CLASSICAL AND MOTIVIC ADAMS CHARTS

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Abstract. This document contains large-format Adams charts that compute 2-complete stable homotopy groups, both in the classical context and in the motivic context over $\mathbb{C}$. The charts are essentially complete through the 59-stem and contain partial results to the 70-stem. In the classical context, we believe that these are the most accurate charts of their kind.

We also include Adams charts for the motivic homotopy groups of the cofiber of τ.

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We also include Adams charts for the motivic homotopy groups of the cofiber of τ.

The charts are intended to be viewed electronically. The author can supply versions that are suitable for printing.

Justifications for these calculations appear in [1].

1. The classical Adams spectral sequence

This chart shows the classical Adams spectral sequence. The E_2-page is complete through the 70-stem, as are the Adams d_2 differentials. The Adams d_3, d_4, and d_5 differentials are complete through the 65-stem, with a few indicated exceptions.

(1) Black dots indicate copies of $\mathbb{F}_2$.
(2) Vertical lines indicate h_0 multiplications.
(3) Lines of slope 1 indicate h_1 multiplications.
(4) Lines of slope 1/3 indicate h_2 multiplications.
(5) Light blue lines of slope -2 indicate Adams d_2 differentials.
(6) Red lines of slope -3 indicate Adams d_3 differentials.
(7) Green lines of slope -4 indicate Adams d_4 differentials.
(8) Blue lines of slope -5 indicate Adams d_5 differentials.
(9) Dashed lines indicate plausible Adams d_3, d_4, and d_5 differentials, but we have not independently verified their existence.

2. The E_∞-page of the classical Adams spectral sequence

This chart indicates the E_∞-page of the classical Adams spectral sequence. The chart is essentially complete through the 59-stem. Because of unknown differentials, the actual E_∞-page beyond the 59-stem is a subquotient of what is shown.

See Section 1 for instructions on interpreting the chart. In addition:

(1) Olive lines indicate hidden 2 extensions.
(2) Purple lines indicate hidden η extensions.
(3) Brown lines indicate hidden ν extensions.
(4) Dashed olive, purple, and brown lines indicate possible hidden extensions.
(5) For clarity, some of the unknown Adams d_3, d_4, and d_5 differentials are also shown on this chart.

3. The cohomology of the motivic Steenrod algebra

This chart shows the cohomology of the motivic Steenrod algebra over $\mathbb{C}$. The chart is complete through the 70-stem.

(1) Black dots indicate copies of $\mathbb{M}_2$.
(2) Red dots indicate copies of $\mathbb{M}_2/\tau$.
(3) Blue dots indicate copies of $\mathbb{M}_2/\tau^2$.
(4) Green dots indicate copies of $\mathbb{M}_2/\tau^3$.
(5) Vertical lines indicate h_0 multiplications. These lines might be black, red, blue, or green, depending on the τ torsion of the target.
(6) Lines of slope 1 indicate h_1 multiplications. These lines might be black, red, blue, or green, depending on the τ torsion of the target.
(7) Lines of slope 1/3 indicate h_2 multiplications. These lines might be black, red, blue, or green, depending on the τ torsion of the target.
(8) Red arrows indicate infinite towers of h_1 multiplications, all of which are annihilated by τ.
(9) Magenta lines indicate that an extension hits τ times a generator. For example, $h_0 \cdot h_0 h_2 = \tau h_1^3$ in the 3-stem.
(10) Orange lines indicate that an extension hits τ^2 times a generator. For example, $h_0 \cdot h_0^3 x = \tau^2 h_0 e_0 g$ in the 37-stem.
(11) Dotted lines indicate that the extension is hidden in the May spectral sequence.
(12) Squares indicate that there is a τ extension that is hidden in the May spectral sequence. For example, $\tau \cdot Pc_0 d_0 = h_0^2 r$ in the 30-stem.

4. The E_2-page of the motivic Adams spectral sequence

This chart indicates the Adams d_2 differentials on the E_2-page of the motivic Adams spectral sequence. The chart is complete through the 70-stem. See Section 3 for instructions on interpreting the chart. In addition:

(1) Blue lines of slope -2 indicate Adams d_2 differentials.
(2) Magenta lines of slope -2 indicate that an Adams d_2 differential hits τ times a generator. For example, $d_2(h_0 c_2) = \tau h_1^2 e_1$ in the 40-stem.
(3) Orange lines of slope -2 indicate that an Adams d_2 differential hits τ^2 times a generator. For example, $d_2(h_0 g) = \tau^2 h_0 e_0 g$ in the 37-stem.

5. The E_3-page of the motivic Adams spectral sequence

This chart indicates the Adams d_3 differentials on the E_3-page of the motivic Adams spectral sequence. The chart is complete through the 65-stem, with indicated exceptions. Beyond the 65-stem, there are several unknown differentials.

