Calculations of Unstable Adams E_2 Terms for Spheres

Edward B. Curtis
Paul Goerss
Mark Mahowald
R. James Milgram

1. Introduction

We have made computer calculations of the E_2 terms of the unstable Adams spectral sequences which converge to the homotopy groups of the spheres. We use the Λ algebra and an algorithm based on EHP sequences to calculate the unstable Adams E_2 term through stem dimension 51 (added in proof: stem 58) for each sphere S^n. After some preliminaries in §2 and §3, we describe the EHP algorithm for Λ algebra calculations in §4. In §5, we show how periodicity can be used to shorten the calculations. The computer programs are sketched in §6. Appendix A has an example of assembly code for Λ algebra manipulations. The results of the calculations of the $E_2(S^n)$ through stem 51 are given in the tables in Appendix B.

In this paper, all spaces and groups are to be localized at the prime 2. There are analogous methods for odd primes. The end of a proof is indicated by ♦.

EHP Sequences

The EHP sequences in homotopy groups of spheres are the following. For each $n \geq 1$, there is a long exact sequence (abbr: LES)

$$(1.1) \quad \ldots \to \pi_{n+q+2}(S^{2n+1}) \xrightarrow{P} \pi_{n+q}(S^n) \xrightarrow{E} \pi_{n+q+1}(S^{n+1}) \xrightarrow{H} \pi_{n+q+1}(S^{2n+1}) \xrightarrow{P} \ldots$$

The calculations of $\pi_*(S^n)$, as carried out by Barratt [B] and Toda [Tod], proceed by a double induction. To calculate the groups in the q-stem $\pi_{n+q}(S^n)$, assume (inductively on q) that the groups of the p-stem $\pi_{n+p}(S^n)$ are known for all $p < q$, and all spheres S^n. The q-stem $\pi_{n+q}(S^n)$ is then calculated (inductively on n) starting from the fact that $\pi_{1+q}(S^1) = 0$ for all $q > 0$. The suspension homomorphism E is affected by $\pi_{n+q+2}(S^{2n+1})$ and $\pi_{n+q+1}(S^{2n+1})$ as follows. The exactness of the EHP sequence implies that the elements in image(P) vanish under suspension, and that the elements in kernel(P) must be adjoined to $\pi_{n+q}(S^n)$ to obtain $\pi_{n+q+1}(S^{n+1})$. It has long been recognized that the difficulty in this approach is that of determining the homomorphism P.

The Unstable Adams Spectral Sequence

For each space X, the (stable or unstable) Adams spectral sequence is a sequence of groups $E_r^{*,*}(X)$, which approximate and with increasing r converge to the (stable or ordinary) homotopy groups of X. For X the stable sphere, Adams himself made calculations of the first few groups of the E_2 term using elementary homological algebra. These calculations were extended by Whitehead, May, Mahowald, Tangora and others; [May], [M,T], [T1]. In [C1], was given a recursive algorithm, based on EHP sequences of the UASS, for calculating the E_2 terms of the finite spheres S^n, and calculations were made in low (≤ 17) stem dimensions. This algorithm was used by Whitehead [GWW], who made pencil and paper calculations which were complete through stem 34. In [T2], [T3] and [T4], Tangora showed how to program the algorithm on a computer. His calculations agreed with, and extended those of Whitehead. More important, Tangora proved the validity of the algorithm, and introduced some simplifications into the computations, which he called shortcuts. We are indebted to Tangora for these shortcuts, as well as his insights into the difficulties that arise in these programs.

The present work uses the same algorithm, and builds on [GWW], [T2], [T3], and [T4]; much of what we have done overlaps those. We will concentrate on the changes we have made. The main difference is that we do not attempt to incorporate the entire algorithm into one program. Instead, there is one main file, some subsidiary files, and several executable programs which operate on these files. The information as it is calculated is stored in the main file (called WFILE, for working file). This file can grow to be quite large. One program (*difftag*) extracts information from WFILE in order to manufacture files G(q), one for each positive integer q, which contain the computed image(P) in the q-stem. Other programs use the G(q) to modify WFILE according to the EHP process. Another difference between our programs and those of Tangora is that our programs are semi-interactive in that we are able to intervene in the construction of the files G(q). In this way, we sometimes save the computer a lot of time.

The tables at the end contain our computations of the unstable Adams E_2 terms through stem 51 for each sphere. The stable Adams E_2 term may also be read from the tables (the untagged terms), and they agree with the tables of Mahowald and Tangora ([M,T] and [T1]).

Some programs to do Λ algebra and Steenrod algebra calculations were initially written in BASIC and in assembly language for an IBM PC; they were used to explore the interactive programs and to do calculations through stem 32; this version is available from the last named author (at Stanford University) if you send him a blank $5^1/2$" floppy disc. We have included a printout of the assembly language Λ-algebra manipulator.

The programs as they are presented here were written in C and run through stem 51 on a VAX 11-750 operating under UNIX. We thank the University of Washington, the NSF, the Digital Equipment Company, for making this computer (uw-entropy) available to us; also the systems programmer, Steve Hubert, for showing us how to use it. The programs have been also been run on a SUN Model 2, as well as an ATT microcomputer operating under UNIX.

2. The Unstable Adams E_2 Term

The E_2 term of the UASS for the sphere S^n may be calculated by homological algebra, as follows. In [6A], it was shown that $E_2^{s,t}(S^n)$ is isomorphic to the homology of a differential module $\Lambda(n)$, obtained as a submodule of the Lambda algebra Λ. Λ is (defined as) the algebra (over Z/2) with a generator λ_i for each integer $i \geq 0$, and

(2.1) relations: whenever $2i < j$,

$$\lambda_i \lambda_j = \sum_{k\geq o} C(j\text{-}2i\text{-}2\text{-}k, k)\, \lambda_{j-i-k-1} \lambda_{2i+k+1}$$

Then Λ becomes a differential algebra, with

$$(2.2) \qquad d\lambda_i = \sum_{k\geq 1} C(i\text{-}k, k)\, \lambda_{i-k} \lambda_{k-1}$$

Here $C(n,q)$ stands for the binomial coefficient reduced mod 2.

For each sequence $I = (i, j, \ldots, m)$ of non-negative integers, λ_I denotes the product $\lambda_i\lambda_j \cdots \lambda_m$. A sequence $I = (i_1, i_2, \ldots i_s)$ is called admissable if for each j, $2i_j \geq i_{j+1}$. It follows immediately from the relations that Λ has for basis (over $\mathbf{Z}/2$) the set of all λ_I, where I is admissable. Λ is bigraded by length and dimension, where

$$\text{length}(\lambda_I) = s$$

$$\dim(\lambda_I) = i_1 + i_2 + \ldots + i_s$$

For each positive integer n, $\Lambda(n)$ is defined to be the submodule of Λ spanned by those λ_I which are admissable and for which $i_1 < n$. One of the main results of [6A] is that $\Lambda(n)$ with its differential serves as an E_1 term for the UASS for spheres. That is,

$$E_2^{*,*}(S^n) = H_*(\Lambda(n))$$

$\Lambda^{s,t}(n)$ will denote the submodule of $\Lambda(n)$ spanned by admissable λ_I of length s, and dimension $t - s$.

The EHP sequence methods and the UASS methods are related by the fact that the $E_2^{*,*}(S^n)$ satisfy EHP sequences similar to those of (1.1). That is, for each n, there is a LES:

$$(2.3) \quad \ldots \to E_2^{*,*}(S^{2n+1}) \xrightarrow{E} E_2^{*,*}(S^n) \xrightarrow{H} E_2^{*,*}(S^{n+1}) \xrightarrow{P} E_2^{*,*}(S^{2n+1}) \to \ldots$$

These LES's come about as follows. For each n, there is a short exact sequence:

$$(2.4) \qquad 0 \to \Lambda(n) \xrightarrow{i} \Lambda(n+1) \xrightarrow{h} \Lambda(2n+1) \to 0$$

where i is the inclusion and h is defined on the admissable basis by

$$h(\lambda_i\lambda_j \cdots \lambda_m) = \begin{cases} \lambda_j \cdots \lambda_m, & \text{if } i = n \\ 0, & \text{if } i < n \end{cases}$$

The EHP sequence (2.3) for the unstable E_2 terms is the LES in homology of this short exact sequence (2.4) of differential modules. Thus the homomorphism P arises from the differential d in Λ. The EHP process for calculating the unstable E_2 terms is more tractable than the EHP process for the homotopy groups of spheres because the homorphism P for the unstable Adams E_2 terms, while still difficult, is more easily computable than the homomorphism P in homotopy. In what follows, we shall show how this can be done.

3. Notation and Conventions

Before presenting the algorithm for calculating $E_2^{*,*}(S^n)$, we will describe some techniques from [C1] and [T4] that have been useful.

Ordering

The monomials λ_I of each fixed bidegree (s, t) in Λ are ordered, lexicographically from the left. This induces a total order on each of the vector spaces $\Lambda^{s,t}$, by first expressing each polynomial as a sum of admissable monomials in decreasing order, and then comparing two such polynomials lexicographically. For a sum of admissable monomials, the term which is largest in the lexicographic order will be called the leading term. In a given homology class, the polynomial in the class which is least in the total order will be called the minimal representative.

If λ_I is the leading term of a minimal representative of some (nonzero) homology class, let c(I) stand for the minimal polynomial which is a cycle and which has λ_I for leading term. We seek a basis of each $E_2^{s,t}(S^n)$ consisting of such basis elements c(I), represented by their leading terms.

Odd Endings

Let Λ_0 be the submodule of Λ generated by all admissable monomials λ_I which end with an odd index. Then Λ_0 is closed under the differential, and the inclusion $\Lambda_0 \to \Lambda$ induces an isomorphism in homology except in stem(0); the tower $\{\lambda_0{}^k\}$ is not present in $H_*(\Lambda_0)$. The inclusions $\Lambda_0(n) \to \Lambda(n)$ also induce isomorphisms except on the towers (which occur in stem(0) and, if n is even, in stem(n-1)). Furthermore, the $\Lambda_0(n)$ satisfy a modified EHP property as follows. For each n, there is a short exact sequence:

$$(3.1) \qquad 0 \to \Lambda_0(n) \xrightarrow{i} \Lambda_0(n+1) \xrightarrow{h} \Lambda_0{}^{\#}(2n+1) \to 0$$

where the unit is included in $\Lambda_0{}^{\#}(2n+1)$ when n is odd, but not included in $\Lambda_0{}^{\#}(2n+1)$ when n is even . Thus all the λ_{2k+1} are present in filtration one of Λ_0, but none of the λ_{2k}. The result of this is that if we may restrict attention to the λ_I with odd endings. With this convention, the spectral sequences converge to the finite summands of the 2-primary components of $\pi_*(S^n)$.

Notation

Henceforth for convenience of notation, Λ will stand for Λ_0, that is, the submodule of (what was previously called) Λ spanned by admissable λ_I with odd endings. The initial of a sequence I is its first index. $\Lambda(n)$ will denote the submodule of Λ spanned by admissable λ_I with odd endings and initial $i_1 < n$.

4.The EHP Process for Λ

We next describe the EHP process for finding a basis of each $E_2^{s,t}(S^n)$. Assume inductively that such a basis has been found in all dimensions $< q$ for all spheres, and also in dimension q for spheres S^m, where m is less than n. To obtain the basis for $E_2^{s,t}(S^n)$, for dimension q, for each filtration $s > 0$, take $t = s + q$, and we must

(i) delete a basis for the image of $P : E_2^{s-2,t-n+1}(S^{2n+1}) \to E_2^{s,t}(S^n)$

(ii) adjoin a basis for the kernel of $P : E_2^{s,t}(S^{2n+1}) \to E_2^{s+1,t}(S^n)$

In our situation, we obtain a first quadrant table $\{T^{*,*}\}$. At each integer lattice point $(t - s, s)$, $T^{s,t}$ will consists of a list of elements called rows. Each row is either a sequence:

$$(i_1, i_2, \ldots, i_s)$$

or a pair of sequences:

$$(i_1, i_2, \ldots, i_s) \leftarrow (j_1, j_2, \ldots, j_{s-1})$$

Here, $\dim(I) = t - s$, and if J is present, $\dim(J) = \dim(I) + 1$. If I appears in the latter form as $I \leftarrow J$, then I is said to be tagged by J. Each I that appears is the leading term of a cycle. As above, let c(I) be the minimal cycle in $\Lambda(i_1+1)$ which has λ_I for leading term, and without ambiguity, let c(I) also stand for the homology class in $H_*(\Lambda(i_1+1))$. The notation $I \leftarrow J$ means that

$$(i_1, i_2, \ldots, i_s) + \text{lower terms} = d\,(j_1, j_2, \ldots, j_{s-1} + \text{lower terms})$$

(The complications arising from the lower terms that may occur on the right hand side is discussed in [T4].)

Constructing the Table

The table $T^{*,*}$ starts out empty, changes at each stage, and when completed, gives a basis for the $E_2^{*,*}(S^n)$ as described by the theorem above. First, each of the odd-indexed lambdas is placed in the table. That is, (2k+1) is placed in $T^{1,2k+2}$; the rest of the table starts out empty. Assume inductively that the table has been made correct, i. e., gives a basis for $E_2^{*,*}(S^n)$, for all spheres S^n in all stems less than q, and for all filtrations s. For each $K = (k_1, \ldots)$ which is in the table in stem (q-1), adjoin to the table all (m, K), where subject to the restrictions that:

(4.1a) If K is untagged, then $2m \geq k_1$

(4.1b) If K is tagged by $N = (n_1, \ldots)$, then $2m \geq k_1$ and $2m < n_1$

If K is in $T^{s,t}$ (where $t - s = q - 1$), the term (m, K) will be placed in $T^{s+1, t+m+1}$. Next consider, in increasing order, each J which is in the table in stem(q+1); suppose J is in $T^{s,t}$, where $t - s = q+1$. We must compute P(J) in $T^{s+1,t}$.

The LTO algorithm.

This algorithm computes P(J). This was implicit in [C1], and was described and proven correct in [T4]. In order to make the present treatment as self-contained as possible, we sketch here the algorithm in the form that we need it. Beause it depends on keeping track of only the leading terms of polynomials in Λ, we follow Tangora in calling it the Leading Term Only (LTO) algorithm.

(1) Calculate d(J) as a sum of terms, each in admissable form; find the leading term of the sum (call it I).

(2a) If I is present in $T^{s+1,t}$ and is not yet untagged, then J tags I; replace I by $I \leftarrow J$ and delete J from $T^{s,t}$

(2b) If I is present in $T^{s+1,t}$ and is already tagged, say $I \leftarrow K$, add d(K) to d(J); reduce mod 2; return to step(1) and continue.

(2c) If I is not present in the table, then the LTO algorithm asserts that some tail of I must be in the table and is tagged; find the shortest tail of I that is tagged, say

$$(i_p, \ldots, i_{s+1}) \leftarrow (m_p, \ldots, m_s)$$

Then take $K = (i_1, \ldots, i_{p-1}, m_p, \ldots, m_s)$; add d(K) to d(J); reduce mod 2; return to step (1) and continue. The LTO algorithm assertion is that eventually, either

$$J + \text{lower terms} \qquad \text{is a cycle}$$

or

$$d(J + \text{lower terms}) = I + \text{lower terms}$$

where I is present in $T^{s+1,t}$. In the first case, we say that J completes to a cycle. In the latter case, J tags I; replace I by $I \leftarrow J$ in $T^{s+1,t}$, and delete J from $T^{s,t}$. When this has been done for all J in $T^{s,t}$, the tags in box $T^{s+1,t}$ are correct, and go on to the next higher filtration. When this has been done for all filtrations s (there are only a finite number, by the vanishing theorm), the table $T^{*,*}$ has been made correct in stem(q), and we go on to the next stem. This completes the inductive step of the LTO algorithm.

Theorem A basis for $E_2^{s,t}(S^n)$ consists of all c(I), where I is in $T^{s,t}$ with initial $i_1 < n$, which are either untagged or which have a tag J with initial $j_1 \geq n$.

The proof of this is in [T4]. ♦

5. Periodicity

There are two types of periodicity that shorten the calculations. The first is horizontal periodicity of bidegree $(2^k, 0)$ in the (t - s, s) plane, analogous to James periodicity for truncated projective spaces. The other is of bidegree (8, 4) along the upper edge, and is a version of Adams periodicity for the unstable Adams E_2 terms.

Horizontal Periodicity.

Suppose that $I = (i_1, i_2, \ldots, i_s)$ is tagged by $J = (j_1\, j_2, \ldots, j_{s-1})$. Let n be the least power of 2 which is greater than the difference of initials $j_1 - i_1$. Let $I^* = (i_1 + n, i_2, \ldots, i_s)$ and $J^* = (j_1 + n, j_2, \ldots, j_{s-1})$. The assertion is that if I^* is a cycle that is not tagged by some term less than J^*, then I^* will be tagged by J^*.

At present we cannot prove the full strength of this periodicity assertion, but we want to use it anyway. For this purpose, we define two integers index and flag as follows. For each admissable sequence $K = (k_1, k_2, \ldots k_s)$, let

$$\begin{aligned} \text{index}(K) &= k_2 - k_1 - 1, && \text{if } 2k_1 < k_2 \\ &= 0 && \text{otherwise} \end{aligned}$$

Suppose that I is tagged by J, with $\Sigma_\alpha I_\alpha = d(\Sigma_\beta J_\beta)$. Then let flag($I \leftarrow J$) be the maximum of index(K), where K appears in any relation that is used to express d($\Sigma_\beta J_\beta$) as a linear combination of admisssable monomials. That is flag(I ←J) is the largest initial that is affected by the relations in the first position.

Lemma Suppose that $I = (i_1, i_2, \ldots, i_s)$ is tagged by $J = (j_1, j_2, \ldots, j_{s-1})$, and suppose that flag(I ←J) < i_1. Let $n = 2^k$ be the least power of 2 for which $2^k > j_1 - i_1$ and take $I^* = (i_1 + n, i_2, \ldots, i_s)$ and $J^* = (j_1 + n, j_2, \ldots, j_{s-1})$. Suppose also that I^* and J^* both appear in the table; and that I^* is not tagged by some term earlier than J^*. Then J^* will tag I^*.

