]$-leader is $\eta A[23]M_1$.

The leading differential $d^{16}(\eta^2\sigma M_1^9) = \eta A[23]M_1$ determines tentative differentials whose cokernel in degrees less than 69 is $Z_2\eta A[23]\{M_1^{15}\overline{M}_2, M_1^{15}\overline{M}_2^2, M_1^{15}\overline{M}_3\}$. The new $\eta A[23]$-leader is $\eta A[23]M_1^{15}\overline{M}_2$.

<u>DEGREE 26:</u> $\nu^2 C[20]$

The leading differential $d^4(\nu C[20]M_1^2) = \nu^2 C[20]$ determines tentative differentials with image $Z_2\nu^2\{1, M_1, M_1^2, M_2, M_1M_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2\nu^2 C[20]\{M_1^3, M_1^2M_2, M_1^3M_2\} \otimes B\langle 4\rangle$, and the $\nu^2 C[20]$-leader is $\nu^2 C[20]M_1^3$.

The leading differential $d^{16}(\beta_1 M_1^{11}) = \nu^2 C[20]M_1^3$ determines tentative differentials whose image in degrees less than 69 has dimensions given by the fourth column in the table below. Since in each row of the table below, the numbers in the third and fourth columns add up to the numbers in the second column, it follows that $E^{18}_{*,26} = 0$ in degrees less than 69.

N	DIM $E^4_{N,26}$	DIM $\nu E^4_{N+4,23}$	DIM $d^{16}(E^{16}_{N+16,11})$
0	1	1	0
2	1	1	0
4	1	1	0
6	2	1	1
8	2	2	0
10	2	1	1
12	3	2	1
14	4	3	1
16	4	4	0
18	5	3	2
20	6	4	2
22	6	4	2
24	7	5	2
26	8	5	3
28	9	7	2
30	11	8	3
32	12	9	3
34	13	8	5
36	15	10	5
38	16	11	5
40	17	12	5
42	20	13	7

<u>DEGREE 28:</u> A[8]C[20]

The leading differential $d^6(2\nu C[20]\bar{M}_2) = A[8]C[20]$ determines tentative differentials with image $Z_2 A[8]C[20]\{1, M_1^2\} \otimes B\langle 4\rangle$, and the A[8]C[20]-leader is $A[8]C[20]M_1$.

The leading differential $d^{18}(\beta_1 M_1^{10}) = A[8]C[20]M_1$ determines tentative differentials whose cokernel in degrees less than 69 is given by the table below. The new A[8]C[20]-leader is $A[8]C[20]M_1\bar{M}_2$.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(8,28)	1 1 0 0	(12,28)	3 1 0 0	(16,28)	5 1 0 0
(20,28)	1 3 0 0		7 1 0 0		
(22,28)	1 1 1 0	(24,28)	3 3 0 0		9 1 0 0
(26,28)	3 1 1 0	(28,28)	5 3 0 0		11 1 0 0
(30,28)	5 1 1 0	(32,28)	7 3 0 0		13 1 0 0
(34,28)	1 3 1 0		7 1 1 0	(36,28)	1 1 2 0
	3 5 0 0		9 3 0 0		15 1 0 0
(38,28)	1 1 0 1		3 3 1 0		9 1 1 0
(40,28	3 1 2 0		5 5 0 0		11 3 0 0
	13 0 1 0		17 1 0 0		

DEGREE 30: A[30]

The leading differential $d^{24}(2\sigma M_1^{12}) = A[30]$ determines tentative differentials with image $Z_2A[30] \otimes Z_2[\langle M_1^4\rangle^2, \langle M_2^2\rangle^2, \langle M_3\rangle^2, \langle M_4\rangle^2, \{M_5\}, \ldots, \{M_n\}, \ldots\}$. Since $\eta A[30] \neq 0$, the remaining elements are $[Z_2A[30]\{M_1^2, \bar{M}_2, M_1^2\bar{M}_2\} \otimes B\langle 4\rangle]$

$$\oplus\ [Z_2A[30]\langle M_4\rangle \otimes Z_2[\langle M_1^4\rangle^2, \langle M_2^2\rangle^2, \langle M_3\rangle^2, \langle M_4\rangle^2, \{M_5\}, \ldots, \{M_n\}, \ldots\}] \oplus$$
$$\bigoplus_{\alpha,\beta,\gamma} Z_2A[30]\langle M_1^4\rangle^{\alpha}\langle M_2^2\rangle^{\beta}\langle M_3\rangle^{\gamma} \otimes B\langle 8\rangle.$$

The last sum is taken over all $0 \le \alpha,\ \beta,\ \gamma \le 1$ with $0 < \alpha\beta\gamma$. The A[30]-leader is $A[30]M_1^2$.

DEGREE 31: A[31]

The leading differential $d^{10}(\nu A[19]M_1^2\bar{M}_2) = A[31]$ determines tentative differentials with image $Z_2A[31]\{1, M_1\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2A[31]\{M_1^2, M_1^3, M_2, M_1M_2, M_1^2M_2, M_1^3M_2\} \otimes B\langle 4\rangle$, and the A[31]-leader is $A[31]M_1^2$.

DEGREE 31: ηA[30]

The leading differential $d^2(A[30]M_1) = \eta A[30]$ determines tentative differentials with image $Z_2\eta A[30] \otimes B\langle 2\rangle$. The cokernel of these differentials is $Z_2\eta A[30]M_1 \otimes B\langle 2\rangle$, and the $\eta A[30]$-leader is $\eta A[30]M_1$.