See Section 3 for instructions on interpreting the chart. In addition:

(1) Blue lines of slope -3 indicate Adams d_3 differentials.
(2) Magenta lines of slope -3 indicate that an Adams d_3 differential hits τ times a generator. For example, $d_3(r) = \tau h_1 d_0^2$ in the 29-stem.
(3) Orange lines of slope -3 indicate that an Adams d_3 differential hits τ^k times a generator for $k \geq 2$. For example, $d_3(Q_2) = \tau^2 gt$ in the 56-stem, and $d_3(\tau W_1) = \tau^4 e_0^4$ in the 68-stem.
(4) Dashed lines indicate plausible Adams d_3 differentials, but we have not independently verified their existence.

6. The E_4-page of the motivic Adams spectral sequence

This chart indicates the Adams d_4 and d_5 differentials on the E_4-page of the motivic Adams spectral sequence. The chart is complete through the 65-stem, with a few indicated exceptions. Beyond the 65-stem, there are several unknown differentials.

Because of unknown d_3 differentials, the actual E_4-page beyond the 65-stem is a subquotient of what is shown.

See Section 3 for instructions on interpreting the chart. In addition:

(1) Purple dots indicate copies of $\mathbb{M}_2/\tau^4$.
(2) Blue lines of slope -4 and -5 indicate Adams d_4 and d_5 differentials.
(3) Magenta lines of slope -5 indicate that an Adams d_5 differential hits τ times a generator. For example, $d_5(\tau P h_2 e_0) = \tau d_0 z$ in the 55-stem.
(4) Orange lines of slope -4 and -5 indicate that an Adams d_4 or d_5 differential hits τ^k times a generator for $k \geq 2$.
(5) Dashed lines indicate plausible Adams d_3, d_4, and d_5 differentials, but we have not independently verified their existence.
(6) For clarity, the unknown Adams d_3 differentials are also shown on this chart.

7. The E_∞-page of the motivic Adams spectral sequence

This chart indicates the E_∞-page of the motivic Adams spectral sequence. The chart is complete through the 59-stem. Because of unknown differentials, the actual E_∞-page beyond the 59-stem is a subquotient of what is shown.

See Section 3 for instructions on interpreting the chart. In addition:

(1) Purple dots indicate copies of $\mathbb{M}_2/\tau^4$.
(2) For clarity, some of the unknown Adams d_3, d_4, and d_5 differentials are also shown on this chart.

8. The E_2-page of the motivic Adams spectral sequence for the cofiber of τ

This chart indicates the E_2-page of the motivic Adams spectral sequence for the cofiber of τ. The chart is complete through the 70-stem, with indicated exceptions.

(1) Black dots indicate copies of $\mathbb{M}_2/\tau$ that are in the image of the inclusion $E_2(S^{0,0}) \to E_2(C\tau)$ of the bottom cell.
(2) Red dots indicate copies of $\mathbb{M}_2/\tau$ that are detected by the projection $E_2(C\tau) \to E_2(S^{0,0})$ to the top cell.
(3) Black lines indicate extensions by h_0, h_1, and h_2 that are in the image of the inclusion of the bottom cell.
(4) Red lines indicate extensions by h_0, h_1, and h_2 that are detected by projection to the top cell.
(5) Blue lines indicate extensions by h_0, h_1, and h_2 that are hidden in the sense that they are not detected by the top cell or the bottom cell.
(6) Arrows of slope 1 indicate infinite towers of h_1 extensions.
(7) Dashed blue lines indicate unknown hidden extensions.
(8) Light blue lines indicate Adams differentials.
(9) Dashed light blue lines indicate unknown Adams differentials. These unknown differentials are also indicated on later charts.

9. The E_3-page of the motivic Adams spectral sequence for the cofiber of τ

This chart indicates the E_3-page of the motivic Adams spectral sequence for the cofiber of τ. The E_3-page is complete through the 70-stem, but the Adams d_3 differentials are complete only through the 64-stem. Beyond the 64-stem, there are a number of unknown differentials.

See Section 8 for instructions on interpreting the chart.

10. The E_4-page of the motivic Adams spectral sequence for the cofiber of τ

This chart indicates the E_4-page of the motivic Adams spectral sequence for the cofiber of τ. The E_4-page is complete through the 64-stem. Beyond the 64-stem, the actual E_4-page is a subquotient of what is shown. The chart shows both Adams d_4 and d_5 differentials.

See Section 8 for instructions on interpreting the chart.