Proof. Let $M = H_*(RP^\infty)$, as a module with the Steenrod algebra acting on the right. As a vector space, M has a generator e_n in each positive dimension n. Consider the chain complex $M \otimes \Lambda$. with differential

$$d(x \otimes \lambda_I) = \Sigma_j (xSq^j) \otimes \lambda_{j-1}\lambda_I + x \otimes d(\lambda_I)$$

For any sequence $I = (i_1, i_2, \ldots i_s)$, let PI stand for $e_{i1} \otimes (i_2, \ldots i_s)$. The map $M \otimes \Lambda \to \Lambda$ which sends PI to I is a map of chain complexes because of the formulas for $d(e_j)$ and $d(\lambda_j)$. For each positive integer m, let $M(m, \infty) = H_*(RP_m{}^\infty)$. The assumption that flag(I ← J) < n implies that $\Sigma_\alpha PI_\alpha = d(\Sigma_\beta PJ_\beta)$ in $M(i_1,\infty) \otimes \Lambda$. James Periodicity for truncated projective spaces implies that $\Sigma_\alpha PI_\alpha{}^* = d(\Sigma_\beta PJ_\beta{}^*)$ in $M(i_1 + n,\infty) \otimes \Lambda$. This shows that $\Sigma_\alpha I_\alpha{}^* = d(\Sigma_\beta J_\beta{}^*)$ in $\Lambda/\Lambda(i_1)$. Hence I^* will be tagged by J^* in Λ. ♦

We use this flag(I ← J) to check validity of the periodicity assertion each time we want to use it. While the program *difftag* is calculating that I is tagged by J, we have *difftag* keep track of this integer flag (I ← J). If flag(I ←J) is smaller than the initial of I, the term with tag $I^* \leftarrow J^*$ is placed in a file called STORE, for use at a later time.

In the simplest version of the program (below), the program *kill* does not make use of this horizontal periodicity. A faster version (also below) takes account of and stores the valid cases of horizontal periodicity, as checked by *difftag* . For this we use two more programs *postpone* , and *perkill* which take account of the (validated) horizontal periodicities of period 2, 4, 8, 16, in increasing order. We have observed that in most cases, *difftag* calculates that flag(I ←J) is less than the initial of I, so that the periodicity is valid; furthermore, we have found no instances (through stem 51), where the strong form of the horizontal periodicity assertion is not valid.

Adams Periodicity.

Along the vanishing line of slope 1/2 , there is a recurrent pattern of period (8, 4). The elements

(4 1 1 2 4 1 1 . . .)
(5 1 2 4 1 1 1)
(6 2 4 1 1 . . .)
(8 4 1 1 . . .)
(12, 1 1 . . .)
(13, 1 . . .)

together with similar previous elements, generate by the EHP process, an almost closed portion of the table. That is, each of the elements has for Hopf Invariant another member of the pattern. The stable survivors in this pattern are given in the following table.

							241124...
					41124...		1234 ...
			1124...		5124...	234...	
		124...	224...	344...	624...		
	24 ...	44...			8...		
4...							
5...							
6...							
8...							

Recurrent Pattern (6.1)

Adams periodicity for the unstable E_2 terms would assert that this pattern, including also the classes that are tagged (suppressed from the table for lack of space), repeats with period (8, 4) along the upper edge. We do not make use of Adams periodicity directly. Rather, in making the calculations near the edge we find that there is no interference from below, and that this pattern results. In making these calculations, we make repeated use of the observations of [T4; §3.9], in particular,

(i) If no untagged cycle at the target is smaller than the leading term of d(I), then I must complete to a cycle.

(ii) If K is a cycle, and i = 1, 3, 7, or 15, then (i, K) is also a cycle.

(iii) (2i-1, j-1, 1, K) is tagged by (2i, j, K), provided that both monomials are admissable, and present in the table.

<u>Use of Relations</u>

The differentials emanating from elements in the verticals in dimensions 4k - 1 can be difficult to compute directly. In these cases, we use the shortcut advocated by [T4; §3.9.7]. For example,

$$d(14, 2\,4\,1\,1\,2\,4\,1\,1\,2\,4\,1\,1\,1) = 2........2\,4\,5\,3\,3\,3$$

where the dots represent 2's. This differential is obtained from:

$$d(39) = 31, 7$$

and repeated multiplications on the left by 0 (i.e., by λ_0). Information of this sort is calculated by a program called *relation* (which is very similar to *difftag*), and is then put into the files by hand.

6. The Programs

The programs manufacture a file called WFILE. This file consists of a list of rows. Each row is either a sequence of integers (called a term):

$$(i_1, i_2, \ldots, i_s)$$

or a pair of such sequences (a term with a tag):

$$(i_1, i_2, \ldots, i_s) \leftarrow (j_1, j_2, \ldots, j_{s-1})$$

WFILE will be built up in stages as follows. A maximum stem dimension N is chosen, and all terms constructed are to have dimension $\leq$ N. We start by placing in WFILE all odd positive integers $i \leq N$; each is a term (consisting of a single integer) on a separate row. Then having completed q-1 stages, the q^{th} stage takes four steps, as follows.

(1) Find all the terms I of dimension q that are to be tagged by some J of dimension q+1, and place these $I \leftarrow J$ in a file called G(q). For this step a program *difftag* calculates for each J, the leading term of d(J).

(2) A program *kill* uses G(q) to tag terms in WFILE. For each occurrence of $I \leftarrow J$ in G(q), it looks for I and J in WFILE, and if it finds them both untagged, it tags I by J and deletes J from WFILE.

(3) A program *showstem* searches WFILE for all terms that have dimension q, and places them in a file called S(q) .

(4) A program *loadstem* uses the terms of S(q) to make new rows according to (4.1), and appends these rows to WFILE.

Once the files G(q) are known through some dimension N, the file WFILE, which will contain the computed E_2 terms to stem N, is made by a shell script. This is a sort of master program which calls on the executable programs *kill, showstem,* and *loadstem* to perform steps (2), (3), (4) for each integer q from 1 to N. This is very straightforward, and no more will be said.

The program *difftag* is more complicated, so we give a sketch of it. Using the differential (2.2), *difftag* first computes d(J) as a linear combination of I's (possibly inadmissable), and places them in a list of rows called LIST; each row is assigned coefficient 1. This LIST is traversed sequentially from the beginning, and each I is tested for inadmissability from the left. If I is inadmissable at position j, then the relations (2.1) are used to express I as a linear combination of K's which are admissable at position j, and these K's are appended to LIST, each with coefficient 1; the coefficient of I is set to 0. Continue (just once) through the list until all rows are admissable (which must occur after a finite number of steps). LIST is next searched sequentially, keeping track of (by a pointer) the largest row I with non-zero coefficient. Initially, I is taken as the first row; if a larger row is encountered, the pointer is changed to point to this one. Each later occurrence of the same I is assigned coefficient 0 and the coefficient of the first occurrence is incremented. This coefficient is now reduced mod 2; if the coefficient becomes 1, I is the leading term, and the search terminates. If the coefficient is 0, the list is searched again for the largest I. If all coefficients become 0, J is a cycle, and the program exits. Otherwise, WFILE is searched for I. If I is found untagged, then J tags I, and the program terminates. If I $\leftarrow$ K is found, then d(K) is appended to LIST; again LIST is searched for the largest term, and the process continues as before. If I is not present, then WFILE is searched for shorter and shorter tails of I until finally some tail of I is found that is tagged, say

$$(i_p, \ldots, i_{s+1}) \leftarrow (m_p, \ldots, m_s)$$

Then K is taken to be $(i_1, \ldots, i_{p-1}, m_p, \ldots, m_s)$ and d(K) is appended to LIST, which is again searched for the largest term, and the process continues as before. ♦

Modifications due to Periodicity

If the programs were run as above, *difftag* would spend a lot of time on terms whose outcome is (correctly) predicted by periodicity. Therefore, we modify *difftag* as follows. The program keeps track (by an integer flag), of the largest initial of a term which is affected by a relation in the first position. If this flag is less than i_1, then $I^* \leftarrow J^*$ is placed in a file called STORE for later use. The valid cases of periodicity are incorporated into the programs as follows. Step (2) is replaced by two steps:

(2a)The program *kill* uses G(q) to tag terms in WFILE as before.

(2b)A program *postpone* uses G(q), to create a file STORE. For each occurrence of I←J in G(q), flagged as valid under horizontal periodicity, *postpone* places in STORE all $I^* \leftarrow J^*$ obtained by increasing the initials by the appropriate power of 2, and multiples thereof.

(2c) A program *perkill* uses G(q) and STORE to tag terms of dimension q in WFILE. For each occurrence of I ← J (in increasing order from either G(q) or STORE), *perkill* looks for I and J in WFILE, and if it finds them both untagged, it tags I by J and deletes J from WFILE.

Steps (3) and (4) proceed as before. ♦

Appendix A: 8086 Assembly code

We also include a printout of the 8086 assembly code for a Λ algebra manipulator, which can be used to put each monomial into admissable form. (Note: in moving this code to other CPU's, beware that they may handle flags differently).

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;the two terms being fixed are stored in lambda2 and lambda1
; the routine expands these two into admissable form.

lamex0a:  mov   al,lambda2              ;the two terms to put into
          mov   ah,lambda1              ;admissable form
          shl   ah,1
          inc   ah
          sub   al,ah                   ;n = lambda2 - 2(lambda1) -1
          mov   variable_j,al
          jc    lamex1                  ;exit
          jz    lamex1
          mov   first_coefficient,ah    ;store it
          mov   dl,lambda1
          add   dl,al                   ;n+i in dl
          mov   second_coefficient,dl
          mov   constant_n,al

lamex2a:  test  constant_n,0ffh         ;Beginning of the loop
          jz    lamex1
          dec   constant_n
```

```
lamex2:     mov     ah,constant_n                      ;put the next terms in the
            mov     al,variable_j                      ;expansion into al,ah
            sub     ah,al                              ;do we continue?
            jc      lamex3
            call    binomial                           ;check  binomial coefficient

lamex3a:    call    load_buffer_with_coefficient       ;if non-zero, store in buffer

lamex3:     dec     variable_j                         ;continue processing
            test    variablej,80h                      ;check the sign
            jz      lamex2                             ;if non_negative continue

lamex1:     ret                                        ;otherwise exit

;the next routine gets the binomial coefficient. Assume that ah=n, al= m
;computes (n,m) mod(2). Returns z set if result is zero, zn set otherwise

