CLASSICAL AND MOTIVIC ADAMS-NOVIKOV CHARTS

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Abstract. This document contains large-format Adams-Novikov charts that compute the classical 2-complete stable homotopy groups. The charts are essentially complete through the 59-stem. We believe that these are the most accurate and extensive charts of their kind. We also include a motivic Adams-Novikov E_∞ chart.

This document contains large-format Adams-Novikov charts that compute the classical 2-complete stable homotopy groups. The charts are essentially complete through the 59-stem. We believe that these are the most accurate and extensive charts of their kind. We also include a motivic Adams-Novikov E_∞ chart.

The charts are intended to be viewed electronically. The author can supply versions that are suitable for printing.

Justifications for these calculations appear in [1].

1. The E_2-page of the Adams-Novikov spectral sequence

This chart shows the E_2-page of the classical Adams-Novikov spectral sequence through the 59-stem, together with all differentials. Beyond the 47-stem, there are some uncertainties, which are indicated on the chart.

For legibility, the chart is split into two pieces. The well-understood α family and its α_1 multiples are depicted on a separate grid.

The names of elements are essentially arbitrary and do not necessarily reflect structure.

(1) Solid dots indicate copies of $\mathbb{Z}/2$.
(2) Open circles indicate copies of $\mathbb{Z}/4$.
(3) Solid squares indicate copies of $\mathbb{Z}/8$.
(4) Open squares indicate copies of $\mathbb{Z}/2^k$ for some $k \geq 4$. The default value of k is 4, unless shown otherwise on the chart. The value $k = \infty$ indicates a copy of $\mathbb{Z}_2$.
(5) Lines of slope 1 indicate α_1 multiplications.
(6) Arrows of slope 1 indicate infinitely many α_1 multiplications.
(7) Lines of slope 1/3 indicate $\alpha_{2/2}$ multiplications.
(8) Magenta lines indicate that an extension equals 2 times a generator. For example, $\alpha^2_{2/2}z_{14}$ equals $2z_{20,4}$ in the 20-stem.
(9) Orange lines indicate that an extension equals 4 times a generator. For example, $\alpha^3_1 z_{20,2}$ equals $4z_{23}$ in the 23-stem.
(10) Red lines of slope -3 indicate Adams-Novikov d_3 differentials.
(11) Blue lines of slope -5 indicate Adams-Novikov d_5 differentials.
(12) Green lines of slope -7 indicate Adams-Novikov d_7 differentials.
(13) Purple lines of slope -9 indicate Adams-Novikov d_9 differentials.
(14) Gray dots indicate elements that may or may not exist in the Adams-Novikov spectral sequence. The first of these occurs in the 54-stem.
(15) Brown dashed boxes indicate possible 2 extensions that we have been unable to verify. The first of these occurs in the 54-stem.
(16) Dashed lines of slope 1 and slope 1/3 indicate possible α_1 and $\alpha_{2/2}$ extensions that we have not been able to verify. The first of these occurs in the 47-stem.
(17) Dashed lines of slope -2 indicate possible d_3 differentials. These differentials occur if and only if their source elements exist. The first of these occurs in the 55-stem.

2. The E_∞-page of the Adams-Novikov spectral sequence

This chart indicates the E_∞-page of the classical Adams-Novikov spectral sequence, together with hidden extensions by 2, η, and ν. Beyond the 48-stem, there are some uncertainties.

See Section 1 for instructions on interpreting the chart. In addition,

(1) Olive lines indicate hidden 2 extensions.
(2) Purple lines indicate hidden η extensions.
(3) Brown lines indicate hidden ν extensions.
(4) Dashed lines indicate possible 2, η, and ν extensions that we have been unable to verify.

3. The E_∞-page of the motivic Adams-Novikov spectral sequence

This chart indicates the E_∞-page of the motivic Adams-Novikov spectral sequence. Beyond the 48-stem, there are some uncertainties.

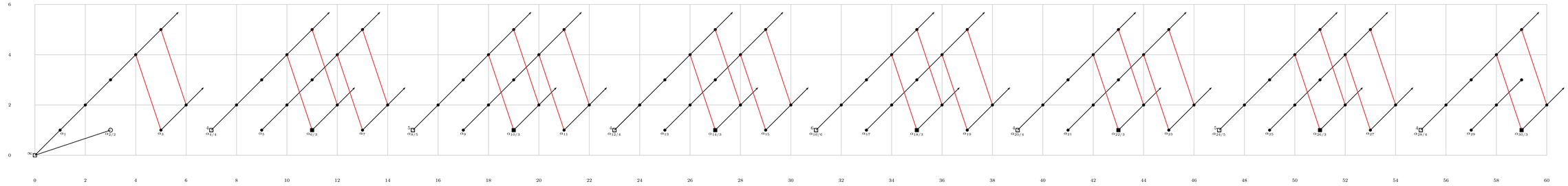
See Section 1 for instructions on interpreting the chart. In addition,

(1) Red dots indicate copies of $\mathbb{Z}/2[\tau]/\tau$.
(2) Blue dots indicate copies of $\mathbb{Z}/2[\tau]/\tau^2$.
(3) Purple dots indicate copies of $\mathbb{Z}/2[\tau]/\tau^4$.
(4) Green dots indicate copies $\mathbb{Z}/2[\tau]/\tau^3$.
(5) Green squares with blue centers indicate copies of $\mathbb{Z}/8[\tau]/(\tau^3, 2\tau^2)$.