11. The E_∞-page of the motivic Adams spectral sequence for the cofiber of τ

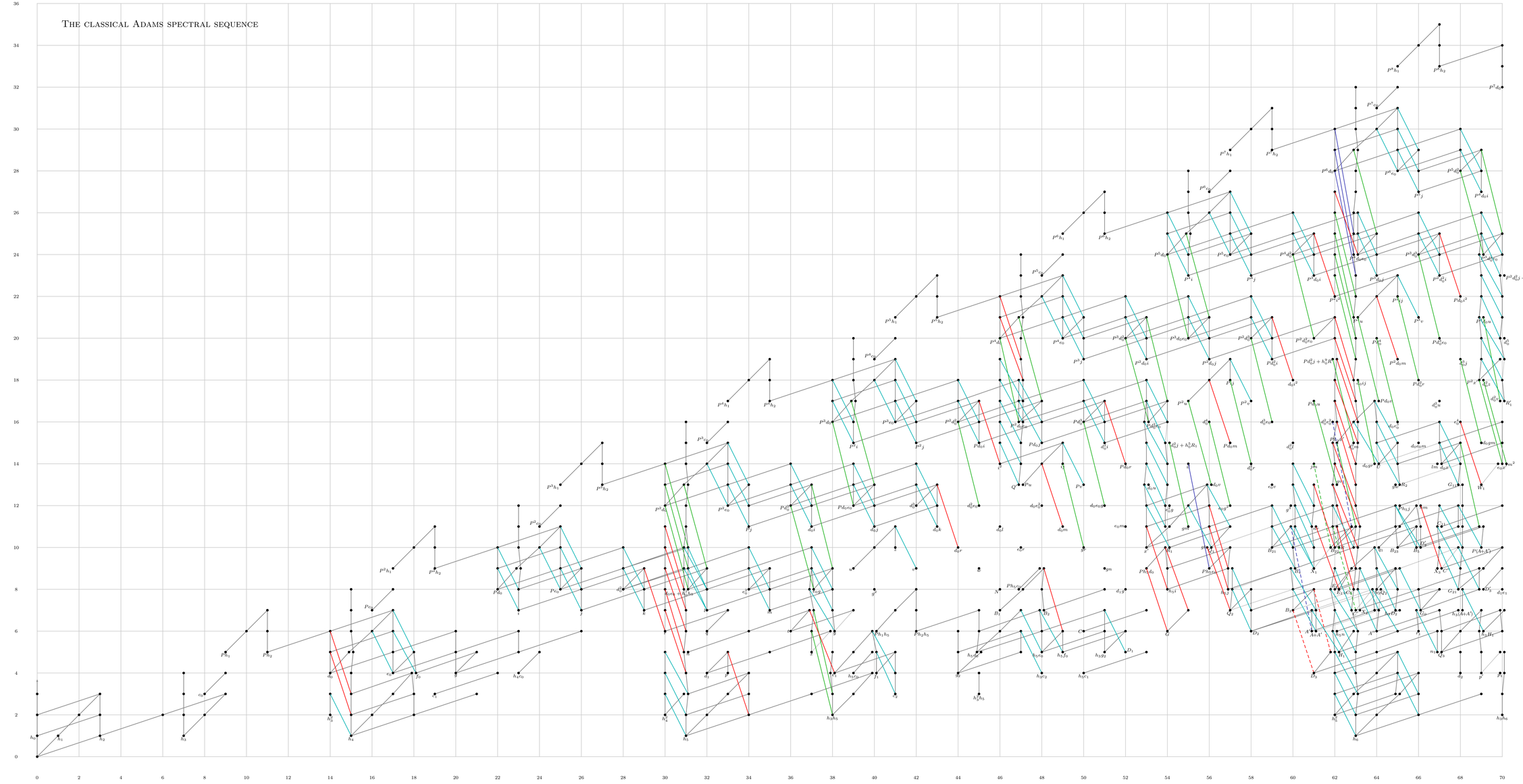
This chart indicates the E_∞-page of the motivic Adams spectral sequence for the cofiber of τ. The chart is complete through the 63-stem, but hidden extensions are only shown through the 59-stem. Beyond the 63-stem, the actual E_∞-page is a subquotient of what is shown.

(1) Black dots indicate copies of $\mathbb{M}_2/\tau$ that are in the image of the inclusion $\pi_{*,*} \to \pi_{*,*}(C\tau)$ of the bottom cell.
(2) Red dots indicate copies of $\mathbb{M}_2/\tau$ that are detected by the projection $\pi_{*,*}(C\tau) \to \pi_{*-1,*+1}$ to the top cell.
(3) Black lines indicate extensions by h_0, h_1, and h_2 that are in the image of the inclusion of the bottom cell.
(4) Red lines indicate extensions by h_0, h_1, and h_2 that are detected by projection to the top cell.
(5) Blue lines indicate extensions by 2, η, and ν that are not hidden in the E_∞-page but are not detected by the top cell or the bottom cell.
(6) Arrows of slope 1 indicate infinite towers of h_1 extensions.
(7) Olive lines indicate 2 extensions that are hidden in the E_∞-page.
(8) Purple lines indicate η extensions that are hidden in the E_∞-page.
(9) Brown lines indicate ν extensions that are hidden in the E_∞-page.
(10) Dashed lines indicate possible extensions.
(11) Dashed light blue lines indicate unknown Adams differentials. These unknown differentials are included for clarity.