binomial:   push    ax
            and     ah,al
            sub     ah,al
            jz      bin1
            sub     ah,ah
            pop     ax
            ret
bin1:       cmp     ah,1
            pop
            ret
```

Appendix B The Tables

Using these programs, we have calculated the unstable Adams E_2 terms for all spheres S^n, and all stems through stem 51. To save space, we have used a compressed notation as follows. The symbol * stands for the sequence 2 4 1 1; a sequence of dots with 2's at each end stands for repeated 2's, where each dot substitutes for a missing 2. Certain subsequences occur so often that it is convenient to write them in compressed form: 6653 for 6 6 5 3; 24333 for 2 4 3 3 3; 45333 for 4 5 3 3 3; 35733 for 3 5 7 3 3; and 59777 for 5 9 7 7 7.

The E_2 term for the UASS for each sphere may be read from the tables as follows. A basis for $E_2^{s,t}(S^n)$ is in one-one correspondence with the terms I which are in $T^{s,t}$ with initial less than n, and which are either untagged or are tagged by J with initial greater than or equal to n.

TABLE of $E_2{}^{s,t}(S^n)$

5						1 2 1 1 1 ← 2 3 1 1
4				1 1 1 1 ← 2 2 1 2 1 1 ← 4 1	2 1 1 1 ← 4 1 1 1 2 1 1 ← 2 3 1	3 1 1 1 ← 4 2 1
3			1 1 1	1 2 1 ← 2 3	3 1 1 ← 5 1	3 2 1 ← 4 3
2		1 1	2 1	3 1 ← 5		3 3
1	1		3			

Stems 1 - 6

6				1 2 4 1 1 1
5			2 4 1 1 1	3 4 1 1 1 ← 4 5 1 1 1 1 2 3 3 ← 8 1 1 1
4	4 1 1 1	5 1 1 1 ← 6 2 1	6 1 1 1 ← 8 1 1 1 2 3 3	7 1 1 1 ← 8 2 1 3 5 1 1 ← 4 6 1 2 2 3 3 ← 9 1 1
3	5 1 1	6 1 1 ← 8 1 5 2 1 ← 6 3 2 3 3	7 1 1 ← 9 1 3 3 3	7 2 1 ← 8 3 4 3 3 ← 10 1 3 6 1 ← 4 7
2	6 1	7 1 ← 9 5 3		7 3 ← 11
1	7			

Stems 7 - 10

8		1 1 1 * 1 ← 2 2 * 1	2 1 1 * 1←4 1 * 1 1 2 1 * 1←2 3 * 1
7	1 1 * 1	2 1 * 1 ← 4 * 1 1 2 * 1 ← 2 4 4 1 1 1	3 1 * 1←5 * 1 1 2 1 1 2 3 3←2 3 1 2 3 3
6	2 * 1 1 1 1 2 3 3 ← 2 2 2 3 3	3 * 1 ← 6 4 1 1 1 2 1 1 2 3 3 ← 4 1 2 3 3 1 2 1 2 3 3 ← 2 3 2 3 3	3 1 1 2 3 3←4 2 2 3 3
5	4 4 1 1 1 2 1 2 3 3 ← 4 2 3 3 1 2 2 3 3 ← 2 4 3 3	5 4 1 1 1 ← 6 5 1 1 3 6 1 1 1 ← 4 7 1 1 3 1 2 3 3 ← 5 2 3 3 1 2 3 3 3 ← 2 3 5 3	3 2 2 3 3←4 4 3 3
4	3 6 1 1 ← 4 7 1 3 2 3 3 ← 6 3 3 2 3 3 3 ← 4 5 3	9 1 1 1 ← 10 2 1 5 5 1 1 ← 6 6 1 3 3 3 3 ← 5 5 3	10 1 1 1←12 1 1 4 3 3 3←8 3 3 3 4 3 3←4 7 3
3	5 3 3 ← 9 3 3 5 3 ← 5 7	10 1 1 ← 12 1 9 2 1 ← 10 3 5 6 1 ← 6 7	11 1 1←13 1 7 3 3←11 3
2		11 1 ← 13	
1			

Stem 11 Stem 12 Stem 13

9	1 2 1 1 * 1←2 3 1 * 1	
8	3 1 1 * 1←4 2 * 1	4 1 1 * 1
7	3 2 * 1←4 4 4 1 1 1	5 1 * 1
6	3 4 4 1 1 1	6 * 1 1 2 3 3 3 3←2 3 5 3 3
5	7 4 1 1 1←8 5 1 1 5 1 2 3 3 2 3 3 3 3←4 5 3 3	8 4 1 1 1 6 1 2 3 3←8 2 3 3 24333
4	11 1 1 1←12 2 1 7 5 1 1←8 6 1 6 2 3 3 5 3 3 3←9 3 3 3 5 3 3←5 7 3	12 1 1 1 7 2 3 3←10 3 3 6 3 3 3←8 5 3
3	11 2 1←12 3 7 6 1←8 7 6 5 3	13 1 1 7 5 3←9 7
2	7 7	14 1
1		15

Stem 14 Stem 15

9		2 4 1 1 * 1
8	5 1 1 * 1←6 2 * 1	6 1 1 * 1←8 1 * 1 1 2 3 4 4 1 1 1
7	6 1 * 1←8 * 1 5 2 * 1←6 4 4 1 1 1 2 3 4 4 1 1 1	7 1 * 1←9 * 1 3 3 4 4 1 1 1←4 5 1 2 3 3 1 1 24333
6	7 * 1←10 4 1 1 1 5 4 4 1 1 1←8 1 2 3 3 1 24333	3 5 1 2 3 3←4 6 2 3 3 2 24333
5	9 4 1 1 1←10 5 1 1 7 1 2 3 3←9 2 3 3 3 4 3 3 3←4 7 3 3	3 6 2 3 3 3 5 3 3 3←4 6 5 3
4	13 1 1 1←14 2 1 9 5 1 1←10 6 1 7 3 3 3←9 5 3	14 1 1 1←16 1 1 8 3 3 3 5 10 1 1←6 11 1 3 6 5 3←4 7 7
3	14 1 1←16 1 13 2 1←14 3 9 6 1←10 7	15 1 1←17 1 11 3 3
2	15 1←17 13 3	
1		

Stem 16 Stem 17

11		1 1 * * 1
10	1 * * 1	2 * * 1 1 1 1 2 3 4 4 1 1 1←2 2 2 3 4 4 1 1 1
9	3 4 1 1 * 1←4 5 1 * 1 1 1 2 3 4 4 1 1 1←8 1 1 * 1	4 4 1 1 * 1 2 1 2 3 4 4 1 1 1←4 2 3 4 4 1 1 1 1 2 2 3 4 4 1 1 1←2 4 3 4 4 1 1 1
8	7 1 1 * 1←8 2 * 1 3 5 1 * 1←4 6 * 1 2 2 3 4 4 1 1 1←9 1 * 1 1 1 1 24333←2 2 24333	3 6 1 * 1←4 7 * 1 3 2 3 4 4 1 1 1←6 3 4 4 1 1 1 2 1 1 24333←4 1 24333 1 2 1 24333←2 3 24333
7	7 2 * 1←8 4 4 1 1 1 4 3 4 4 1 1 1←10 * 1 3 6 * 1←4 8 4 1 1 1 2 1 24333←4 24333 1 2 24333←2 3 6 2 3 3	5 3 4 4 1 1 1←6 5 1 2 3 3 3 5 4 4 1 1 1←4 7 1 2 3 3 3 1 24333←5 24333
6	7 4 4 1 1 1←12 4 1 1 1 3 6 1 2 3 3←4 7 2 3 3 3 24333←5 6 2 3 3	5 5 1 2 3 3←6 6 2 3 3
5	11 4 1 1 1←12 5 1 1 9 1 2 3 3←16 1 1 1 5 10 1 1 1←6 11 1 1 4 5 3 3 3 3 6 3 3 3←4 7 5 3	10 1 2 3 3←12 2 3 3
4	15 1 1 1←16 2 1 11 5 1 1←12 6 1 10 2 3 3←17 1 1 9 3 3 3 5 7 3 3	11 2 3 3←14 3 3 10 3 3 3←12 5 3 5 6 5 3←6 7 7
3	15 2 1←16 3 12 3 3←18 1 11 6 1←12 7 10 5 3	13 3 3←17 3 11 5 3←13 7 5 7 7
2	15 3←19 11 7	
1		
	Stem 18	Stem 19

12	1 1 1 * * 1 ← 2 2 * * 1	2 1 1 * * 1 ← 4 1 * * 1 1 2 1 * * 1 ← 2 3 * * 1
11	2 1 * * 1 ← 4 * * 1 1 2 * * 1 ← 2 4 4 1 1 * 1	3 1 * * 1 ← 5 * * 1 1 2 1 1 2 3 4 4 1 1 1 ← 2 3 1 2 3 4 4 1 1 1
10	3 * * 1 ← 6 4 1 1 * 1 2 1 1 2 3 4 4 1 1 1 ← 4 1 2 3 4 4 1 1 1 1 2 1 2 3 4 4 1 1 1 ← 2 3 2 3 4 4 1 1 1	3 1 1 2 3 4 4 1 1 1 ← 4 2 2 3 4 4 1 1 1
9	5 4 1 1 * 1 ← 6 5 1 * 1 3 6 1 1 * 1 ← 4 7 1 * 1 3 1 2 3 4 4 1 1 1 ← 5 2 3 4 4 1 1 1 1 2 1 1 24333 ← 2 3 1 24333	3 2 2 3 4 4 1 1 1 ← 4 4 3 4 4 1 1 1
8	9 1 1 * 1 ← 10 2 * 1 5 5 1 * 1 ← 6 6 * 1 3 1 1 24333 ← 4 2 24333	10 1 1 * 1 ← 12 1 * 1 4 1 1 24333 ← 8 3 4 4 1 1 1 3 4 3 4 4 1 1 1 ← 4 7 4 4 1 1 1
7	10 1 * 1 ← 12 * 1 9 2 * 1 ← 10 4 4 1 1 1 5 6 * 1 ← 6 8 4 1 1 1 3 2 24333 ← 4 3 6 2 3 3	11 1 * 1 ← 13 * 1 7 3 4 4 1 1 1 ← 8 5 1 2 3 3 5 1 24333 ← 11 4 4 1 1 1 1 2 45333 ← 2 4 7 3 3 3
6	11 * 1 ← 14 4 1 1 1 9 4 4 1 1 1 ← 12 1 2 3 3 5 8 4 1 1 1 ← 6 12 1 1 1 3 3 6 2 3 3 ← 4 8 3 3 3 2 45333	7 5 1 2 3 3 ← 8 6 2 3 3 6 24333 ← 13 1 2 3 3 3 45333 ← 4 5 7 3 3
5	13 4 1 1 1 ← 14 5 1 1 11 1 2 3 3 ← 13 2 3 3 4 7 3 3 3	7 6 2 3 3 ← 14 2 3 3 3 5 7 3 3
4	17 1 1 1 ← 18 2 1 13 5 1 1 ← 14 6 1 11 3 3 3 ← 13 5 3 6 6 5 3	18 1 1 1 ← 20 1 1 12 3 3 3 ← 16 3 3 7 6 5 3 ← 8 7 7
3	18 1 1 ← 20 1 17 2 1 ← 18 3 13 6 1 ← 14 7	19 1 1 ← 21 1 15 3 3 ← 19 3 7 7 7
2	19 1 ← 21	
1		
	Stem 20	Stem 21

13	1 2 1 1 * * 1 ← 2 3 1 * * 1	
12	3 1 1 * * 1 ← 4 2 * * 1	4 1 1 * * 1
11	3 2 * * 1 ← 4 4 4 1 1 * 1	5 1 * * 1
10	3 4 4 1 1 * 1	6 * * 1
9	7 4 1 1 * 1 ← 8 5 1 * 1 5 1 2 3 4 4 1 1 1	8 4 1 1 * 1 6 1 2 3 4 4 1 1 1 ← 8 2 3 4 4 1 1 1 2 4 1 1 24333
8	11 1 1 * 1 ← 12 2 * 1 7 5 1 * 1 ← 8 6 * 1 6 2 3 4 4 1 1 1 5 1 1 24333 ← 6 2 24333	12 1 1 * 1 7 2 3 4 4 1 1 1 ← 10 3 4 4 1 1 1 6 1 1 24333 ← 8 1 24333 1 2 2 45333 ← 2 4 45333
7	11 2 * 1 ← 12 4 4 1 1 1 7 6 * 1 ← 8 8 4 1 1 1 6 1 24333 ← 8 24333 5 2 24333 ← 6 3 6 2 3 3 2 2 45333 ← 14 * 1	13 1 * 1 9 3 4 4 1 1 1 ← 10 5 1 2 3 3 7 1 24333 ← 9 24333 5 7 4 4 1 1 1 ← 6 9 1 2 3 3 3 2 45333 ← 4 4 7 3 3 3
6	7 8 4 1 1 1 ← 8 12 1 1 1 7 24333 ← 9 6 2 3 3 5 3 6 2 3 3 ← 6 8 3 3 3 4 45333 ← 16 4 1 1 1	9 5 1 2 3 3 ← 10 6 2 3 3 5 9 1 2 3 3 ← 6 10 2 3 3 5 45333 ← 6 5 7 3 3 3 4 7 3 3 3 ← 4 6653 2 35733
5	15 4 1 1 1 ← 16 5 1 1 7 12 1 1 1 ← 8 13 1 1 5 8 3 3 3 ← 20 1 1 1	14 1 2 3 3 ← 16 2 3 3 5 10 2 3 3 ← 6 12 3 3 5 9 3 3 3 ← 6 10 5 3 5 5 7 3 3 ← 10 6 5 3 3 6653
4	19 1 1 1 ← 20 2 1 15 5 1 1 ← 16 6 1 13 3 3 3 ← 17 3 3 8 6 5 3 ← 21 1 1 7 13 1 1 ← 8 14 1 3 5 7 7	15 2 3 3 ← 18 3 3 14 3 3 3 ← 16 5 3 9 6 5 3 ← 10 7 7 7 14 1 1 ← 8 15 1 6 11 3 3 ← 8 13 3 5 10 5 3 ← 6 11 7 4 5 7 7
3	19 2 1 ← 20 3 15 6 1 ← 16 7 14 5 3 ← 22 1 7 14 1 ← 8 15	15 5 3 ← 17 7 9 7 7 ← 21 3 7 13 3 ← 9 15
2	15 7 ← 23	
1		
	Stem 22	Stem 23

13		2 4 1 1 * * * 1
12	5 1 1 * * 1 ← 6 2 * * 1	6 1 1 * * 1 ← 8 1 * * 1 1 2 3 4 4 1 1 * 1
11	6 1 * * 1 ← 8 * * 1 5 2 * * 1 ← 6 4 4 1 1 * 1 2 3 4 4 1 1 * 1	7 1 * * 1 ← 9 * * 1 3 3 4 4 1 1 * 1 ← 4 5 1 2 3 4 4 1 1 1 1 1 * 24333
10	7 * * 1 ← 10 4 1 1 * 1 5 4 4 1 1 * 1 ← 8 1 2 3 4 4 1 1 1 1 * 24333	3 5 1 2 3 4 4 1 1 1 ← 4 6 2 3 4 4 1 1 1 2 * 24333
9	9 4 1 1 * 1 ← 10 5 1 * 1 7 1 2 3 4 4 1 1 1 ← 9 2 3 4 4 1 1 1 3 4 1 1 24333 ← 4 5 1 24333	3 6 2 3 4 4 1 1 1 1 2 2 2 45333 ← 2 4 2 45333
8	13 1 1 * 1 ← 14 2 * 1 9 5 1 * 1 ← 10 6 * 1 7 1 1 24333 ← 8 2 24333 3 5 1 24333 ← 4 6 24333 2 2 2 45333 ← 9 1 24333	14 1 1 * 1 ← 16 1 * 1 8 1 1 24333 5 10 1 * 1 ← 6 11 * 1 3 6 1 24333 ← 4 7 24333 3 2 2 45333 ← 4 4 45333
7	14 1 * 1 ← 16 * 1 13 2 * 1 ← 14 4 4 1 1 1 9 6 * 1 ← 10 8 4 1 1 1 7 2 24333 ← 8 3 6 2 3 3 4 2 45333 ← 10 24333 3 6 24333 ← 4 7 6 2 3 3 1 2 35733 ← 2 3 6653	15 1 * 1 ← 17 * 1 11 3 4 4 1 1 1 ← 12 5 1 2 3 3 5 9 4 4 1 1 1 ← 6 11 1 2 3 3 5 2 45333 ← 6 4 7 3 3 3 3 4 45333 ← 4 5 8 3 3 3 2 2 35733 ← 8 45333
6	15 * 1 ← 18 4 1 1 1 13 4 4 1 1 1 ← 16 1 2 3 3 9 8 4 1 1 1 ← 10 12 1 1 1 7 3 6 2 3 3 ← 8 8 3 3 3 6 45333 ← 11 6 2 3 3 5 10 1 2 3 3 ← 6 11 2 3 3 3 35733 ← 5 6653	11 5 1 2 3 3 ← 12 6 2 3 3 7 9 1 2 3 3 ← 8 10 2 3 3 7 45333 ← 8 5 7 3 3 5 4 7 3 3 3 ← 6 6653 4 35733 ← 9 8 3 3 3 3 5 8 3 3 3 ← 4 8 6 5 3 1 2 3 5 7 7 ← 8 9 3 3 3
5	17 4 1 1 1 ← 18 5 1 1 15 1 2 3 3 ← 17 2 3 3 9 12 1 1 1 ← 10 13 1 1 7 14 1 1 1 ← 8 15 1 1 7 8 3 3 3 ← 16 3 3 3 6 9 3 3 3 ← 8 11 3 3 5 10 3 3 3 ← 6 11 5 3 2 3 5 7 7	7 10 2 3 3 ← 8 12 3 3 7 9 3 3 3 ← 8 10 5 3 7 5 7 3 3 ← 12 6 5 3 3 3 5 7 7 ← 17 3 3 3 2 4 5 7 7 ← 9 11 3 3
4	21 1 1 1 ← 22 2 1 17 5 1 1 ← 18 6 1 15 3 3 3 ← 17 5 3 9 13 1 1 ← 10 14 1 7 11 3 3 ← 9 13 3 5 5 7 7 ← 19 3 3	22 1 1 1 ← 24 1 1 11 6 5 3 ← 12 7 7 7 12 3 3 ← 8 15 3 7 10 5 3 ← 8 11 7 6 5 7 7 ← 18 5 3 4 7 7 7 ← 10 13 3
3	22 1 1 ← 24 1 21 2 1 ← 22 3 17 6 1 ← 18 7 9 14 1 ← 10 15	23 1 1 ← 25 1 11 7 7 ← 19 7 7 11 7 ← 11 15
2	23 1 ← 25	
1		

Stem 24 Stem 25

8	15 1 1 * 1←16 2 * 1 11 5 1 * 1←12 6 * 1 10 2 3 4 4 1 1 1←17 1 * 1 9 1 1 24333←10 2 24333 5 5 1 24333←6 6 24333 4 2 2 45333 1 2 2 35733←2 4 35733	11 2 3 4 4 1 1 1←14 3 4 4 1 1 1 10 1 1 24333←12 1 24333 5 2 2 45333←6 4 45333 3 4 2 45333←4 6 45333 2 2 2 35733←8 2 45333 1 2 3 35733←2 3 5 5 7 3 3
7	15 2 * 1←16 4 4 1 1 1 12 3 4 4 1 1 1←18 * 1 11 6 * 1←12 8 4 1 1 1 10 1 24333←12 24333 9 2 24333←10 3 6 2 3 3 6 2 45333 5 6 24333←6 7 6 2 3 3 3 2 35733←4 3 6653 2 3 35733←4 5 5 7 3 3 1 1 2 3 5 7 7←2 2 4 5 7 7	13 3 4 4 1 1 1←14 5 1 2 3 3 11 1 24333←13 24333 7 2 45333←8 4 7 3 3 3 5 4 45333←6 5 8 3 3 3 4 2 35733←10 45333 3 6 45333←4 7 8 3 3 3 2 1 2 3 5 7 7←4 2 3 5 7 7 1 2 2 3 5 7 7←2 3 4 5 7 7
6	15 4 4 1 1 1←20 4 1 1 1 11 8 4 1 1 1←12 12 1 1 1 11 24333←13 6 2 3 3 9 3 6 2 3 3←10 8 3 3 3 5 7 6 2 3 3←6 12 3 3 3 5 35733←9 5 7 3 3 3 5 5 7 3 3←5 8 6 5 3 3 3 6653 2 2 3 5 7 7←4 4 5 7 7 1 2 4 5 7 7←2 4 7 7 7	13 5 1 2 3 3←14 6 2 3 3 9 45333←10 5 7 3 3 7 4 7 3 3 3←8 6653 6 35733←11 8 3 3 3 5 5 8 3 3 3←6 8 6 5 3 3 6 9 3 3 3←4 7 11 3 3 3 2 3 5 7 7←5 4 5 7 7 2 3 3 5 7 7←4 5 5 7 7