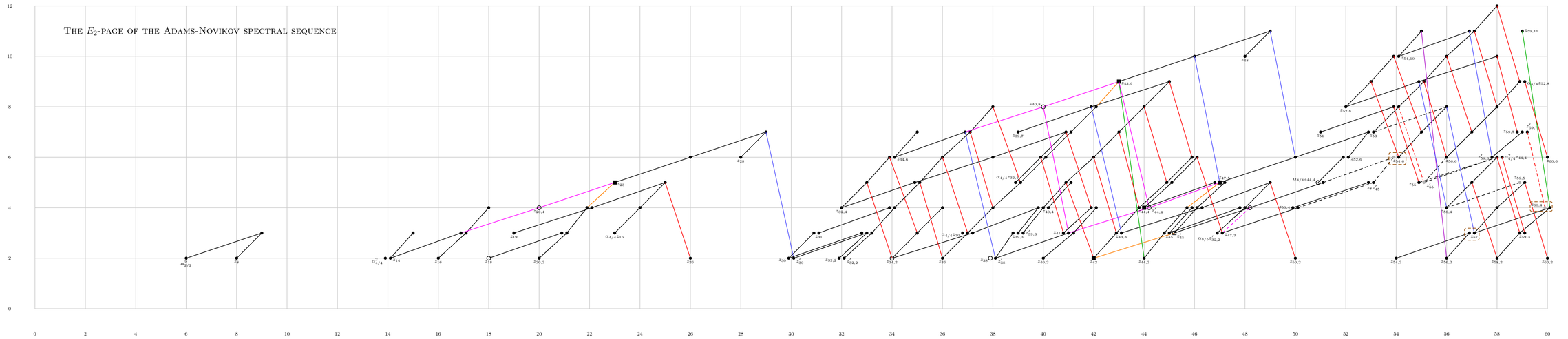
References

[1] Daniel C. Isaksen, *Stable stems* (2014), preprint.

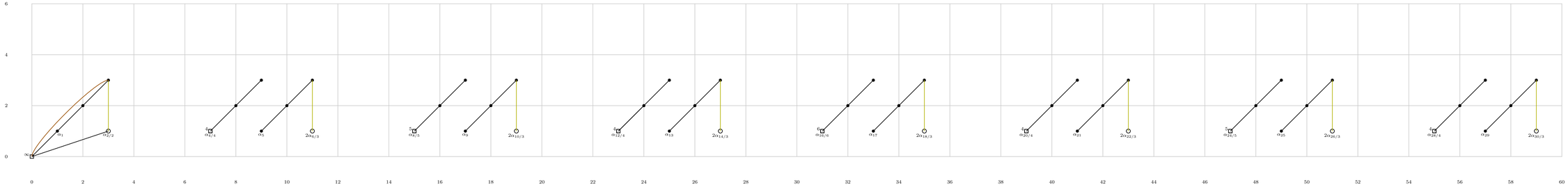
The α family in the Adams-Novikov spectral sequence



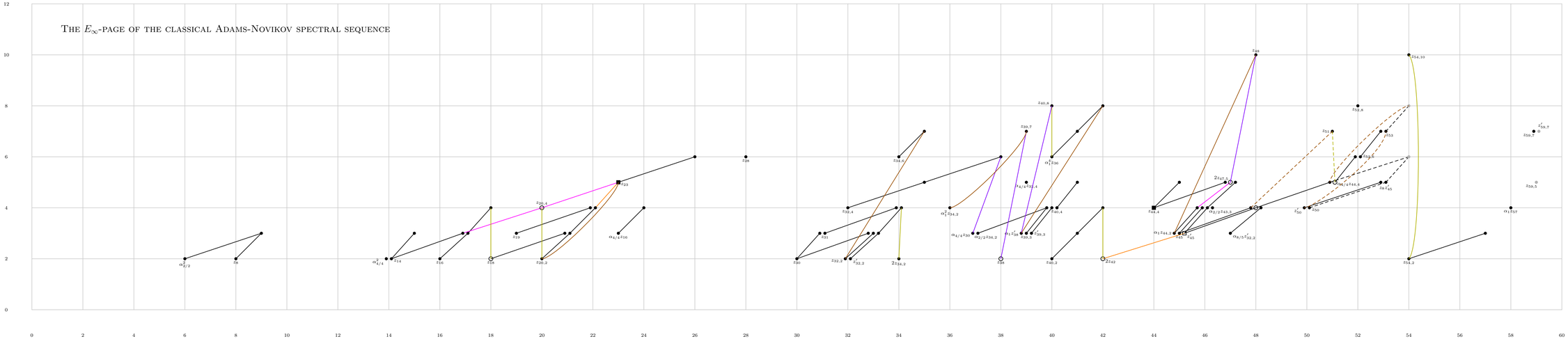
The E_2-page of the Adams-Novikov spectral sequence



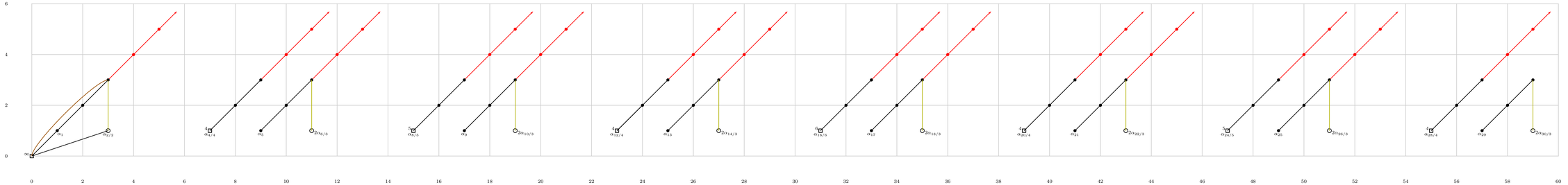
The α family in the classical Adams-Novikov E_∞-page



The E_∞-page of the classical Adams-Novikov spectral sequence



The α family in the motivic Adams-Novikov E_∞-page



The E_∞-page of the motivic Adams-Novikov spectral sequence