References

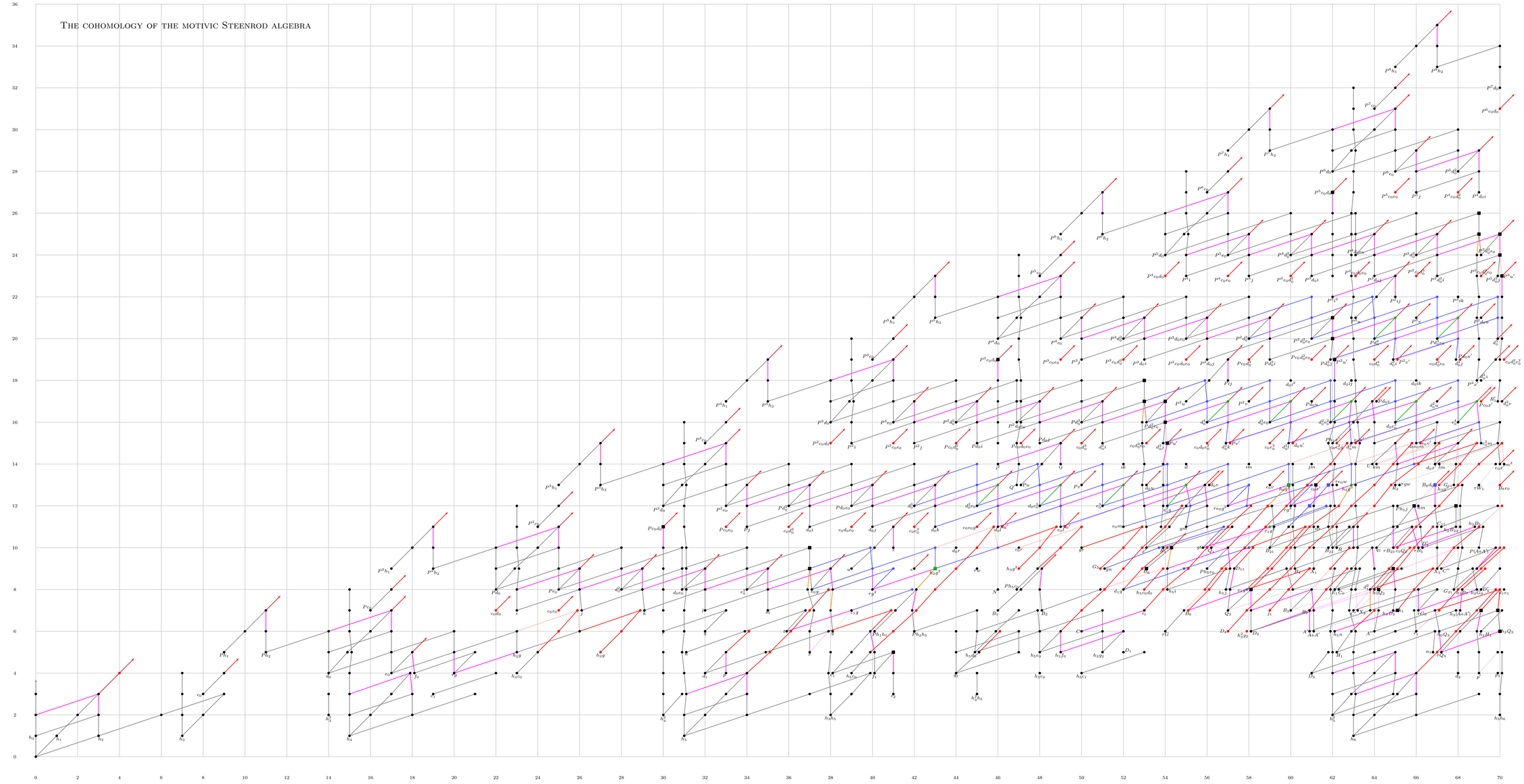
[1] Daniel C. Isaksen, *Stable stems* (2014), preprint, available at `arXiv:1407.8418`.