5	19 4 1 1 1←20 5 1 1 17 1 2 3 3←24 1 1 1 11 12 1 1 1←12 13 1 1 6 11 3 3 3←8 13 3 3 4 3 5 7 7←8 5 7 7 3 6 11 3 3←4 7 13 3 3 4 5 7 7←6 7 7 7	18 1 2 3 3←20 2 3 3 9 9 3 3 3←10 10 5 3 7 6653←14 6 5 3 5 3 5 7 7←9 5 7 7 3 5 5 7 7←5 9 7 7
4	23 1 1 1←24 2 1 19 5 1 1←20 6 1 18 2 3 3←25 1 1 11 13 1 1←12 14 1 7 13 3 3←9 15 3 7 5 7 7←13 7 7 5 7 7 7←9 11 7	19 2 3 3←22 3 3 18 3 3 3←20 5 3 13 6 5 3←14 7 7 10 11 3 3←12 13 3 9 10 5 3←10 11 7
3	23 2 1←24 3 20 3 3←26 1 19 6 1←20 7 11 14 1←12 15	21 3 3←25 3 19 5 3←21 7 11 13 3←13 15
2	23 3←27	
1		

Stem 26 (filtrations ≦ 8) Stem 27 (filtrations ≦ 8)

15		1 1 * * * 1
14	1 * * * 1	2 * * * 1 1 1 1 2 3 4 4 1 1 * 1←2 2 2 3 4 4 1 1 * 1
13	3 4 1 1 * * 1←4 5 1 * * 1 1 1 2 3 4 4 1 1 * 1←8 1 1 * * 1	4 4 1 1 * * 1 2 1 2 3 4 4 1 1 * 1←4 2 3 4 4 1 1 * 1 1 2 2 3 4 4 1 1 * 1←2 4 3 4 4 1 1 * 1
12	7 1 1 * * 1←8 2 * * 1 3 5 1 * * 1←4 6 * * 1 2 2 3 4 4 1 1 * 1←9 1 * * 1 1 1 1 * 24333←2 2 * 24333	3 6 1 * * 1←4 7 * * 1 3 2 3 4 4 1 1 * 1←6 3 4 4 1 1 * 1 2 1 1 * 24333←4 1 * 24333 1 2 1 * 24333←2 3 * 24333
11	7 2 * * 1←8 4 4 1 1 * 1 4 3 4 4 1 1 * 1←10 * * 1 3 6 * * 1←4 8 4 1 1 * 1 2 1 * 24333←4 * 24333 1 2 * 24333←2 3 6 2 3 4 4 1 1 1	5 3 4 4 1 1 * 1←6 5 1 2 3 4 4 1 1 1 3 5 4 4 1 1 * 1←4 7 1 2 3 4 4 1 1 1 3 1 * 24333←5 * 24333
10	7 4 4 1 1 * 1←12 4 1 1 * 1 3 6 1 2 3 4 4 1 1 1←4 7 2 3 4 4 1 1 1 3 * 24333←5 6 2 3 4 4 1 1 1	5 5 1 2 3 4 4 1 1 1←6 6 2 3 4 4 1 1 1 1 2 2 2 2 45333←2 4 2 2 45333
9	11 4 1 1 * 1←12 5 1 * 1 9 1 2 3 4 4 1 1 1←16 1 1 * 1 5 10 1 1 * 1←6 11 1 * 1 3 6 1 1 24333←4 7 1 24333 2 2 2 2 45333	10 1 2 3 4 4 1 1 1←12 2 3 4 4 1 1 1 3 2 2 2 45333←4 4 2 45333

Stem 26 (filtrations ≥ 9) Stem 27 (filtrations ≥ 9)

7	18 1 * 1←20 * 1 17 2 * 1←18 4 4 1 1 1 13 6 * 1←14 8 4 1 1 1 11 2 24333←12 3 6 2 3 3 5 2 35733←6 3 6653 3 4 35733←4 7 5 7 3 3 3 1 2 3 5 7 7←4 2 4 5 7 7 2 3 3 6653←8 35733 1 2 3 3 5 7 7←2 3 5 5 7 7	19 1 * 1←21 * 1 15 3 4 4 1 1 1←16 5 1 2 3 3 13 1 24333←19 4 4 1 1 1 9 2 45333←10 4 7 3 3 3 7 4 45333←8 5 8 3 3 3 6 2 35733 5 6 45333←6 7 8 3 3 3 3 5 35733←5 7 5 7 3 3 3 3 3 6653←9 35733 1 2 4 3 5 7 7←2 3 6 5 7 7
6	19 * 1←22 4 1 1 1 17 4 4 1 1 1←20 1 2 3 3 13 8 4 1 1 1←14 12 1 1 1 11 3 6 2 3 3←12 8 3 3 3 7 35733←9 6653 5 3 6653←11 5 7 3 3 3 3 3 5 7 7←5 5 5 7 7 3 2 4 5 7 7←4 4 7 7 7 2 4 3 5 7 7←4 6 5 7 7	15 5 1 2 3 3←16 6 2 3 3 14 24333←21 1 2 3 3 11 45333←12 5 7 3 3 9 4 7 3 3 3←10 6653 7 5 8 3 3 3←8 8 6 5 3 5 7 8 3 3 3←8 13 3 3 3 5 2 3 5 7 7←13 8 3 3 3 4 3 3 5 7 7←8 3 5 7 7 3 6 11 3 3 3←4 7 13 3 3 3 4 3 5 7 7←4 7 5 7 7
5	21 4 1 1 1←22 5 1 1 19 1 2 3 3←21 2 3 3 13 12 1 1 1←14 13 1 1 10 9 3 3 3←12 11 3 3 7 12 3 3 3←8 15 3 3 6 3 5 7 7←10 5 7 7 3 6 5 7 7←6 9 7 7 3 4 7 7 7←4 7 11 7	15 6 2 3 3←22 2 3 3 11 9 3 3 3←12 10 5 3 7 13 3 3 3←8 14 5 3 7 8 6 5 3←9 15 3 3 7 3 5 7 7←11 5 7 7 6 4 5 7 7←16 6 5 3 3 5 7 7 7←5 7 11 7
4	25 1 1 1←26 2 1 21 5 1 1←22 6 1 19 3 3 3←21 5 3 13 13 1 1←14 14 1 11 11 3 3←13 13 3 7 7 7 7←11 11 7	26 1 1 1←28 1 1 20 3 3 3←24 3 3 15 6 5 3←16 7 7 11 10 5 3←12 11 7 9 18 1 1←10 19 1 8 7 7 7←22 5 3 7 14 5 3←8 15 7
3	26 1 1←28 1 25 2 1←26 3 21 6 1←22 7 13 14 1←14 15	27 1 1←29 1 23 3 3←27 3 15 7 7←23 7
2	27 1←29	
1		

Stem 28 (filtrations ≤ 8)   Stem 29 (filtrations ≤ 8)

16	1 1 1 * * * 1←2 2 * * * 1	2 1 1 * * * 1←4 1 * * * 1 1 2 1 * * * 1←2 3 * * * 1
15	2 1 * * * 1←4 * * * 1 1 2 * * * 1←2 4 4 1 1 * * 1	3 1 * * * 1←5 * * * 1 1 2 1 1 2 3 4 4 1 1 * 1←2 3 1 2 3 4 4 1 1 * 1
14	3 * * * 1←6 4 1 1 * * 1 2 1 1 2 3 4 4 1 1 * 1←4 1 2 3 4 4 1 1 * 1 1 2 1 2 3 4 4 1 1 * 1←2 3 2 3 4 4 1 1 * 1	3 1 1 2 3 4 4 1 1 * 1←4 2 2 3 4 4 1 1 * 1
13	5 4 1 1 * * 1←6 5 1 * * 1 3 6 1 1 * * 1←4 7 1 * * 1 3 1 2 3 4 4 1 1 * 1←5 2 3 4 4 1 1 * 1 1 2 1 1 * 24333←2 3 1 * 24333	3 2 2 3 4 4 1 1 * 1←4 4 3 4 4 1 1 * 1
12	9 1 1 * * 1←10 2 * * 1 5 5 1 * * 1←6 6 * * 1 3 1 1 * 24333←4 2 * 24333	10 1 1 * * 1←12 1 * * 1 4 1 1 * 24333←8 3 4 4 1 1 * 1 3 4 3 4 4 1 1 * 1←4 7 4 4 1 1 * 1
11	10 1 * * 1←12 * * 1 9 2 * * 1←10 4 4 1 1 * 1 5 6 * * 1←6 8 4 1 1 * 1 3 2 * 24333←4 3 6 2 3 4 4 1 1 1	11 1 * * 1←13 * * 1 7 3 4 4 1 1 * 1←8 5 1 2 3 4 4 1 1 1 5 1 * 24333←11 4 4 1 1 * 1 1 2...2 45333←2 4 2 2 2 45333
10	11 * * 1←14 4 1 1 * 1 9 4 4 1 1 * 1←12 1 2 3 4 4 1 1 1 5 8 4 1 1 * 1←6 12 1 1 * 1 3 3 6 2 3 4 4 1 1 1←4 8 1 1 24333 2...2 45333	7 5 1 2 3 4 4 1 1 1←8 6 2 3 4 4 1 1 1 6 * 24333←13 1 2 3 4 4 1 1 1 3 2..2 45333←4 4 2 2 45333
9	13 4 1 1 * 1←14 5 1 * 1 11 1 2 3 4 4 1 1 1←13 2 3 4 4 1 1 1 4 2 2 2 45333 1 2 2 2 35733←2 4 2 35733	7 6 2 3 4 4 1 1 1←14 2 3 4 4 1 1 1 3 4 2 2 45333←4 6 2 45333 2..2 35733
8	17 1 1 * 1←18 2 * 1 13 5 1 * 1←14 6 * 1 11 1 1 24333←12 2 24333 6 2 2 45333 3 2 2 35733←4 4 35733 1 2 1 2 3 5 7 7←2 3 2 3 5 7 7	18 1 1 * 1←20 1 * 1 12 1 1 24333←16 3 4 4 1 1 1 3 6 2 45333 1 2 3 3 6653←4 5 35733

Stem 28 (filtrations ≥ 9) Stem 29 (filtrations ≥ 9)

8	19 1 1 * 1 ← 20 2 * 1 15 5 1 * 1 ← 16 6 * 1 13 1 1 24333 ← 14 2 24333 7 13 1 * 1 3 4 2 35733 ← 4 6 35733 2 2 3 3 6653 ← 8 2 35733	20 1 1 * 15 2 3 4 4 1 1 1 ← 18 3 4 4 1 1 14 1 1 24333 ← 16 1 24333 7 14 1 * 1 ← 8 15 * 1 5 10 1 24333 ← 6 11 24333 5 6 2 45333 3 2 3 3 6653 ← 4 7 35733 2 3 3 3 6653 ← 4 5 3 6653 1 2 3 3 3 5 7 7 ← 2 3 5 3 5 7 7
7	19 2 * 1 ← 20 4 4 1 1 1 15 6 * 1 ← 16 8 4 1 1 1 14 1 24333 ← 16 24333 13 2 24333 ← 14 3 6 2 3 3 10 2 45333 7 2 35733 ← 8 3 6653 4 3 3 6653 ← 10 35733 3 6 35733 ← 4 7 6653 2 3 3 3 5 7 7 ← 4 5 3 5 7 7	21 1 * 1 17 3 4 4 1 1 1 ← 18 5 1 2 3 3 15 1 24333 ← 17 24333 11 2 45333 ← 12 4 7 3 3 3 7 13 4 4 1 1 1 ← 8 15 1 2 3 3 5 3 3 6653 ← 8 2 3 5 7 7 3 5 3 6653 ← 5 7 6653 2 4 3 3 5 7 7 ← 4 6 3 5 7 7
6	15 8 4 1 1 1 ← 16 12 1 1 1 15 24333 ← 17 6 2 3 3 13 3 6 2 3 3 ← 14 8 3 3 3 12 45333 7 14 1 2 3 3 ← 8 15 2 3 3 7 3 6653 ← 11 6653 6 2 3 5 7 7 ← 8 4 5 7 7 5 3 3 5 7 7 ← 6 6 5 7 7 3 5 3 5 7 7 ← 5 7 5 7 7	22 * 1 17 5 1 2 3 3 ← 18 6 2 3 3 13 45333 ← 14 5 7 3 3 11 4 7 3 3 3 ← 12 6653 9 5 8 3 3 3 ← 10 8 6 5 3 7 7 8 3 3 3 ← 8 15 3 3 3 7 2 3 5 7 7 ← 9 4 5 7 7 6 3 3 5 7 7 ← 8 5 5 7 7 3 6 3 5 7 7 ← 6 7 5 7 7 2 3 5 7 7 7 ← 10 3 5 7 7
5	23 4 1 1 1 ← 24 5 1 1 15 12 1 1 1 ← 16 13 1 1 12 9 3 3 3 9 18 1 1 1 ← 10 19 1 1 7 14 3 3 3 ← 8 15 5 3 7 4 5 7 7 ← 10 7 7 7 6 5 5 7 7 ← 8 9 7 7 5 6 5 7 7 ← 6 11 7 7 4 5 7 7 7 ← 12 5 7 7	24 4 1 1 1 22 1 2 3 3 ← 24 2 3 3 13 9 3 3 3 ← 14 10 5 3 13 5 7 3 3 ← 18 6 5 3 9 8 6 5 3 ← 10 14 5 3 9 3 5 7 7 7 5 5 7 7 ← 9 9 7
4	27 1 1 1 ← 28 2 1 23 5 1 1 ← 24 6 1 21 3 3 3 ← 25 3 3 15 13 1 1 ← 16 14 1 13 11 3 3 9 7 7 7 ← 17 7 7 7 9 7 7 ← 9 15 7	28 1 1 1 23 2 3 3 ← 26 3 3 22 3 3 3 ← 24 5 3 17 6 5 3 ← 18 7 7 14 11 3 3 ← 16 13 3 13 10 5 3 ← 14 11 7 9 14 5 3 ← 10 15 7
3	27 2 1 ← 28 3 23 6 1 ← 24 7 15 14 1 ← 16 15 14 13 3	29 1 1 23 5 3 ← 25 7 15 13 3 ← 17 15 13 11 7
2	15 15	30 1
1		31

Stem 30 (filtrations $\leq$ 8)   Stem 31 (filtrations $\leq$ 8)

17	1 2 1 1 * * * 1 ← 2 3 1 * * * 1	
16	3 1 1 * * * 1 ← 4 2 * * * 1	4 1 1 * * * 1
15	3 2 * * * 1 ← 4 4 4 1 1 * * 1	5 1 * * * 1
14	3 4 4 1 1 * * 1	6 * * * 1
13	7 4 1 1 * * 1 ← 8 5 1 * * 1 5 1 2 3 4 4 1 1 * 1	8 4 1 1 * * 1 6 1 2 3 4 4 1 1 * 1 ← 8 2 3 4 4 1 1 * 1 * * 24333
12	11 1 1 * * 1 ← 12 2 * * 1 7 5 1 * * 1 ← 8 6 * * 1 6 2 3 4 4 1 1 * 1 5 1 1 * 24333 ← 6 2 * 24333	12 1 1 * * 1 7 2 3 4 4 1 1 * 1 ← 10 3 4 4 1 1 * 1 6 1 1 * 24333 ← 8 1 * 24333 1 2....2 45333 ← 2 4 2..2 45333
11	11 2 * * 1 ← 12 4 4 1 1 * 1 7 6 * * 1 ← 8 8 4 1 1 * 1 6 1 * 24333 ← 8 * 24333 5 2 * 24333 ← 6 3 6 2 3 4 4 1 1 1 2....2 45333	13 1 * * 1 9 3 4 4 1 1 * 1 ← 10 5 1 2 3 4 4 1 1 1 7 1 * 24333 ← 9 * 24333 5 7 4 4 1 1 * 1 ← 6 9 1 2 3 4 4 1 1 1 3 2...2 45333 ← 4 4 2 2 2 45333
10	7 8 4 1 1 * 1 ← 8 12 1 1 * 1 7 * 24333 ← 9 6 2 3 4 4 1 1 1 5 3 6 2 3 4 4 1 1 1 ← 6 8 1 1 24333 4 2..2 45333 1 2..2 35733 ← 2 3 6 2 45333	14 * * 1 9 5 1 2 3 4 4 1 1 1 ← 10 6 2 3 4 4 1 1 1 5 9 1 2 3 4 4 1 1 1 ← 6 10 2 3 4 4 1 1 1 3 4 2 2 2 45333 ← 4 6 2 2 45333 5 2..2 45333 ← 6 4 2 2 45333 2...2 35733
9	15 4 1 1 * 1 ← 16 5 1 * 1 7 12 1 1 * 1 ← 8 13 1 * 1 5 8 1 1 24333 3 2 2 2 35733 ← 4 4 2 35733 1 1 2 3 3 6653 ← 2 3 5 35733	16 4 1 1 * 1 14 1 2 3 4 4 1 1 1 ← 16 2 3 4 4 1 1 1 5 10 2 3 4 4 1 1 1 ← 6 12 3 4 4 1 1 1 5 4 2 2 45333 ← 6 6 2 45333 3 6 2 2 45333 1 2 2 3 3 6653 ← 2 4 3 3 6653
	Stem 30 (filtrations ≥ 9)	Stem 31 (filtrations ≥ 9)

	Stem 32 (filtrations ≤ 8)	Stem 33 (filtrations ≤ 8)
8	21 1 1 * 1 ← 22 2 * 1 17 5 1 * 1 ← 18 6 * 1 15 1 1 24333 ← 16 2 24333 9 13 1 * 1 ← 17 1 24333 3 6 2 35733 3 3 3 3 6653 ← 4 5 2 3 5 7 7 1 2 4 3 3 5 7 7 ← 2 3 6 3 5 7 7	22 1 1 * 1 ← 24 1 * 1 7 12 3 4 4 1 1 1 ← 8 15 4 4 1 1 1 7 6 2 45333 ← 14 2 45333 4 3 3 3 6653 ← 8 3 3 6653 3 4 3 3 6653 ← 4 7 3 6653 1 1 2 3 5 7 7 7 ← 2 2 4 5 7 7 7
7	22 1 * 1 ← 24 * 1 21 2 * 1 ← 22 4 4 1 1 1 17 6 * 1 ← 18 8 4 1 1 1 15 2 24333 ← 16 3 6 2 3 3 12 2 45333 ← 18 24333 9 2 35733 ← 10 3 6653 6 3 3 6653 5 6 35733 ← 6 7 6653 3 5 7 8 3 3 3 ← 4 7 8 6 5 3 3 5 2 3 5 7 7 ← 4 6 4 5 7 7 3 4 3 3 5 7 7 ← 4 7 3 5 7 7 1 2 3 5 7 7 7 ← 8 3 3 5 7 7	23 1 * 1 ← 25 * 1 19 3 4 4 1 1 1 ← 20 5 1 2 3 3 13 2 45333 ← 14 4 7 3 3 3 10 2 35733 ← 16 45333 7 3 3 6653 ← 11 3 6653 5 5 3 6653 3 6 2 3 5 7 7 ← 4 7 4 5 7 7 2 2 3 5 7 7 7 ← 4 4 5 7 7 7 1 2 4 5 7 7 7 ← 2 4 7 7 7 7
6	23 * 1 ← 26 4 1 1 1 21 4 4 1 1 1 ← 24 1 2 3 3 17 8 4 1 1 1 ← 18 12 1 1 1 15 3 6 2 3 3 ← 16 8 3 3 3 14 45333 ← 19 6 2 3 3 11 35733 ← 13 6653 9 3 6653 7 3 3 5 7 7 ← 8 6 5 7 7 3 6 4 5 7 7 ← 4 8 7 7 7 2 4 5 7 7 7 ← 9 5 5 7 7	19 5 1 2 3 3 ← 20 6 2 3 3 15 45333 ← 16 5 7 3 3 13 4 7 3 3 3 ← 14 6653 12 35733 ← 17 8 3 3 3 9 2 3 5 7 7 ← 16 9 3 3 3 5 10 9 3 3 3 ← 6 11 11 3 3 3 6 5 5 7 7 ← 4 7 9 7 7 3 4 5 7 7 7 ← 5 8 7 7 7
5	25 4 1 1 1 ← 26 5 1 1 23 1 2 3 3 ← 25 2 3 3 17 12 1 1 1 ← 18 13 1 1 15 8 3 3 3 ← 24 3 3 3 14 9 3 3 3 ← 16 11 3 3 7 6 5 7 7 ← 8 11 7 7 5 10 11 3 3 ← 6 11 13 3 4 7 7 7 7 ← 10 9 7 7	15 9 3 3 3 ← 16 10 5 3 15 5 7 3 3 ← 20 6 5 3 11 3 5 7 7 10 4 5 7 7 ← 17 11 3 3 5 7 7 7 7 ← 9 11 7 7
4	29 1 1 1 ← 30 2 1 25 5 1 1 ← 26 6 1 23 3 3 3 ← 25 5 3 17 13 1 1 ← 18 14 1 15 11 3 3 ← 17 13 3 13 5 7 7 11 7 7 7 ← 19 7 7 7 11 7 7 ← 11 15 7	30 1 1 1 ← 32 1 1 19 6 5 3 ← 20 7 7 15 10 5 3 ← 16 11 7 14 5 7 7 12 7 7 7 ← 18 13 3
3	30 1 1 ← 32 1 29 2 1 ← 30 3 25 6 1 ← 26 7 17 14 1 ← 18 15	31 1 1 ← 33 1 27 3 3 15 11 7 ← 19 15
2	31 1 ← 33 29 3	
1		

Stem 32 (filtrations ≤ 8) Stem 33 (filtrations ≤ 8)

	Stem 32 (filtrations ≧ 9)	Stem 33 (filtrations ≧ 9)
17		2 4 1 1 * * * 1
16	5 1 1 * * * 1 ← 6 2 * * * 1	6 1 1 * * * 1 ← 8 1 * * * 1 1 2 3 4 4 1 1 * * 1
15	6 1 * * * 1 ← 8 * * * 1 5 2 * * * 1 ← 6 4 4 1 1 * * 1 2 3 4 4 1 1 * * 1	7 1 * * * 1 ← 9 * * * 1 3 3 4 4 1 1 * * 1 ← 4 5 1 2 3 4 4 1 1 * 1 1 1 * * 24333
14	7 * * * 1 ← 10 4 1 1 * * 1 5 4 4 1 1 * * 1 ← 8 1 2 3 4 4 1 1 * 1 1 * * 24333	3 5 1 2 3 4 4 1 1 * 1 ← 4 6 2 3 4 4 1 1 * 1 2 * * 24333
13	9 4 1 1 * * 1 ← 10 5 1 * * 1 7 1 2 3 4 4 1 1 * 1 ← 9 2 3 4 4 1 1 * 1 3 4 1 1 * 24333 ← 4 5 1 * 24333	3 6 2 3 4 4 1 1 * 1 1 2.....2 45333 ← 2 4 2...2 45333
12	13 1 1 * * 1 ← 14 2 * * 1 9 5 1 * * 1 ← 10 6 * * 1 7 1 1 * 24333 ← 8 2 * 24333 3 5 1 * 24333 ← 4 6 * 24333 2.....2 45333 ← 9 1 * 24333	14 1 1 * * 1 ← 16 1 * * 1 8 1 1 * 24333 5 10 1 * * 1 ← 6 11 * * 1 3 6 1 * 24333 ← 4 7 * 24333 3 2....2 45333 ← 4 4 2..2 45333
11	14 1 * * 1 ← 16 * * 1 13 2 * * 1 ← 14 4 4 1 1 * 1 9 6 * * 1 ← 10 8 4 1 1 * 1 7 2 * 24333 ← 8 3 6 2 3 4 4 1 1 1 4 2...2 45333 ← 10 * 24333 3 6 * 24333 ← 4 7 6 2 3 4 4 1 1 1 1 2...2 35733 ← 2 3 6 2 2 45333	15 1 * * 1 ← 17 * * 1 11 3 4 4 1 1 * 1 ← 12 5 1 2 3 4 4 1 1 1 5 9 4 4 1 1 * 1 ← 6 11 1 2 3 4 4 1 1 1 5 2...2 45333 ← 6 4 2 2 2 45333 3 4 2..2 45333 ← 4 5 8 1 1 24333 2....2 35733 ← 8 2..2 45333
10	15 * * 1 ← 18 4 1 1 * 1 13 4 4 1 1 * 1 ← 16 1 2 3 4 4 1 1 1 9 8 4 1 1 * 1 ← 10 12 1 1 * 1 7 3 6 2 3 4 4 1 1 1 ← 8 8 1 1 24333 6 2..2 45333 ← 11 6 2 3 4 4 1 1 1 5 10 1 2 3 4 4 1 1 1 ← 6 11 2 3 4 4 1 1 1 3 2..2 35733 ← 4 3 6 2 45333	11 5 1 2 3 4 4 1 1 1 ← 12 6 2 3 4 4 1 1 1 7 9 1 2 3 4 4 1 1 1 ← 8 10 2 3 4 4 1 1 1 7 2..2 45333 ← 8 4 2 2 45333 5 4 2 2 2 45333 ← 6 6 2 2 45333 4 2..2 35733 ← 9 8 1 1 24333 3 5 8 1 1 24333 ← 4 7 13 1 * 1 1 2 2 2 3 3 6653 ← 2 3 6 2 35733
9	17 4 1 1 * 1 ← 18 5 1 * 1 15 1 2 3 4 4 1 1 1 ← 17 2 3 4 4 1 1 1 9 12 1 1 * 1 ← 10 13 1 * 1 7 14 1 1 * 1 ← 8 15 1 * 1 7 8 1 1 24333 ← 16 1 1 24333 5 10 1 1 24333 ← 6 11 1 24333 3 3 6 2 45333 ← 4 6 2 35733 2 2 2 3 3 6653 1 2 3 3 3 6653 ← 2 3 5 3 6653	7 10 2 3 4 4 1 1 1 ← 8 12 3 4 4 1 1 1 7 4 2 2 45333 ← 8 6 2 45333 5 6 2 2 45333 ← 11 13 1 * 1 3 2 2 3 3 6653 ← 4 4 3 3 6653

Stem 32 (filtrations ≧ 9) Stem 33 (filtrations ≧ 9)

	23 1 1 * 1←24 2 * 1	
	19 5 1 * 1←20 6 * 1	
	18 2 3 4 4 1 1 1←25 1 * 1	19 2 3 4 4 1 1 1←22 3 4 4 1 1 1
	17 1 1 24333←18 2 24333	18 1 1 24333←20 1 24333
	5 6 2 35733	9 6 2 45333←16 2 45333
	3 5 3 3 6653←4 7 2 3 5 7 7	5 10 2 45333←6 12 45333
	2 1 2 3 5 7 7 7←4 2 3 5 7 7 7	3 6 3 3 6653
8	1 2 2 3 5 7 7 7←2 3 4 5 7 7 7	3 1 2 3 5 7 7 7←4 2 4 5 7 7 7
	23 2 * 1←24 4 4 1 1 1	21 3 4 4 1 1 1←22 5 1 2 3 3
	20 3 4 4 1 1 1←26 * 1	19 1 24333←21 24333
	19 6 * 1←20 8 4 1 1 1	15 2 45333←16 4 7 3 3 3
	18 1 24333←20 24333	12 2 35733←18 45333
	17 2 24333←18 3 6 2 3 3	9 15 4 4 1 1 1←10 17 1 2 3 3
	11 2 35733←12 3 6653	9 3 3 6653←12 2 3 5 7 7
	5 5 2 3 5 7 7←6 6 4 5 7 7	6 5 2 3 5 7 7
	3 6 3 3 5 7 7←4 7 5 5 7 7	5 7 3 6653←6 12 9 3 3 3
7	3 2 3 5 7 7 7←5 4 5 7 7 7	3 2 4 5 7 7 7←4 4 7 7 7 7
	23 4 4 1 1 1←28 4 1 1 1	
	19 8 4 1 1 1←20 12 1 1 1	21 5 1 2 3 3←22 6 2 3 3
	19 24333←21 6 2 3 3	17 45333←18 5 7 3 3
	17 3 6 2 3 3←18 8 3 3 3	15 4 7 3 3 3←16 6653
	13 35733←17 5 7 3 3	14 35733←19 8 3 3 3
	10 2 3 5 7 7←12 4 5 7 7	11 2 3 5 7 7←13 4 5 7 7
	5 7 3 5 7 7	9 17 1 2 3 3←10 18 2 3 3
6	5 6 4 5 7 7←6 8 7 7 7	3 4 7 7 7 7←4 7 11 7 7
		26 1 2 3 3←28 2 3 3
	27 4 1 1 1←28 5 1 1	17 9 3 3 3←18 10 5 3
	25 1 2 3 3←32 1 1 1	15 6653←22 6 5 3
	19 12 1 1 1←20 13 1 1	13 3 5 7 7←17 5 7 7
	12 3 5 7 7←16 5 7 7	9 18 2 3 3←10 20 3 3
5	11 4 5 7 7←14 7 7 7	59777
	31 1 1 1←32 2 1	
	27 5 1 1←28 6 1	27 2 3 3←30 3 3
	26 2 3 3←33 1 1	26 3 3 3←28 5 3
	25 3 3 3	21 6 5 3←22 7 7
	19 13 1 1←20 14 1	18 11 3 3←20 13 3
	15 5 7 7←21 7 7	17 10 5 3←18 11 7
4	13 7 7 7←17 11 7	11 22 1 1←12 23 1
	31 2 1←32 3	
	28 3 3←34 1	
	27 6 1←28 7	29 3 3←33 3
	26 5 3	27 5 3←29 7
3	19 14 1←20 15	19 13 3←21 15
	31 3←35	
2	27 7	
1		

Stem 34 (filtrations ≤ 8)   Stem 35 (filtrations ≤ 8)

19		1 1 * * * * 1
18	1 * * * * 1	2 * * * * 1 1 1 1 2 3 4 4 1 1 * * 1←2 2 2 3 4 4 1 1 * * 1
17	3 4 1 1 * * * 1←4 5 1 * * * 1 1 1 2 3 4 4 1 1 * * 1←8 1 1 * * * 1	4 4 1 1 * * * 1 2 1 2 3 4 4 1 1 * * 1←4 2 3 4 4 1 1 * * 1 1 2 2 3 4 4 1 1 * * 1←2 4 3 4 4 1 1 * * 1
16	7 1 1 * * * 1←8 2 * * * 1 3 5 1 * * * 1←4 6 * * * 1 2 2 3 4 4 1 1 * * 1←9 1 * * * 1 1 1 1 * * 24333←2 2 * * 24333	3 6 1 * * * 1←4 7 * * * 1 3 2 3 4 4 1 1 * * 1←6 3 4 4 1 1 * * 1 2 1 1 * * 24333←4 1 * * 24333 1 2 1 * * 24333←2 3 * * 24333
15	7 2 * * * 1←8 4 4 1 1 * * 1 4 3 4 4 1 1 * * 1←10 * * * 1 3 6 * * * 1←4 8 4 1 1 * * 1 2 1 * * 24333←4 * * 24333 1 2 * * 24333←2 3 6 2 3 4 4 1 1 * 1	5 3 4 4 1 1 * * 1←6 5 1 2 3 4 4 1 1 * 1 3 5 4 4 1 1 * * 1←4 7 1 2 3 4 4 1 1 * 1 3 1 * * 24333←5 * * 24333
14	7 4 4 1 1 * * 1←12 4 1 1 * * 1 3 6 1 2 3 4 4 1 1 * 1←4 7 2 3 4 4 1 1 * 1 3 * * 24333←5 6 2 3 4 4 1 1 * 1	5 5 1 2 3 4 4 1 1 * 1←6 6 2 3 4 4 1 1 * 1 1 2......2 45333←2 4 2....2 45333
13	11 4 1 1 * * 1←12 5 1 * * 1 9 1 2 3 4 4 1 1 * 1←16 1 1 * * 1 5 10 1 1 * * 1←6 11 1 * * 1 3 6 1 1 * 24333←4 7 1 * 24333 2......2 45333	10 1 2 3 4 4 1 1 * 1←12 2 3 4 4 1 1 * 1 3 2.....2 45333←4 4 2...2 45333
12	15 1 1 * * 1←16 2 * * 1 11 5 1 * * 1←12 6 * * 1 10 2 3 4 4 1 1 * 1←17 1 * * 1 9 1 1 * 24333←10 2 * 24333 5 5 1 * 24333←6 6 * 24333 4 2....2 45333 1 2....2 35733←2 4 2..2 35733	11 2 3 4 4 1 1 * 1←14 3 4 4 1 1 * 1 10 1 1 * 24333←12 1 * 24333 5 2....2 45333←6 4 2..2 45333 3 4 2...2 45333←4 6 2..2 45333 2.....2 35733←8 2...2 45333
11	15 2 * * 1←16 4 4 1 1 * 1 12 3 4 4 1 1 * 1←18 * * 1 11 6 * * 1←12 8 4 1 1 * 1 10 1 * 24333←12 * 24333 9 2 * 24333←10 3 6 2 3 4 4 1 1 1 6 2...2 45333 5 6 * 24333←6 7 6 2 3 4 4 1 1 1 3 2...2 35733←4 3 6 2 2 45333	13 3 4 4 1 1 * 1←14 5 1 2 3 4 4 1 1 1 11 1 * 24333←13 * 24333 7 2...2 45333←8 4 2 2 2 45333 5 4 2..2 45333←6 5 8 1 1 24333 4 2...2 35733←10 2..2 45333 3 6 2..2 45333←4 7 8 1 1 24333 1 2..2 3 3 6653←2 3 5 6 2 45333
10	15 4 4 1 1 * 1←20 4 1 1 * 1 11 8 4 1 1 * 1←12 12 1 1 * 1 11 * 24333←13 6 2 3 4 4 1 1 1 9 3 6 2 3 4 4 1 1 1←10 8 1 1 24333 5 7 6 2 3 4 4 1 1 1←6 12 1 1 24333 5 2..2 35733←6 3 6 2 45333 3 3 6 2 2 45333←4 5 6 2 45333 2..2 3 3 6653	13 5 1 2 3 4 4 1 1 1←14 6 2 3 4 4 1 1 1 9 2..2 45333←10 4 2 2 45333 7 4 2 2 2 45333←8 6 2 2 45333 6 2..2 35733←11 8 1 1 24333 5 5 8 1 1 24333←6 7 13 1 * 1 3 2 2 2 3 3 6653←4 3 6 2 35733
9	19 4 1 1 * 1←20 5 1 * 1 17 1 2 3 4 4 1 1 1←24 1 1 * 1 11 12 1 1 * 1←12 13 1 * 1 5 3 6 2 45333←6 6 2 35733 3 5 6 2 45333	18 1 2 3 4 4 1 1 1←20 2 3 4 4 1 1 1 9 4 2 2 45333←10 6 2 45333 7 6 2 2 45333←13 13 1 * 1 5 7 13 1 * 1←6 10 2 45333 3 3 6 2 35733←4 6 3 3 6653 2 4 3 3 3 6653 1 2 1 2 3 5 7 7 7←2 3 2 3 5 7 7 7

Stem 34 (filtrations ≥ 9) Stem 35 (filtrations ≥ 9)

9	21 4 1 1 * 1←22 5 1 * 1 19 1 2 3 4 4 1 1 1←21 2 3 4 4 1 1 1 13 12 1 1 * 1←14 13 1 * 1 7 12 1 1 24333←8 13 1 24333 7 3 6 2 45333←8 6 2 35733 5 5 6 2 45333←11 6 2 45333 3 4 3 3 3 6653←4 5 5 3 6653	15 6 2 3 4 4 1 1 1←22 2 3 4 4 1 1 1 11 4 2 2 45333←12 6 2 45333 7 7 13 1 * 1←8 10 2 45333 5 9 13 1 * 1←6 12 2 45333 5 3 6 2 35733←6 6 3 3 6653 3 5 6 2 35733
8	25 1 1 * 1←26 2 * 1 21 5 1 * 1←22 6 * 1 19 1 1 24333←20 2 24333 7 13 1 24333←8 14 24333 7 6 2 35733←14 2 35733 3 5 5 3 6653←5 9 3 6653	26 1 1 * 1←28 1 * 1 20 1 1 24333←24 3 4 4 1 1 1 9 18 1 * 1←10 19 * 1 7 14 1 24333←8 15 24333 7 10 2 45333←8 12 45333 5 6 3 3 6653 4 7 3 3 6653
7	26 1 * 1←28 * 1 25 2 * 1←26 4 4 1 1 1 21 6 * 1←22 8 4 1 1 1 19 2 24333←20 3 6 2 3 3 13 2 35733←14 3 6653 10 3 3 6653←16 35733 7 14 24333←8 15 6 2 3 3	27 1 * 1←29 * 1 23 3 4 4 1 1 1←24 5 1 2 3 3 21 1 24333←27 4 4 1 1 1 17 2 45333←18 4 7 3 3 3 11 3 3 6653←17 35733 9 17 4 4 1 1 1←10 19 1 2 3 3 7 12 45333 3 5 7 3 5 7 7
6	27 * 1←30 4 1 1 1 25 4 4 1 1 1←28 1 2 3 3 21 8 4 1 1 1←22 12 1 1 1 19 3 6 2 3 3←20 8 3 3 3 15 35733←17 6653 13 3 6653←19 5 7 3 3 9 18 1 2 3 3←10 19 2 3 3 7 6 4 5 7 7←8 8 7 7 7 5 9 3 5 7 7 3 5 7 7 7 7←5 7 11 7 7	23 5 1 2 3 3←24 6 2 3 3 22 24333←29 1 2 3 3 19 45333←20 5 7 3 3 17 4 7 3 3 3←18 6653 13 2 3 5 7 7 11 17 1 2 3 3←12 18 2 3 3 7 12 9 3 3 3←8 13 11 3 3 6 9 3 5 7 7←16 3 5 7 7 4 5 7 7 7 7←8 9 7 7 7
5	29 4 1 1 1←30 5 1 1 27 1 2 3 3←29 2 3 3 21 12 1 1 1←22 13 1 1 18 9 3 3 3←20 11 3 3 14 3 5 7 7←18 5 7 7 11 22 1 1 1←12 23 1 1 9 18 3 3 3←10 19 5 3 7 8 7 7 7←8 15 7 7 6 9 7 7 7←16 7 7 7	23 6 2 3 3←30 2 3 3 19 9 3 3 3←20 10 5 3 15 3 5 7 7←19 5 7 7 14 4 5 7 7 11 18 2 3 3←12 20 3 3 7 13 11 3 3←8 14 13 3 7 9 7 7 7←9 15 7 7
4	33 1 1 1←34 2 1 29 5 1 1←30 6 1 27 3 3 3←29 5 3 21 13 1 1←22 14 1 19 11 3 3←21 13 3 15 7 7 7←19 11 7	34 1 1 1←36 1 1 28 3 3 3←32 3 3 23 6 5 3←24 7 7 19 10 5 3←20 11 7 11 20 3 3←12 23 3 7 14 13 3←8 15 15
3	34 1 1←36 1 33 2 1←34 3 29 6 1←30 7 21 14 1←22 15	35 1 1←37 1 31 3 3←35 3 23 7 7
2	35 1←37	
1		

Stem 36 (filtrations ≤ 9) Stem 37 (filtrations ≤ 9)

20	1 1 1 * * * * 1←2 2 * * * * 1	2 1 1 * * * * 1←4 1 * * * * 1 1 2 1 * * * * 1←2 3 * * * * 1
19	2 1 * * * * 1←4 * * * * 1 1 2 * * * * 1←2 4 4 1 1 * * * 1	3 1 * * * * 1←5 * * * * 1 1 2 1 1 2 3 4 4 1 1 * * 1←2 3 1 2 3 4 4 1 1 * * 1
18	3 * * * * 1←6 4 1 1 * * * 1 2 1 1 2 3 4 4 1 1 * * 1←4 1 2 3 4 4 1 1 * * 1 1 2 1 2 3 4 4 1 1 * * 1←2 3 2 3 4 4 1 1 * * 1	3 1 1 2 3 4 4 1 1 * * 1←4 2 2 3 4 4 1 1 * * 1
17	5 4 1 1 * * * 1←6 5 1 * * * 1 3 6 1 1 * * * 1←4 7 1 * * * 1 3 1 2 3 4 4 1 1 * * 1←5 2 3 4 4 1 1 * * 1 1 2 1 1 * * 24333←2 3 1 * * 24333	3 2 2 3 4 4 1 1 * * 1←4 4 3 4 4 1 1 * * 1
16	9 1 1 * * * 1←10 2 * * * 1 5 5 1 * * * 1←6 6 * * * 1 3 1 1 * * 24333←4 2 * * 24333	10 1 1 * * * 1←12 1 * * * 1 4 1 1 * * 24333←8 3 4 4 1 1 * * 1 3 4 3 4 4 1 1 * * 1←4 7 4 4 1 1 * * 1
15	10 1 * * * 1←12 * * * 1 9 2 * * * 1←10 4 4 1 1 * * 1 5 6 * * * 1←6 8 4 1 1 * * 1 3 2 * * 24333←4 3 6 2 3 4 4 1 1 * 1	11 1 * * * 1←13 * * * 1 7 3 4 4 1 1 * * 1←8 5 1 2 3 4 4 1 1 * 1 5 1 * * 24333←11 4 4 1 1 * * 1 1 2.......2 45333←2 4 2......2 45333