The classical Adams spectral sequence

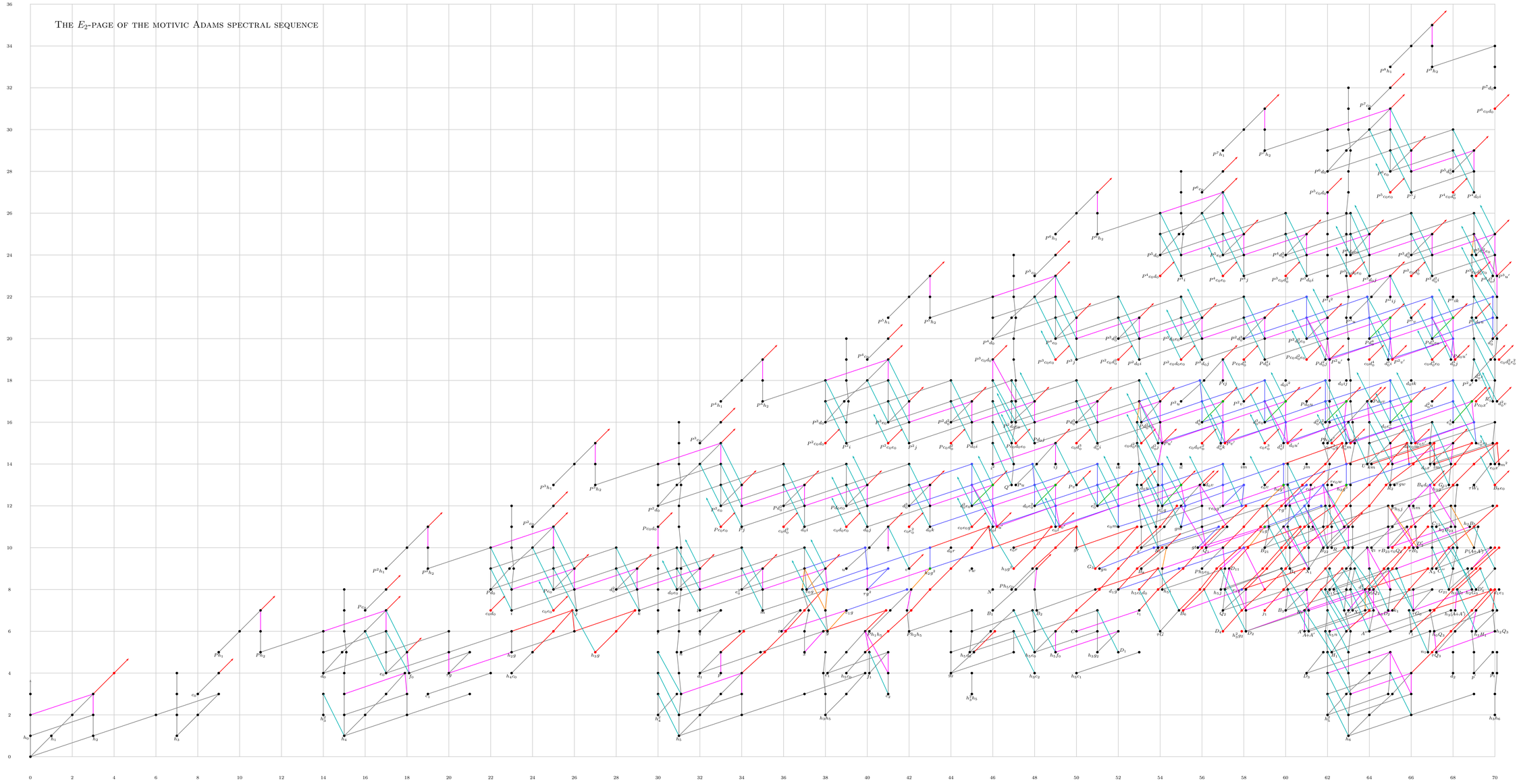


The E_∞-page of the classical Adams spectral sequence

The cohomology of the motivic Steenrod algebra