14	11 * * * 1←14 4 1 1 * * 1 9 4 4 1 1 * * 1←12 1 2 3 4 4 1 1 * 1 5 8 4 1 1 * * 1←6 12 1 1 * * 1 3 3 6 2 3 4 4 1 1 * 1←4 8 1 1 * 24333 2.......2 45333	7 5 1 2 3 4 4 1 1 * 1←8 6 2 3 4 4 1 1 * 1 6 * * 24333←13 1 2 3 4 4 1 1 * 1 3 2......2 45333←4 4 2....2 45333
13	13 4 1 1 * * 1←14 5 1 * * 1 11 1 2 3 4 4 1 1 * 1←13 2 3 4 4 1 1 * 1 4 2.....2 45333 1 2.....2 35733←2 4 2...2 35733	7 6 2 3 4 4 1 1 * 1←14 2 3 4 4 1 1 * 1 3 4 2....2 45333←4 6 2...2 45333 2......2 35733
12	17 1 1 * * 1←18 2 * * 1 13 5 1 * * 1←14 6 * * 1 11 1 1 * 24333←12 2 * 24333 6 2....2 45333 3 2....2 35733←4 4 2..2 35733	18 1 1 * * 1←20 1 * * 1 12 1 1 * 24333←16 3 4 4 1 1 * 1 7 2....2 45333←8 4 2..2 45333 3 6 2...2 45333 1 2...2 3 3 6653←2 4 2 2 2 3 3 6653
11	18 1 * * 1←20 * * 1 17 2 * * 1←18 4 4 1 1 * 1 13 6 * * 1←14 8 4 1 1 * 1 11 2 * 24333←12 3 6 2 3 4 4 1 1 1 5 2...2 35733←6 3 6 2 2 45333 3 4 2..2 35733←4 5 6 2 2 45333 2...2 3 3 6653←8 2..2 35733	19 1 * * 1←21 * * 1 15 3 4 4 1 1 * 1←16 5 1 2 3 4 4 1 1 1 13 1 * 24333←19 4 4 1 1 * 1 9 2...2 45333←10 4 2 2 2 45333 7 4 2..2 45333←8 5 8 1 1 24333 6 2...2 35733 5 6 2..2 45333←6 7 8 1 1 24333 3 2..2 3 3 6653←4 3 5 6 2 45333
10	19 * * 1←22 4 1 1 * 1 17 4 4 1 1 * 1←20 1 2 3 4 4 1 1 1 13 8 4 1 1 * 1←14 12 1 1 * 1 11 3 6 2 3 4 4 1 1 1←12 8 1 1 24333 7 2..2 35733←8 3 6 2 45333 5 3 6 2 2 45333←6 5 6 2 45333 4 2 2 2 3 3 6653←9 6 2 2 45333 3 5 6 2 2 45333←4 7 6 2 45333 1 24333 6653←2 3 6 3 3 6653	15 5 1 2 3 4 4 1 1 1←16 6 2 3 4 4 1 1 1 14 * 24333←21 1 2 3 4 4 1 1 1 11 2..2 45333←12 4 2 2 45333 9 4 2 2 2 45333←10 6 2 2 45333 7 5 8 1 1 24333←8 7 13 1 * 1 5 7 8 1 1 24333←6 9 13 1 * 1 5 2 2 2 3 3 6653←6 3 6 2 35733 3 3 5 6 2 45333←4 5 6 2 35733 2 24333 6653

Stem 36 (filtrations ≥ 10)   Stem 37 (filtrations ≥ 10)

7	27 2 * 1←28 4 4 1 1 1 23 6 * 1←24 8 4 1 1 1 22 1 24333←24 24333 21 2 24333←22 3 6 2 3 3 18 2 45333←30 * 1 15 2 35733←16 3 6653 12 3 3 6653←18 35733 9 14 24333←10 15 6 2 3 3 6 11 35733←8 13 5 7 3 3 6 9 3 6653 5 9 2 3 5 7 7←6 10 4 5 7 7 4 5 7 3 5 7 7←8 9 3 5 7 7 2 3 5 7 7 7 7←4 59777	25 3 4 4 1 1 1←26 5 1 2 3 3 23 1 24333←25 24333 19 2 45333←20 4 7 3 3 3 13 3 3 6653←16 2 3 5 7 7 7 14 45333←8 15 8 3 3 3 7 9 3 6653←10 12 9 3 3 3 5 10 2 3 5 7 7←6 11 4 5 7 7 5 5 7 3 5 7 7←9 9 3 5 7 7 3 5 9 3 5 7 7 2 4 5 7 7 7 7←4 6 9 7 7 7
6	23 8 4 1 1 1←24 12 1 1 1 23 24333←25 6 2 3 3 21 3 6 2 3 3←22 8 3 3 3 20 45333←32 4 1 1 1 15 3 6653←19 6653 14 2 3 5 7 7←16 4 5 7 7 9 15 6 2 3 3←10 20 3 3 3 8 12 9 3 3 3 7 13 5 7 3 3←10 8 7 7 7 7 9 3 5 7 7←17 3 5 7 7 5 10 4 5 7 7←6 12 7 7 7 3 59777	25 5 1 2 3 3←26 6 2 3 3 21 45333←22 5 7 3 3 19 4 7 3 3 3←20 6653 15 2 3 5 7 7←17 4 5 7 7 9 12 9 3 3 3←10 13 11 3 3 7 14 9 3 3 3←8 15 11 3 3 6 11 3 5 7 7←8 13 5 7 7 3 6 9 7 7 7←8 11 7 7 7
5	31 4 1 1 1←32 5 1 1 23 12 1 1 1←24 13 1 1 21 8 3 3 3←36 1 1 1 20 9 3 3 3 15 4 5 7 7←18 7 7 7 10 19 3 3 3←12 21 3 3 9 8 7 7 7←10 15 7 7 7 14 11 3 3←8 15 13 3 6 11 7 7 7←8 13 11 7	30 1 2 3 3←32 2 3 3 21 9 3 3 3←22 10 5 3 21 5 7 3 3←26 6 5 3 9 13 11 3 3←10 14 13 3 7 13 5 7 7 7 11 7 7 7←9 13 11 7
4	35 1 1 1←36 2 1 31 5 1 1←32 6 1 29 3 3 3←33 3 3 24 6 5 3←37 1 1 23 13 1 1←24 14 1 21 11 3 3 17 7 7 7 11 21 3 3←13 23 3 7 13 11 7←9 15 15	31 2 3 3←34 3 3 30 3 3 3←32 5 3 25 6 5 3←26 7 7 22 11 3 3←24 13 3 21 10 5 3←22 11 7 20 5 7 7 9 14 13 3←10 15 15
3	35 2 1←36 3 31 6 1←32 7 30 5 3←38 1 23 14 1←24 15 22 13 3	31 5 3←33 7 25 7 7←37 3 23 13 3←25 15 21 11 7
2	31 7←39 23 15	
1		

Stem 38 (filtrations ≤ 7)   Stem 39 (filtrations ≤ 7)

12	19 1 1 * * 1←20 2 * * 1 15 5 1 * * 1←16 6 * * 1 13 1 1 * 24333←14 2 * 24333 8 2....2 45333←21 1 * * 1 7 13 1 * * 1←8 14 * * 1 3 4 2...2 35733←4 6 2..2 3 5 7 3 3 2....2 3 3 6653←8 2...2 35733	15 2 3 4 4 1 1 * 1←18 3 4 4 1 1 * 1 14 1 1 * 24333←16 1 * 24333 9 2....2 45333←10 4 2..2 45333 7 14 1 * * 1←8 15 * * 1 5 10 1 * 24333←6 11 * 24333 5 6 2...2 45333 3 2...2 3 3 6653←4 4 2 2 2 3 3 6653
11	19 2 * * 1←20 4 4 1 1 * 1 15 6 * * 1←16 8 4 1 1 * 1 14 1 * 24333←16 * 24333 13 2 * 24333←14 3 6 2 3 4 4 1 1 1 10 2...2 45333←22 * * 1 7 14 * * 1←8 16 4 1 1 * 1 7 2...2 35733←8 3 6 2 2 45333 4 2..2 3 3 6653←10 2..2 35733 3 6 2..2 35733←4 7 6 2 2 45333 1 2 24333 6653←2 3 5 6 2 35733	17 3 4 4 1 1 * 1←18 5 1 2 3 4 4 1 1 1 15 1 * 24333←17 * 24333 11 2...2 45333←12 4 2 2 2 45333 9 4 2..2 45333←10 5 8 1 1 24333 7 13 4 4 1 1 * 1←8 15 1 2 3 4 4 1 1 1 5 2..2 3 3 6653←6 3 5 6 2 45333 3 4 2 2 2 3 3 6653←4 5 5 6 2 45333 2 2 24333 6653←8 2 2 2 3 3 6653
10	15 8 4 1 1 * 1←16 12 1 1 * 1 15 * 24333←17 6 2 3 4 4 1 1 1 13 3 6 2 3 4 4 1 1 1←14 8 1 1 24333 12 2..2 45333←24 4 1 1 * 1 9 2..2 35733←10 3 6 2 45333 7 14 1 2 3 4 4 1 1 1←8 15 2 3 4 4 1 1 1 7 3 6 2 2 45333←8 5 6 2 45333 6 2 2 2 3 3 6653←11 6 2 2 45333 5 5 6 2 2 45333←6 7 6 2 45333 3 24333 6653←4 3 6 3 3 6653	17 5 1 2 3 4 4 1 1 1←18 6 2 3 4 4 1 1 1 13 2..2 45333←14 4 2 2 45333 11 4 2 2 2 45333←12 6 2 2 45333 9 5 8 1 1 24333←10 7 13 1 * 1 7 7 8 1 1 24333←8 9 13 1 * 1 7 2 2 2 3 3 6653←8 3 6 2 35733 5 3 5 6 2 45333←6 5 6 2 35733 4 24333 6653←9 5 6 2 45333 3 5 5 6 2 45333←4 7 6 2 35733 1 2 3 5 5 3 6653←2 3 6 5 2 3 5 7 7
9	23 4 1 1 * 1←24 5 1 * 1 15 12 1 1 * 1←16 13 1 * 1 13 8 1 1 24333←28 1 1 * 1 9 18 1 1 * 1←10 19 1 * 1 9 3 6 2 45333←10 6 2 35733 7 14 1 1 24333←8 15 1 24333 7 5 6 2 45333←13 6 2 45333 5 7 6 2 45333←6 10 2 35733 3 3 6 3 3 6653←4 6 5 2 3 5 7 7 2 3 5 5 3 6653	22 1 2 3 4 4 1 1 1←24 2 3 4 4 1 1 1 13 4 2 2 45333←14 6 2 45333 9 7 13 1 * 1←10 10 2 45333 7 9 13 1 * 1←8 12 2 45333 7 3 6 2 3 5 7 3 3←8 6 3 3 6653 5 5 6 2 35733←11 6 2 3 5 7 3 3 2 4 7 3 3 6653
8	27 1 1 * 1←28 2 * 1 23 5 1 * 1←24 6 * 1 21 1 1 24333←22 2 24333 15 13 1 * 1←29 1 * 1 9 13 1 24333←10 14 24333 9 6 2 35733←16 2 3 5 7 3 3 5 10 2 35733←6 12 3 5 7 3 3 5 7 3 3 6653←6 9 2 3 5 7 7 5 5 5 3 6653←9 12 45333 3 6 5 2 3 5 7 7	23 2 3 4 4 1 1 1←26 3 4 4 1 1 1 22 1 1 24333←24 1 24333 9 10 2 45333←10 12 45333 7 12 2 45333←8 14 45333 7 6 3 3 6653←14 3 3 6653 6 5 5 3 6653←8 9 3 6653 2 3 5 7 3 5 7 7←4 5 9 3 5 7 7 1 2 3 5 7 7 7 7←2 3 59777

Stem 38 (filtrations 8 to 12) Stem 39 (filtrations 8 to 12)

21	1 2 1 1 * * * * 1←2 3 1 * * * * 1	
20	3 1 1 * * * * 1←4 2 * * * * 1	4 1 1 * * * * 1
19	3 2 * * * * 1←4 4 4 1 1 * * * 1	5 1 * * * * 1
18	3 4 4 1 1 * * * 1	6 * * * * 1
17	7 4 1 1 * * * 1←8 5 1 * * * 1 5 1 2 3 4 4 1 1 * * 1	8 4 1 1 * * * 1 6 1 2 3 4 4 1 1 * * 1←8 2 3 4 4 1 1 * * 1 2 4 1 1 * * 24333
16	11 1 1 * * * 1←12 2 * * * 1 7 5 1 * * * 1←8 6 * * * 1 6 2 3 4 4 1 1 * * 1 5 1 1 * * 24333←6 2 * * 24333	12 1 1 * * * 1 7 2 3 4 4 1 1 * * 1←10 3 4 4 1 1 * * 1 6 1 1 * * 24333←8 1 * * 24333 1 2.......2 45333←2 4 2......2 45333
15	11 2 * * * 1←12 4 4 1 1 * * 1 7 6 * * * 1←8 8 4 1 1 * * 1 6 1 * * 24333←8 * * 24333 5 2 * * 24333←6 3 6 2 3 4 4 1 1 * 1 2........2 45333←14 * * * 1	13 1 * * * 1 9 3 4 4 1 1 * * 1←10 5 1 2 3 4 4 1 1 * 1 7 1 * * 24333←9 * * 24333 5 7 4 4 1 1 * * 1←6 9 1 2 3 4 4 1 1 * 1 3 2.......2 45333←4 4 2.....2 45333
14	7 8 4 1 1 * * 1←8 12 1 1 * * 1 7 * * 24333←9 6 2 3 4 4 1 1 * 1 5 3 6 2 3 4 4 1 1 * 1←6 8 1 1 * 24333 4 2......2 45333←16 4 1 1 * * 1 1 2......2 35733←2 3 6 2...2 45333	9 5 1 2 3 4 4 1 1 * 1←10 6 2 3 4 4 1 1 * 1 5 9 1 2 3 4 4 1 1 * 1←6 10 2 3 4 4 1 1 * 1 5 2......2 45333←6 4 2....2 45333 3 4 2.....2 45333←4 6 2....2 45333 2.......2 3 5 7 3 3
13	15 4 1 1 * * 1←16 5 1 * * 1 7 12 1 1 * * 1←8 13 1 * * 1 5 8 1 1 * 24333←20 1 1 * * 1 3 2.....2 35733←4 4 2...2 3 5 7 3 3	14 1 2 3 4 4 1 1 * 1←16 2 3 4 4 1 1 * 1 5 10 2 3 4 4 1 1 * 1←6 12 3 4 4 1 1 * 1 5 4 2....2 45333←6 6 2...2 45333 3 6 2....2 45333 1 2....2 3 3 6653←2 4 2..2 3 3 6653

Stem 38 (filtrations ≥ 13) Stem 39 (filtrations ≥ 13)

7	30 1 * 1←32 * 1 29 2 * 1←30 4 4 1 1 1 25 6 * 1←26 8 4 1 1 1 23 2 24333←24 3 6 2 3 3 20 2 45333←26 24333 17 2 35733←18 3 6653 7 12 35733←8 15 5 7 3 3 7 9 2 3 5 7 7←8 10 4 5 7 7 6 5 7 3 5 7 7←10 9 3 5 7 7 3 6 9 3 5 7 7←8 11 3 5 7 7 3 4 5 7 7 7 7←4 7 9 7 7 7	31 1 * 1←33 * 1 27 3 4 4 1 1 1←28 5 1 2 3 3 21 2 45333←22 4 7 3 3 3 18 2 35733←24 45333 15 3 3 6653←19 3 6653 11 12 45333 9 14 45333←10 15 8 3 3 3 7 13 35733←9 15 5 7 3 3 7 5 7 3 5 7 7←11 9 3 5 7 7 3 3 59777←6 6 9 7 7 7
6	31 * 1←34 4 1 1 1 29 4 4 1 1 1←32 1 2 3 3 25 8 4 1 1 1←26 12 1 1 1 23 3 6 2 3 3←24 8 3 3 3 22 45333←27 6 2 3 3 19 35733←21 6653 17 3 6653 7 11 3 5 7 7←8 14 5 7 7 7 10 4 5 7 7←8 12 7 7 7 6 12 3 5 7 7←9 13 5 7 7 5 59777	27 5 1 2 3 3←28 6 2 3 3 23 45333←24 5 7 3 3 21 4 7 3 3 3←22 6653 20 35733←25 8 3 3 3 17 2 3 5 7 7←24 9 3 3 3 11 12 9 3 3 3←12 13 11 3 3 9 15 8 3 3 3←12 21 3 3 3 7 12 3 5 7 7←8 15 5 7 7 6 59777←10 13 5 7 7 5 6 9 7 7 7←8 13 7 7 7 3 6 11 7 7 7←4 7 13 11 7
5	33 4 1 1 1←34 5 1 1 31 1 2 3 3←33 2 3 3 25 12 1 1 1←26 13 1 1 23 8 3 3 3←32 3 3 3 22 9 3 3 3←24 11 3 3 18 3 5 7 7 11 20 3 3 3←12 23 3 3 7 14 5 7 7 7 12 7 7 7←8 15 11 7	23 9 3 3 3←24 10 5 3 23 5 7 3 3←28 6 5 3 19 3 5 7 7←33 3 3 3 18 4 5 7 7←25 11 3 3 11 21 3 3 3←13 23 3 3 11 13 11 3 3←12 14 13 3 9 11 7 7 7 7 13 7 7 7←9 15 11 7
4	37 1 1 1←38 2 1 33 5 1 1←34 6 1 31 3 3 3←33 5 3 25 13 1 1←26 14 1 23 11 3 3←25 13 3 21 5 7 7←35 3 3 19 7 7 7 11 15 7 7	38 1 1 1←40 1 1 27 6 5 3←28 7 7 23 10 5 3←24 11 7 22 5 7 7←34 5 3 20 7 7 7←26 13 3 13 26 1 1←14 27 1 11 14 13 3←12 15 15 10 13 11 7
3	38 1 1←40 1 37 2 1←38 3 33 6 1←34 7 25 14 1←26 15	39 1 1←41 1 27 7 7←35 7 23 11 7←27 15 11 15 15
2	39 1←41	
1		

Stem 40 (filtrations ≤ 7) Stem 41 (filtrations ≤ 7)

12	21 1 1 * * 1←22 2 * * 1 17 5 1 * * 1←18 6 * * 1 15 1 1 * 24333←16 2 * 24333 10 2....2 45333←17 1 * 24333 9 13 1 * * 1←10 14 * * 1 3 6 2...2 35733 1 2 2 24333 6653←2 4 24333 6653	22 1 1 * * 1←24 1 * * 1 11 2....2 45333←12 4 2..2 45333 7 12 3 4 4 1 1 * 1←8 15 4 4 1 1 * 1 7 6 2...2 45333←14 2...2 45333 3 4 2..2 3 3 6653←4 6 2 2 2 3 3 6653 2..2 4 3 3 3 6653←8 2..2 3 3 6653
11	22 1 * * 1←24 * * 1 21 2 * * 1←22 4 4 1 1 * 1 17 6 * * 1←18 8 4 1 1 * 1 15 2 * 24333←16 3 6 2 3 4 4 1 1 1 12 2....4 5 3 3 3←18 * 24333 9 14 * * 1←10 16 4 1 1 * 1 9 2...2 35733←10 3 6 2 2 45333 6 2..2 3 3 6653 5 6 2..2 35733←6 7 6 2 2 45333 3 2 24333 6653←4 3 5 6 2 35733	23 1 * * 1←25 * * 1 19 3 4 4 1 1 * 1←20 5 1 2 3 4 4 1 1 1 13 2...2 45333←14 4 2 2 2 45333 11 4 2..2 45333←12 5 8 1 1 24333 10 2...2 35733←16 2..2 45333 7 2..2 3 3 6653←8 3 5 6 2 45333 4 2 24333 6653←10 2 2 2 3 3 6653 3 6 2 2 2 3 3 6653←4 7 5 6 2 45333 1 2 2 3 5 5 3 6653←2 3 4 7 3 3 6653 1 1 2 4 7 3 3 6653
10	23 * * 1←26 4 1 1 * 1 21 4 4 1 1 * 1←24 1 2 3 4 4 1 1 1 17 8 4 1 1 * 1←18 12 1 1 * 1 15 3 6 2 3 4 4 1 1 1←16 8 1 1 24333 14 2..2 45333←19 6 2 3 4 4 1 1 1 11 2..2 35733←12 3 6 2 45333 9 16 4 1 1 * 1←10 20 1 1 * 1 9 3 6 2 2 45333←10 5 6 2 45333 5 7 6 2 2 45333←6 9 6 2 45333 5 24333 6653←6 3 6 3 3 6653 3 3 5 6 2 35733←4 5 6 3 3 6653 2 2 3 5 5 3 6653←4 4 7 3 3 6653 1 2 4 7 3 3 6653	19 5 1 2 3 4 4 1 1 1←20 6 2 3 4 4 1 1 1 15 2..2 45333←16 4 2 2 45333 13 4 2 2 2 45333←14 6 2 2 45333 12 2..2 35733←17 8 1 1 24333 11 5 8 1 1 24333←12 7 13 1 * 1 9 2 2 2 3 3 6653←10 3 6 2 35733 7 3 5 6 2 45333←8 5 6 2 35733 6 24333 6653←11 5 6 2 45333 5 5 5 6 2 45333←6 7 6 2 35733 3 2 3 5 5 3 6653←4 3 6 5 2 3 5 7 7 2 2 4 7 3 3 6653
9	25 4 1 1 * 1←26 5 1 * 1 23 1 2 3 4 4 1 1 1←25 2 3 4 4 1 1 1 17 12 1 1 * 1←18 13 1 * 1 15 8 1 1 24333←24 1 1 24333 11 3 6 2 45333←12 6 2 35733 5 9 6 2 45333←6 12 2 35733 5 3 6 3 3 6653←6 6 5 2 3 5 7 7 3 5 6 3 3 6653←4 7 12 45333 3 4 7 3 3 6653←8 5 5 3 6653 1 2 3 5 7 3 5 7 7←2 3 5 9 3 5 7 7	15 4 2 2 45333←16 6 2 45333 13 6 2 2 45333←19 13 1 * 1 11 7 13 1 * 1←12 10 2 45333 9 3 6 2 35733←10 6 3 3 6653 7 5 6 2 35733←13 6 2 35733 5 7 6 2 35733←6 10 3 3 6653 3 5 5 5 3 6653←4 6 9 3 6653 3 3 6 5 2 3 5 7 7←5 7 12 45333 1 2 4 5 7 3 5 7 7←2 3 6 9 3 5 7 7
8	29 1 1 * 1←30 2 * 1 25 5 1 * 1←26 6 * 1 23 1 1 24333←24 2 24333 17 13 1 * 1←25 1 24333 7 10 2 35733←8 12 35733 7 5 5 3 6653←9 9 3 6653 5 9 3 3 6653←6 11 2 3 5 7 7 5 6 5 2 3 5 7 7 3 3 5 7 3 5 7 7←5 5 9 3 5 7 7 2 4 5 7 3 5 7 7←4 6 9 3 5 7 7 1 2 4 5 7 7 7 7←2 3 6 9 7 7 7	30 1 1 * 1←32 1 * 1 15 6 2 45333←22 2 45333 11 10 2 45333←12 12 45333 9 6 3 3 6653←16 3 3 6653 5 10 3 3 6653←8 13 35733 4 3 5 7 3 5 7 7←8 5 7 3 5 7 7 3 6 11 35733←4 7 13 5 7 3 3 3 6 9 3 6653←4 8 12 9 3 3 3 3 4 5 7 3 5 7 7←4 7 9 3 5 7 7

Stem 40 (filtrations 8 to 12)   Stem 41 (filtrations 8 to 12)

21		2 4 1 1 * * * * 1
20	5 1 1 * * * * 1←6 2 * * * * 1	6 1 1 * * * * 1←8 1 * * * * 1 1 2 3 4 4 1 1 * * * 1
19	6 1 * * * * 1←8 * * * * 1 5 2 * * * * 1←6 4 4 1 1 * * * 1 2 3 4 4 1 1 * * * 1	7 1 * * * * 1←9 * * * * 1 3 3 4 4 1 1 * * * 1←4 5 1 2 3 4 4 1 1 * * 1 1 1 * * * 24333
18	7 * * * * 1←10 4 1 1 * * * 1 5 4 4 1 1 * * * 1←8 1 2 3 4 4 1 1 * * 1 1 * * * 24333	3 5 1 2 3 4 4 1 1 * * 1←4 6 2 3 4 4 1 1 * * 1 2 * * * 24333
17	9 4 1 1 * * * 1←10 5 1 * * * 1 7 1 2 3 4 4 1 1 * * 1←9 2 3 4 4 1 1 * * 1 3 4 1 1 * * 24333←4 5 1 * * 24333	3 6 2 3 4 4 1 1 * * 1 1 2.........2 45333←2 4 2.......2 45333
16	13 1 1 * * * 1←14 2 * * * 1 9 5 1 * * * 1←10 6 * * * 1 7 1 1 * * 24333←8 2 * * 24333 3 5 1 * * 24333←4 6 * * 24333 2.........2 45333←9 1 * * 24333	14 1 1 * * * 1←16 1 * * * 1 8 1 1 * * 24333 5 10 1 * * * 1←6 11 * * * 1 3 6 1 * * 24333←4 7 * * 24333 3 2........2 45333←4 4 2......2 45333
15	14 1 * * * 1←16 * * * 1 13 2 * * * 1←14 4 4 1 1 * * 1 9 6 * * * 1←10 8 4 1 1 * * 1 7 2 * * 24333←8 3 6 2 3 4 4 1 1 * 1 4 2.......2 45333←10 * * 24333 3 6 * * 24333←4 7 6 2 3 4 4 1 1 * 1 1 2.......2 35733←2 3 6 2....2 45333	15 1 * * * 1←17 * * * 1 11 3 4 4 1 1 * * 1←12 5 1 2 3 4 4 1 1 * 1 5 9 4 4 1 1 * * 1←6 11 1 2 3 4 4 1 1 * 1 5 2.......2 45333←6 4 2.....2 45333 3 4 2......2 45333←4 5 8 1 1 * 24333 2........2 35733←8 2......2 45333
14	15 * * * 1←18 4 1 1 * * 1 13 4 4 1 1 * * 1←16 1 2 3 4 4 1 1 * 1 9 8 4 1 1 * * 1←10 12 1 1 * * 1 7 3 6 2 3 4 4 1 1 * 1←8 8 1 1 * 24333 6 2......2 45333←11 6 2 3 4 4 1 1 * 1 5 10 1 2 3 4 4 1 1 * 1←6 11 2 3 4 4 1 1 * 1 3 2......2 35733←4 3 6 2...2 45333	11 5 1 2 3 4 4 1 1 * 1←12 6 2 3 4 4 1 1 * 1 7 9 1 2 3 4 4 1 1 * 1←8 10 2 3 4 4 1 1 * 1 7 2......2 45333←8 4 2....2 45333 5 4 2.....2 45333←6 6 2....2 45333 4 2......2 35733←9 8 1 1 * 24333 3 5 8 1 1 * 24333←4 8 2....2 45333 1 2.....2 3 3 6653←2 3 6 2...2 35733
13	17 4 1 1 * * 1←18 5 1 * * 1 15 1 2 3 4 4 1 1 * 1←17 2 3 4 4 1 1 * 1 9 12 1 1 * * 1←10 13 1 * * 1 7 14 1 1 * * 1←8 15 1 * * 1 7 8 1 1 * 24333←16 1 1 * 24333 5 10 1 1 * 24333←6 11 1 * 24333 3 3 6 2...2 45333←4 6 2...2 35733 2.....2 3 3 6653	7 10 2 3 4 4 1 1 * 1←8 12 3 4 4 1 1 * 1 7 4 2....2 45333←8 6 2...2 45333 5 6 2....2 45333←12 2....2 45333 3 2....2 3 3 6653←4 4 2..2 3 3 6653

Stem 40 (filtrations ≥ 13)   Stem 41 (filtrations ≥ 13)

7	31 2 * 1←32 4 4 1 1 1 28 34411 1←34 * 1 27 6 * 1←28 8 4 1 1 1 26 1 24333←28 24333 25 2 24333←26 3 6 2 3 3 19 2 35733←20 3 6653 11 22 * 1←12 24 4 1 1 1 10 9 3 6653 7 14 35733←8 15 6653 6 5 9 3577←12 9 3577 5 6 9 3577←8 13 3577 4 3 59777←8 59777 3 6 11 3577←4 7 13 5 7 7 3 3 6 9 7 7 7←4 7 11 7 7 7	 29 34411 1←30 5 1 2 3 3 27 1 24333←29 24333 23 2 45333←24 4 7 3 3 3 20 2 35733←26 45333 17 3 3 6653←20 23577 11 21 4 4 1 1 1←12 23 1 2 3 3 11 9 3 6653←14 12 9 3 3 3 7 13 3 6653←9 15 6653 7 5 9 3577←13 9 3577 6 6 9 3577←8 14 3577 5 3 59777←9 59777 3 6 12 3577←4 7 14 5 7 7 3 5 59777←5 7 13 5 7 7
6	31 4 4 1 1 1←36 4 1 1 1 27 8 4 1 1 1←28 12 1 1 1 27 24333←29 6 2 3 3 25 3 6 2 3 3←26 8 3 3 3 21 35733←25 5 7 3 3 18 23577←20 4 5 7 7 12 12 9 3 3 3 11 22 1 2 3 3←12 23 2 3 3 9 11 3577←10 14 5 7 7 7 13 3577←9 15 5 7 7 7 59777←11 13 5 7 7	29 5 1 2 3 3←30 6 2 3 3 25 45333←26 5 7 3 3 23 4 7 3 3 3←24 6653 22 35733←27 8 3 3 3 19 23577←21 4 5 7 7 13 12 9 3 3 3←14 13 11 3 3 11 15 8 3 3 3←12 23 3 3 3 10 11 3577←12 13 5 7 7 7 14 3577←10 15 5 7 7 7 6 9 7 7 7←8 15 7 7 7 5 10 19 3 3 3←6 11 21 3 3
5	35 4 1 1 1←36 5 1 1 33 1 2 3 3←40 1 1 1 27 12 1 1 1←28 13 1 1 20 3577←24 5 7 7 19 4 5 7 7←22 7 7 7 13 26 1 1 1←14 27 1 1 11 22 3 3 3←12 23 5 3 9 14 5 7 7←12 13 11 7	 34 1 2 3 3←36 2 3 3 25 9 3 3 3←26 10 5 3 23 6653←30 6 5 3 21 3577←25 5 7 7 13 13 11 3 3←14 14 13 3
4	39 1 1 1←40 2 1 35 5 1 1←36 6 1 34 2 3 3←41 1 1 27 13 1 1←28 14 1 23 5 7 7←29 7 7 21 7 7 7←25 11 7 11 13 11 7←13 15 15	 35 2 3 3←38 3 3 34 3 3 3←36 5 3 29 6 5 3←30 7 7 26 11 3 3←28 13 3 25 10 5 3←26 11 7 13 14 13 3←14 15 15
3	39 2 1←40 3 36 3 3←42 1 35 6 1←36 7 27 14 1←28 15	 37 3 3←41 3 35 5 3←37 7 27 13 3←29 15
2	39 3←43	
1		
	Stem 42 (filtrations ≦ 7)	Stem 43 (filtrations ≦ 7)

11	23 2 * * 1←24 4 4 1 1 * 1 20 34411 * 1←26 * * 1 19 6 * * 1←20 8 4 1 1 * 1 18 1 * 24333←20 * 24333 17 2 * 24333←18 3 6 2 34411 1 11 14 * * 1←12 16 4 1 1 * 1 11 2...2 35733←12 3 6 2 2 45333 5 2 24333 6653←6 3 5 6 2 35733 3 4 24333 6653←4 5 5 6 2 35733 2 1 24733 6653←4 24733 6653 1 2 24733 6653←8 24333 6653	21 34411 * 1←22 5 1 2 34411 1 19 1 * 24333←21 * 24333 15 2...2 45333←16 4 2 2 2 45333 13 4 2..2 45333←14 5 8 1 1 24333 12 2...2 35733←18 2..2 45333 9 15 4 4 1 1 * 1←10 17 1 2 34411 1 9 2..2 3 3 6653←10 3 5 6 2 45333 6 2 24333 6653 5 6 2 2 2 3 3 6653←6 7 5 6 2 45333 3 1 24733 6653←5 24733 6653
10	23 4 4 1 1 * 1←28 4 1 1 * 1 19 8 4 1 1 * 1←20 12 1 1 * 1 19 * 24333←21 6 2 34411 1 17 3 6 2 34411 1←18 8 1 1 24333 13 2..2 35733←14 3 6 2 45333 11 16 4 1 1 * 1←12 20 1 1 * 1 11 3 6 2 2 45333←12 5 6 2 45333 7 24333 6653←8 3 6 3 3 6653 5 3 5 6 2 35733←6 5 6 3 3 6653 4 2 3 5 5 3 6653←9 5 6 2 35733 3 5 5 6 2 35733←4 7 6 3 3 6653 3 24733 6653←6 4 7 3 3 6653	21 5 1 2 34411 1←22 6 2 34411 1 17 2..2 45333←18 4 2 2 45333 15 4 2 2 2 45333←16 6 2 2 45333 14 2..2 35733←19 8 1 1 24333 13 5 8 1 1 24333←14 7 13 1 * 1 11 2 2 2 3 3 6653←12 3 6 2 35733 9 17 1 2 34411 1←10 18 2 34411 1 9 3 5 6 2 45333←10 5 6 2 35733 5 7 5 6 2 45333←6 9 6 2 35733 5 2 3 5 5 3 6653←6 3 6 5 23577 3 3 4 7 3 3 6653←4 5 6 5 23577 2 3 3 6 5 23577←4 7 5 5 3 6653 1 2 3 3 5 7 3577←2 3 5 5 7 3577
9	27 4 1 1 * 1←28 5 1 * 1 25 1 2 34411 1←32 1 1 * 1 19 12 1 1 * 1←20 13 1 * 1 13 3 6 2 45333←14 6 2 35733 11 20 1 1 * 1←12 21 1 * 1 7 9 6 2 45333←8 12 2 35733 7 3 6 3 3 6653←8 6 5 23577 5 5 6 3 3 6653←6 7 12 45333 5 4 7 3 3 6653←11 6 3 3 6653 4 5 5 5 3 6653 3 6 5 5 3 6653←4 7 9 3 6653 2 3 3 5 7 3577←4 5 5 7 3577	26 1 2 34411 1←28 2 34411 1 17 4 2 2 45333←18 6 2 45333 15 6 2 2 45333←21 13 1 * 1 13 7 13 1 * 1←14 10 2 45333 11 3 6 2 35733←12 6 3 3 6653 9 18 2 34411 1←10 20 34411 1 5 9 6 2 35733←6 12 3 3 6653 5 3 6 5 23577←6 6 9 3 6653 3 5 6 5 23577←9 6 5 23577 2 4 3 5 7 3577←4 6 5 7 3577
8	31 1 1 * 1←32 2 * 1 27 5 1 * 1←28 6 * 1 26 2 34411 1←33 1 * 1 25 1 1 24333←26 2 24333 11 21 1 * 1←12 22 * 1 9 5 5 3 6653 7 12 2 35733←8 14 35733 7 6 5 23577←13 12 45333 5 3 5 7 3577←9 5 7 3577 3 5 5 7 3577←5 7 9 3577 3 3 5 9 3577←5 8 12 9 3 3 3	27 2 34411 1←30 34411 1 26 1 1 24333←28 1 24333 17 6 2 45333←24 2 45333 13 10 2 45333←14 12 45333 11 22 1 * 1←12 23 * 1 10 5 5 3 6653←12 9 3 6653 9 18 1 24333←10 19 24333 7 10 3 3 6653←8 15 35733 6 11 3 3 6653←8 13 3 6653 6 3 5 7 3577←8 5 9 3577 5 6 9 3 6653←6 8 12 9 3 3 3 4 3 5 9 3577←10 5 7 3577 3 6 5 7 3577←6 7 9 3577 3 3 6 9 3577←4 6 12 3577 2 3 3 59777←4 5 59777

Stem 42 (filtrations 8 to 11) Stem 43 (filtrations 8 to 11)

15	15 2 * * * 1←16 4 4 1 1 * * 1 12 34411 * * 1←18 * * * 1 11 6 * * * 1←12 8 4 1 1 * * 1 10 1 * * 24333←12 * * 24333 9 2 * * 24333←10 3 6 2 34411 * 1 6 2.......2 45333 5 6 * * 24333←6 7 6 2 34411 * 1 3 2.......2 35733←4 3 6 2....2 45333	13 34411 * * 1←14 5 1 2 34411 * 1 11 1 * * 24333←13 * * 24333 7 2.......2 45333←8 4 2.....2 45333 5 4 2......2 45333←6 5 8 1 1 * 24333 4 2......2 35733←10 2......2 45333 3 6 2......2 45333←4 7 8 1 1 * 24333 1 2......2 3 3 6653←2 3 5 6 2...2 45333
14	15 4 4 1 1 * * 1←20 4 1 1 * * 1 11 8 4 1 1 * * 1←12 12 1 1 * * 1 11 * * 24333←13 6 2 34411 * 1 9 3 6 2 34411 * 1←10 8 1 1 * 24333 5 7 6 2 34411 * 1←6 12 1 1 * 24333 5 2......2 35733←6 3 6 2...2 45333 3 3 6 2....2 45333←4 5 6 2...2 45333 2......2 3 3 6653	13 5 1 2 34411 * 1←14 6 2 34411 * 1 9 2......2 45333←10 4 2....2 45333 7 4 2.....2 45333←8 6 2....2 45333 6 2......2 35733←11 8 1 1 * 24333 5 5 8 1 1 * 24333←6 8 2....2 45333 3 2.....2 3 3 6653←4 3 6 2...2 35733