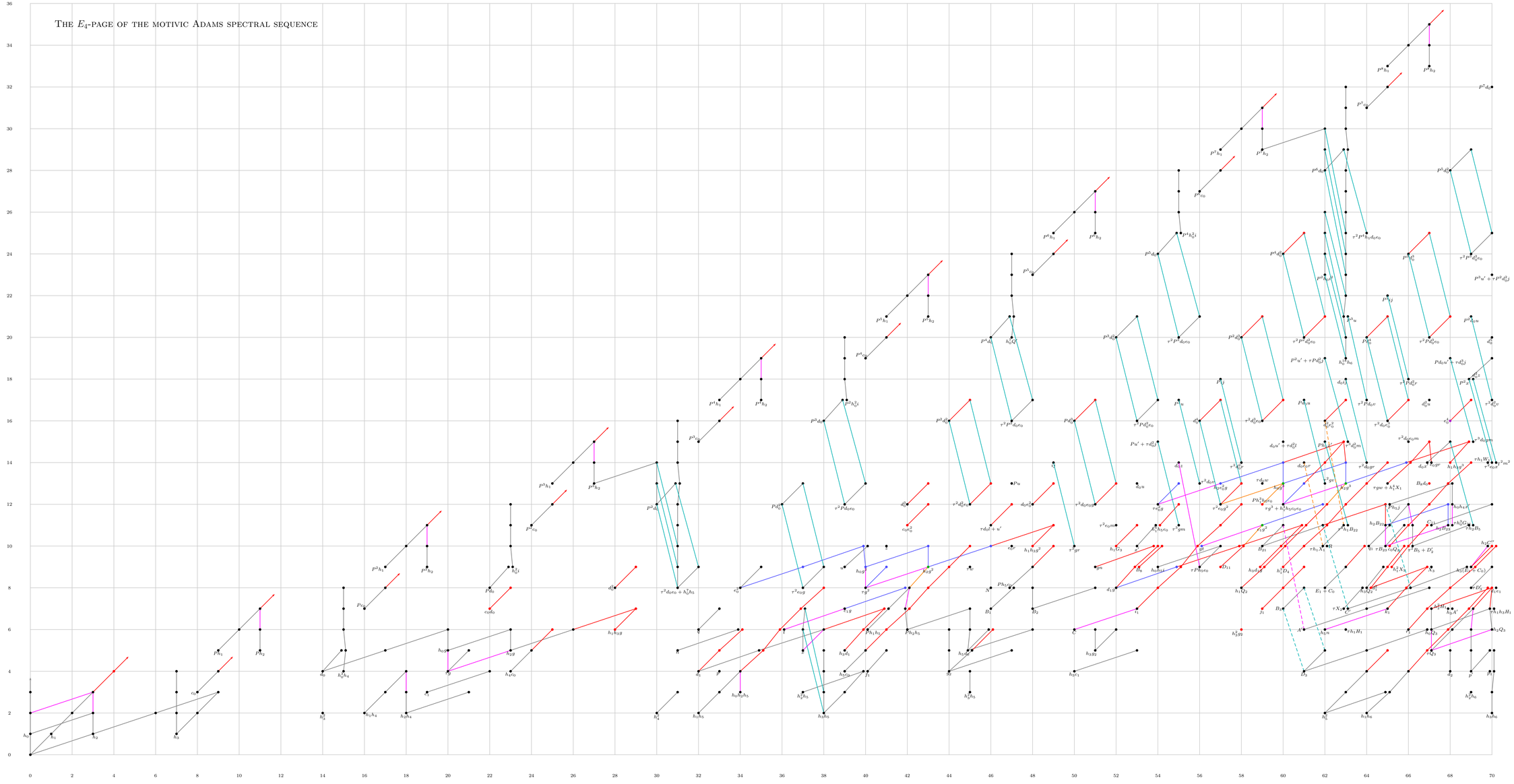


The E_2-page of the motivic Adams spectral sequence



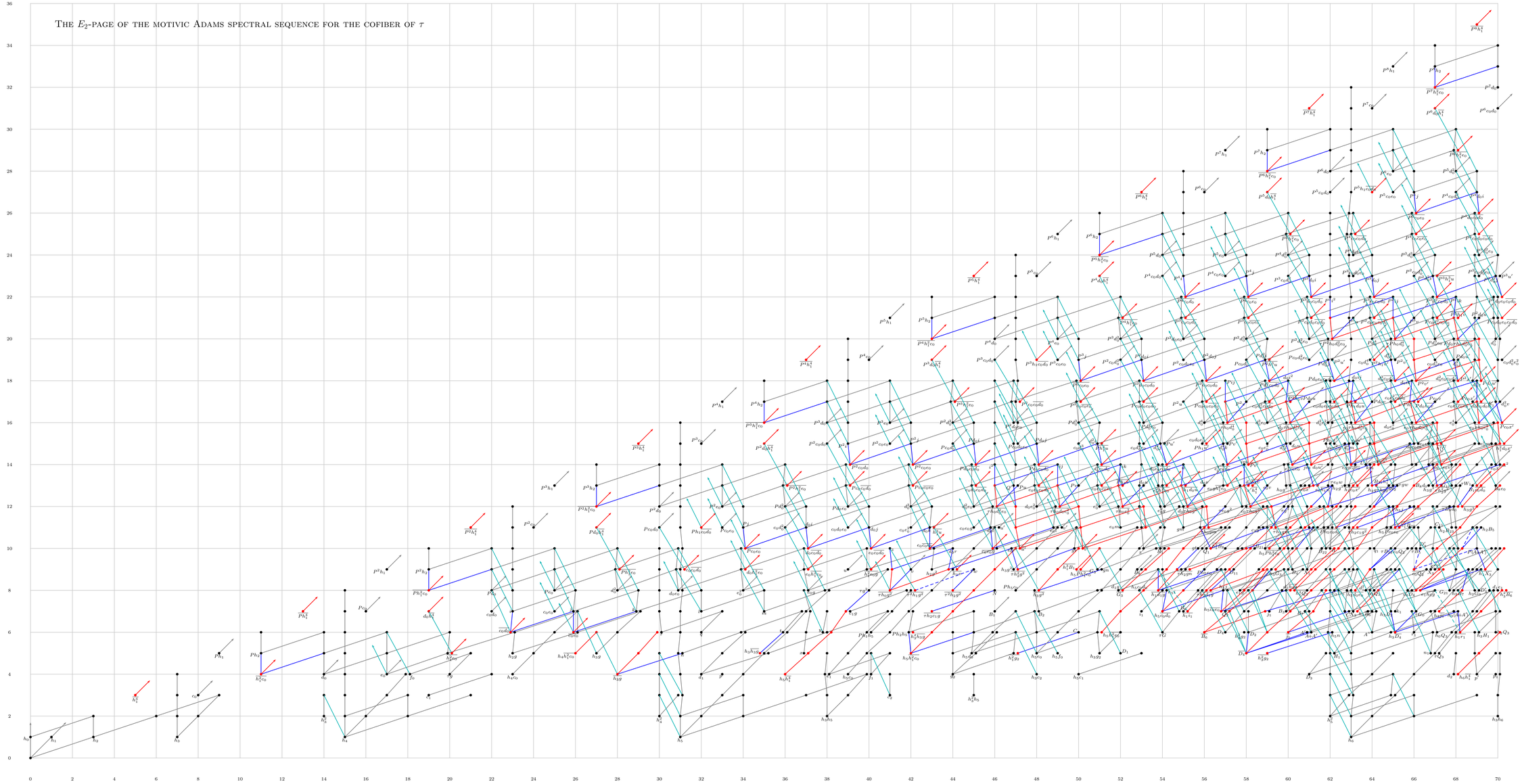
The E_3-page of the motivic Adams spectral sequence

The E_4-page of the motivic Adams spectral sequence

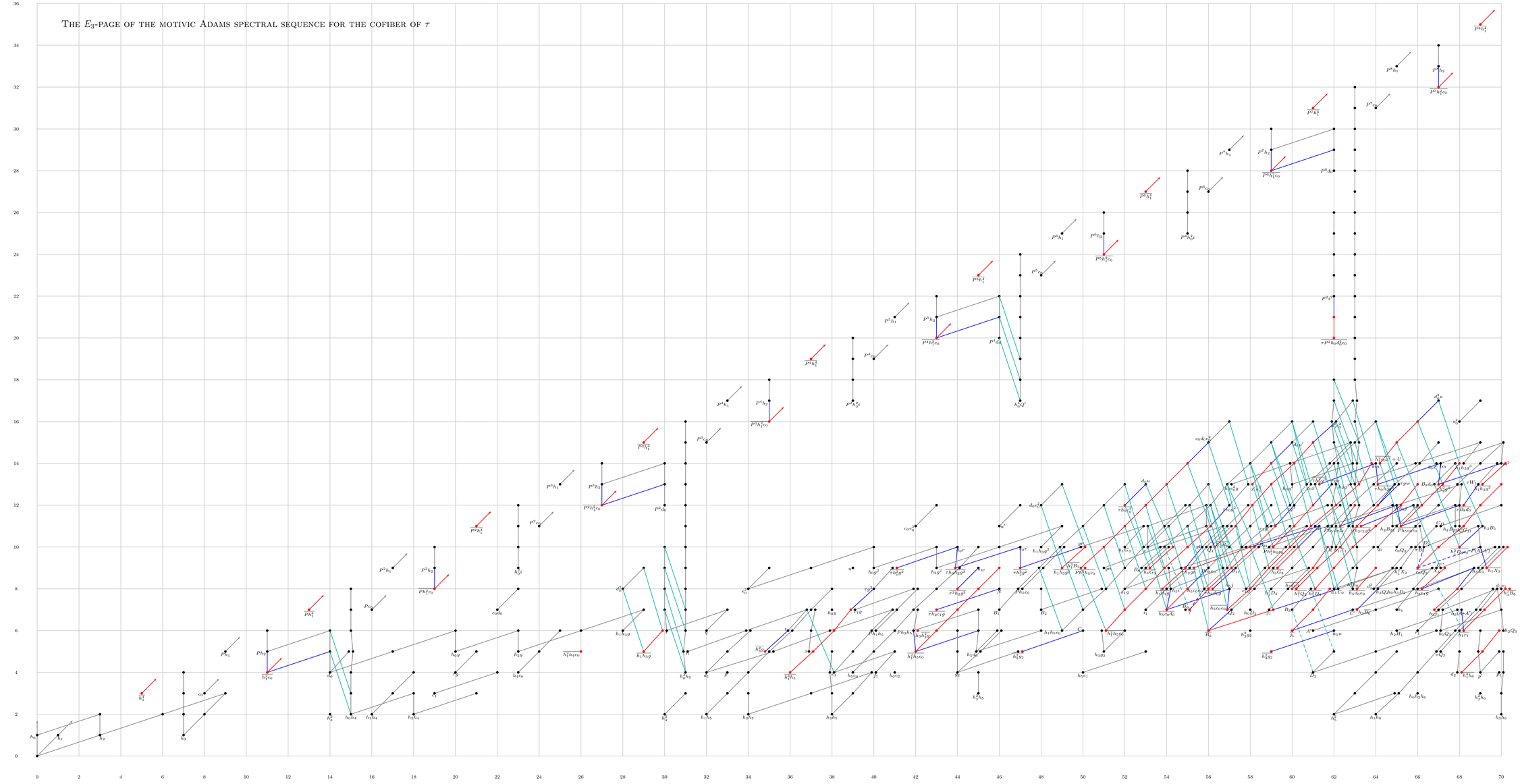


The E_∞-page of the motivic Adams spectral sequence

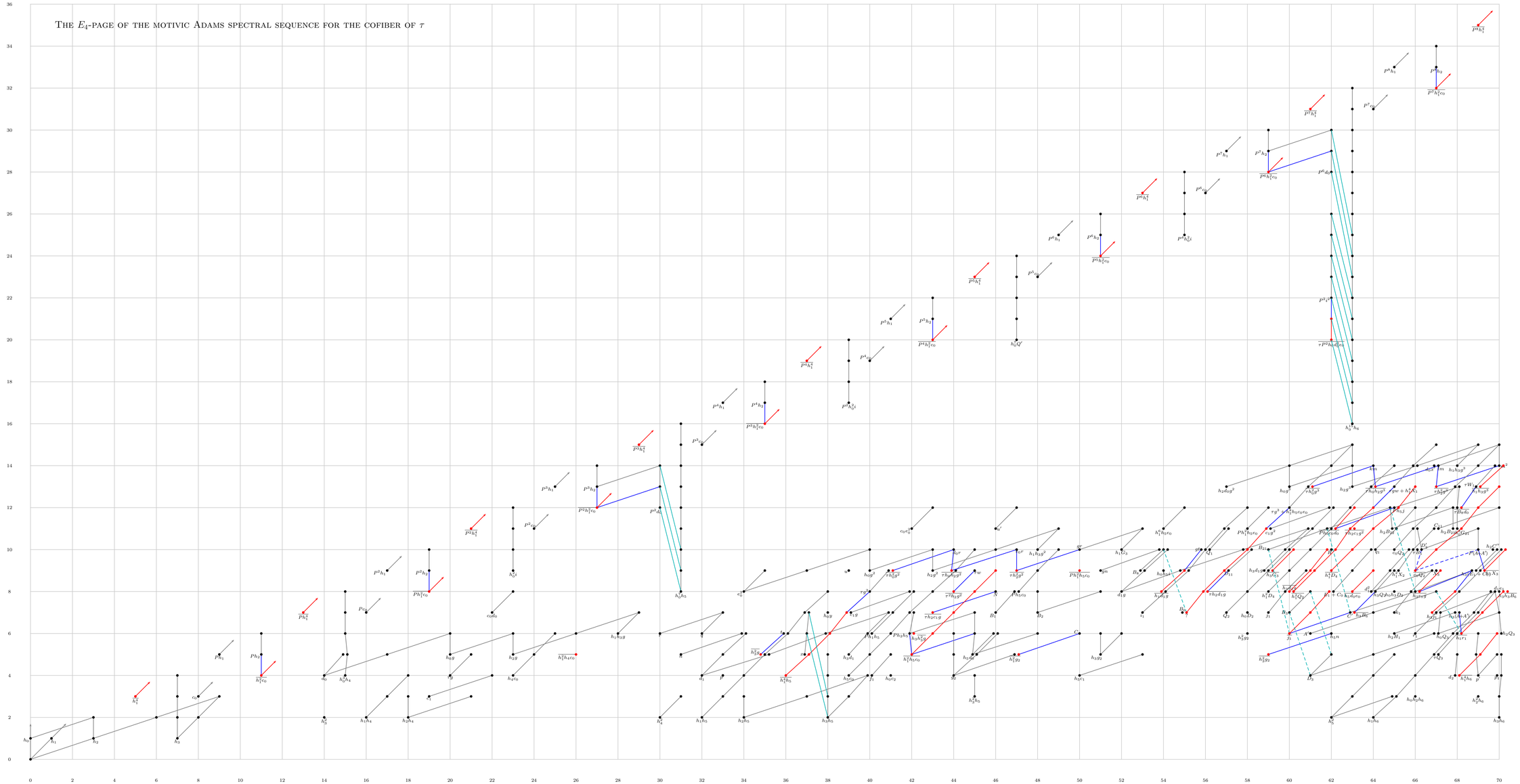
The E_2-page of the motivic Adams spectral sequence for the cofiber of τ



The E_3-page of the motivic Adams spectral sequence for the cofiber of τ



The E_4-page of the motivic Adams spectral sequence for the cofiber of τ



The E_∞-page of the motivic Adams spectral sequence for the cofiber of τ