13	19 4 1 1 * * 1←20 5 1 * * 1 17 1 2 34411 * 1←24 1 1 * * 1 11 12 1 1 * * 1←12 13 1 * * 1 5 3 6 2...2 45333←6 6 2...2 35733 3 5 6 2...2 45333 1 2..2 4 3 3 3 6653←2 4 2 24333 6653	18 1 2 34411 * 1←20 2 34411 * 1 9 4 2....2 45333←10 6 2...2 45333 7 6 2....2 45333←14 2....2 45333 5 8 2....2 45333←6 10 2...2 45333 3 3 6 2...2 35733←4 6 2..2 3 3 6653 2...2 4 3 3 3 6653
12	23 1 1 * * 1←24 2 * * 1 19 5 1 * * 1←20 6 * * 1 18 2 34411 * 1←25 1 * * 1 17 1 1 * 24333←18 2 * 24333 11 13 1 * * 1←12 14 * * 1 5 6 2...2 35733 3 2 2 24333 6653←4 4 24333 6653 1 1 1 24733 6653←2 2 24733 6653	19 2 34411 * 1←22 34411 * 1 18 1 1 * 24333←20 1 * 24333 13 2....2 45333←14 4 2..2 45333 9 6 2...2 45333←16 2...2 45333 5 10 2...2 45333←6 12 2..2 45333 3 6 2..2 3 3 6653 2 1 1 24733 6653←4 1 24733 6653 1 2 1 24733 6653←2 3 24733 6653 1 1 2 24733 6653←2 4 2 3 5 5 3 6653

Stem 42 (filtrations 12 to 15)

Stem 43 (filtrations 12 to 15)

23		1 1 * * * * * 1
22	1 * * * * * 1	2 * * * * * 1 1 1 1 2 34411 * * * 1←2 2 2 34411 * * * 1
21	3 4 1 1 * * * * 1←4 5 1 * * * * 1 1 1 2 34411 * * * 1←8 1 1 * * * * 1	4 4 1 1 * * * * 1 2 1 2 34411 * * * 1←4 2 34411 * * * 1 1 2 2 34411 * * * 1←2 4 34411 * * * 1
20	7 1 1 * * * * 1←8 2 * * * * 1 3 5 1 * * * * 1←4 6 * * * * 1 2 2 34411 * * * 1←9 1 * * * * 1 1 1 1 * * * 24333←2 2 * * * 24333	3 6 1 * * * * 1←4 7 * * * * 1 3 2 34411 * * * 1←6 34411 * * * 1 2 1 1 * * * 24333←4 1 * * * 24333 1 2 1 * * * 24333←2 3 * * * 24333
19	7 2 * * * * 1←8 4 4 1 1 * * * 1 4 34411 * * * 1←10 * * * * 1 3 6 * * * * 1←4 8 4 1 1 * * * 1 2 1 * * * 24333←4 * * * 24333 1 2 * * * 24333←2 3 6 2 34411 * * 1	5 34411 * * * 1←6 5 1 2 34411 * * 1 3 5 4 4 1 1 * * * 1←4 7 1 2 34411 * * 1 3 1 * * * 24333←5 * * * 24333
18	7 4 4 1 1 * * * 1←12 4 1 1 * * * 1 3 6 1 2 34411 * * 1←4 7 2 34411 * * 1 3 * * * 24333←5 6 2 34411 * * 1	5 5 1 2 34411 * * 1←6 6 2 34411 * * 1 1 2..........2 45333←2 4 2........2 45333
17	11 4 1 1 * * * 1←12 5 1 * * * 1 9 1 2 34411 * * 1←16 1 1 * * * 1 5 10 1 1 * * * 1←6 11 1 * * * 1 3 6 1 1 * * 24333←4 7 1 * * 24333 2..........2 45333	10 1 2 34411 * * 1←12 2 34411 * * 1 3 2.........2 45333←4 4 2.......2 45333
16	15 1 1 * * * 1←16 2 * * * 1 11 5 1 * * * 1←12 6 * * * 1 10 2 34411 * * 1←17 1 * * * 1 9 1 1 * * 24333←10 2 * * 24333 5 5 1 * * 24333←6 6 * * 24333 4 2........2 45333 1 2........2 35733←2 4 2......2 35733	11 2 34411 * * 1←14 34411 * * 1 10 1 1 * * 24333←12 1 * * 24333 5 2........2 45333←6 4 2......2 45333 3 4 2.......2 45333←4 6 2......2 45333 2.........2 35733←8 2.......2 45333

Stem 42 (filtrations ≥ 16)   Stem 43 (filtrations ≥ 16)

7	34 1 * 1←36 * 1 33 2 * 1←34 4 4 1 1 1 29 6 * 1←30 8 4 1 1 1 27 2 24333←28 3 6 2 3 3 21 2 35733←22 3 6653 18 3 3 6653←24 35733 13 22 * 1←14 24 4 1 1 1 9 14 35733←10 15 6653 7 13 2 3577←8 14 4 5 7 7 7 6 9 3577←8 15 3577 6 3 59777←10 59777 3 6 59777←6 7 13 5 7 7 3 5 6 9 7 7 7←4 7 13 7 7 7	35 1 * 1←37 * 1 31 34411 1←32 5 1 2 3 3 29 1 24333←35 4 4 1 1 1 25 2 45333←26 4 7 3 3 3 19 3 3 6653←25 35733 15 12 45333←28 45333 7 14 2 3577←8 15 4 5 7 7 7 8 12 9 3 3 3 7 7 9 3577←9 15 3577 7 3 59777←9 14 4 5 7 7 5 5 59777←6 7 14 5 7 7
6	35 * 1←38 4 1 1 1 33 4 4 1 1 1←36 1 2 3 3 29 8 4 1 1 1←30 12 1 1 1 27 3 6 2 3 3←28 8 3 3 3 23 35733←25 6653 21 3 6653←27 5 7 3 3 13 24 4 1 1 1←14 28 1 1 1 11 11 3577←12 14 5 7 7 7 14 4 5 7 7	31 5 1 2 3 3←32 6 2 3 3 30 24333←37 1 2 3 3 27 45333←28 5 7 3 3 25 4 7 3 3 3←26 6653 21 2 3577←29 8 3 3 3 15 12 9 3 3 3←16 13 11 3 3 14 9 3577←24 3577 12 11 3577 9 18 9 3 3 3←10 19 11 3 3 5 7 14 5 7 7←6 11 15 7 7
5	37 4 1 1 1←38 5 1 1 35 1 2 3 3←37 2 3 3 29 12 1 1 1←30 13 1 1 26 9 3 3 3←28 11 3 3 22 3577←26 5 7 7 11 14 5 7 7 9 18 11 3 3←10 19 13 3	31 6 2 3 3←38 2 3 3 27 9 3 3 3←28 10 5 3 23 3577←27 5 7 7 22 4 5 7 7←32 6 5 3 15 13 11 3 3←16 14 13 3 13 13 5 7 7 9 15 7 7 7
4	41 1 1 1←42 2 1 37 5 1 1←38 6 1 35 3 3 3←37 5 3 29 13 1 1←30 14 1 27 11 3 3←29 13 3 23 7 7 7←27 11 7 13 13 11 7	42 1 1 1←44 1 1 36 3 3 3←40 3 3 31 6 5 3←32 7 7 27 10 5 3←28 11 7 24 7 7 7←38 5 3 15 14 13 3←16 15 15 14 13 11 7
3	42 1 1←44 1 41 2 1←42 3 37 6 1←38 7 29 14 1←30 15	43 1 1←45 1 39 3 3←43 3 31 7 7←39 7 15 15 15
2	43 1←45	
1		
	Stem 44 (filtrations ≤ 7)	Stem 45 (filtrations ≤ 7)

	27 * * 1←30 4 1 1 * 1	
	25 4 4 1 1 * 1←28 1 2 34411 1	23 5 1 2 34411 1←24 6 2 34411 1
	21 8 4 1 1 * 1←22 12 1 1 * 1	22 * 24333←29 1 2 34411 1
	19 3 6 2 34411 1←20 8 1 1 24333	19 2..2 45333←20 4 2 2 45333
	15 2..2 35733←16 3 6 2 45333	17 4 2 2 2 45333←18 6 2 2 45333
	13 16 4 1 1 * 1←14 20 1 1 * 1	15 5 8 1 1 24333←16 7 13 1 * 1
	13 3 6 2 2 45333←14 5 6 2 45333	13 2 2 2 3 3 6653←14 3 6 2 35733
	12 2 2 2 3 3 6653←17 6 2 2 45333	11 17 1 2 34411 1←12 18 2 34411 1
	9 18 1 2 34411 1←10 19 2 34411 1	11 3 5 6 2 45333←12 5 6 2 35733
	9 24333 6653←10 3 6 3 3 6653	10 24333 6653←21 8 1 1 24333
	7 3 5 6 2 35733←8 5 6 3 3 6653	7 13 8 1 1 24333←8 15 13 1 * 1
	6 2 3 5 5 3 6653←8 4 7 3 3 6653	7 2 3 5 5 3 6653←8 3 6 5 2 3577
	5 5 5 6 2 35733←6 7 6 3 3 6653	3 5 4 7 3 3 6653←4 7 6 5 2 3577
	2 4 5 5 5 3 6653	3 4 5 5 5 3 6653←6 5 6 5 2 3577
10	1 2 4 3 5 7 3577←2 3 6 5 7 3577	1 2 3 3 5 9 3577←4 5 3 5 7 3577
	29 4 1 1 * 1←30 5 1 * 1	
	27 1 2 34411 1←29 2 34411 1	
	21 12 1 1 * 1←22 13 1 * 1	
	15 3 6 2 45333←16 6 2 35733	
	13 20 1 1 * 1←14 21 1 * 1	23 6 2 34411 1←30 2 34411 1
	13 5 6 2 45333←19 6 2 45333	19 4 2 2 45333←20 6 2 45333
	11 22 1 1 * 1←12 23 1 * 1	15 7 13 1 * 1←16 10 2 45333
	9 18 1 1 24333←10 19 1 24333	13 3 6 2 35733←14 6 3 3 6653
	9 3 6 3 3 6653←10 6 5 2 3577	11 18 2 34411 1←12 20 34411 1
	7 5 6 3 3 6653←8 7 12 45333	11 5 6 2 35733←23 13 1 * 1
	7 4 7 3 3 6653←12 5 5 3 6653	7 9 6 2 35733←8 12 3 3 6653
	5 7 6 3 3 6653←8 11 3 3 6653	7 3 6 5 2 3577←8 6 9 3 6653
	3 5 10 3 3 6653←4 7 13 35733	5 5 6 5 2 3577
	3 4 3 5 7 3577←4 7 5 7 3577	3 5 3 5 7 3577←4 6 5 9 3577
	2 3 3 5 9 3577←8 3 5 7 3577	2 4 3 5 9 3577←5 7 5 7 3577
9	1 2 3 3 59777←2 3 5 59777	1 2 4 3 59777←2 3 6 59777
	33 1 1 * 1←34 2 * 1	34 1 1 * 1←36 1 * 1
	29 5 1 * 1←30 6 * 1	28 1 1 24333←32 34411 1
	27 1 1 24333←28 2 24333	15 10 2 45333←16 12 45333
	15 6 2 35733←22 2 35733	13 6 3 3 6653←26 2 45333
	13 21 1 * 1←14 22 * 1	11 20 34411 1←12 23 4 4 1 1 1
	11 5 5 3 6653←13 9 3 6653	7 12 3 3 6653←8 15 3 6653
	7 11 3 3 6653←8 13 2 3577	7 6 9 3 6653←8 8 12 9 3 3 3
	7 7 12 45333←9 13 3 6653	6 3 5 9 3577←10 5 9 3577
	7 3 5 7 3577←9 5 9 3577	3 6 5 9 3577←8 7 9 3577
	5 3 5 9 3577←11 5 7 3577	3 5 6 9 3577←4 7 13 3577
8	2 4 3 59777←4 6 59777	3 4 3 59777←4 7 59777

Stem 44 (filtrations 8 to 10) Stem 45 (filtrations 8 to 10)

15	18 1 * * * 1←20 * * * 1 17 2 * * * 1←18 4 4 1 1 * * 1 13 6 * * * 1←14 8 4 1 1 * * 1 11 2 * * 24333←12 3 6 2 34411 * 1 5 2.......2 35733←6 3 6 2....2 45333 3 4 2......2 35733←4 5 6 2....2 45333 2.......2 3 3 6653←8 2......2 35733	19 1 * * * 1←21 * * * 1 15 34411 * * 1←16 5 1 2 34411 * 1 13 1 * * 24333←19 4 4 1 1 * * 1 9 2.......2 45333←10 4 2.....2 45333 7 4 2......2 45333←8 5 8 1 1 * 24333 6 2.......2 35733 5 6 2......2 45333←6 7 8 1 1 * 24333 3 2......2 3 3 6653←4 3 5 6 2...2 45333
14	19 * * * 1←22 4 1 1 * * 1 17 4 4 1 1 * * 1←20 1 2 34411 * 1 13 8 4 1 1 * * 1←14 12 1 1 * * 1 11 3 6 2 34411 * 1←12 8 1 1 * 24333 7 2......2 35733←8 3 6 2...2 45333 5 3 6 2....2 45333←6 5 6 2...2 45333 4 2.....2 3 3 6653←9 6 2....2 45333 3 5 6 2....2 45333←4 7 6 2...2 45333 1 2...2 4 3 3 3 6653←2 3 6 2..2 3 3 6653	15 5 1 2 34411 * 1←16 6 2 34411 * 1 14 * * 24333←21 1 2 34411 * 1 11 2......2 45333←12 4 2....2 45333 9 4 2.....2 45333←10 6 2....2 45333 7 5 8 1 1 * 24333←8 8 2....2 45333 5 7 8 1 1 * 24333←6 10 2....2 45333 5 2.....2 3 3 6653←6 3 6 2...2 35733 3 3 5 6 2...2 45333←4 5 6 2...2 35733 2....2 4 3 3 3 6653←13 8 1 1 * 24333
13	21 4 1 1 * * 1←22 5 1 * * 1 19 1 2 34411 * 1←21 2 34411 * 1 13 12 1 1 * * 1←14 13 1 * * 1 7 12 1 1 * 24333←8 13 1 * 24333 7 3 6 2...2 45333←8 6 2...2 35733 5 5 6 2...2 45333←11 6 2...2 45333 3 2..2 4 3 3 3 6653←4 4 2 24333 6653 1 2 1 1 24733 6653←2 3 1 24733 6653	15 6 2 34411 * 1←22 2 34411 * 1 11 4 2....2 45333←12 6 2...2 45333 7 8 2....2 45333←8 10 2...2 45333 5 10 2....2 45333←6 12 2...2 45333 5 3 6 2...2 35733←6 6 2..2 3 3 6653 3 5 6 2...2 35733←16 2....2 45333 1 2 1 2 24733 6653←2 3 2 24733 6653
12	25 1 1 * * 1←26 2 * * 1 21 5 1 * * 1←22 6 * * 1 19 1 1 * 24333←20 2 * 24333 13 13 1 * * 1←14 14 * * 1 7 13 1 * 24333←8 14 * 24333 7 6 2...2 35733←14 2...2 35733 3 4 2 24333 6653←4 6 24333 6653 3 1 1 24733 6653←4 2 24733 6653 2 1 2 24733 6653←5 1 24733 6653	26 1 1 * * 1←28 1 * * 1 20 1 1 * 24333←24 34411 * 1 15 2....2 45333←16 4 2..2 45333 9 18 1 * * 1←10 19 * * 1 7 14 1 * 24333←8 15 * 24333 7 10 2...2 45333←8 12 2..2 45333 5 6 2..2 3 3 6653←18 2...2 45333 4 1 1 24733 6653 3 1 2 24733 6653←4 4 2 3 5 5 3 6653
11	26 1 * * 1←28 * * 1 25 2 * * 1←26 4 4 1 1 * 1 21 6 * * 1←22 8 4 1 1 * 1 19 2 * 24333←20 3 6 2 34411 1 13 14 * * 1←14 16 4 1 1 * 1 13 2...2 35733←14 3 6 2 2 45333 10 2..2 3 3 6653←16 2..2 35733 7 14 * 24333←8 15 6 2 34411 1 7 2 24333 6653←8 3 5 6 2 35733 3 6 24333 6653←4 7 5 6 2 35733 3 2 24733 6653←6 24733 6653 1 2 3 3 6 5 2 3577←2 3 5 6 5 2 3577	27 1 * * 1←29 * * 1 23 34411 * 1←24 5 1 2 34411 1 21 1 * 24333←27 4 4 1 1 * 1 17 2...2 45333←18 4 2 2 2 45333 15 4 2..2 45333←16 5 8 1 1 24333 11 2..2 3 3 6653←12 3 5 6 2 45333 9 17 4 4 1 1 * 1←10 19 1 2 34411 1 8 2 24333 6653←20 2..2 45333 7 12 2..2 45333←8 13 8 1 1 24333 3 4 2 3 5 5 3 6653←4 5 4 7 3 3 6653 1 2 4 5 5 5 3 6653←4 4 5 5 5 3 6653

Stem 44 (filtrations 11 to 15)

Stem 45 (filtrations 11 to 15)

24	1 1 1 * * * * * 1←2 2 * * * * * 1	2 1 1 * * * * * 1←4 1 * * * * * 1 1 2 1 * * * * * 1←2 3 * * * * * 1
23	2 1 * * * * * 1←4 * * * * * 1 1 2 * * * * * 1←2 4 4 1 1 * * * * 1	3 1 * * * * * 1←5 * * * * * 1 1 2 1 1 2 34411 * * * 1←2 3 1 2 34411 * * * 1
22	3 * * * * * 1←6 4 1 1 * * * * 1 2 1 1 2 34411 * * * 1←4 1 2 34411 * * * 1 1 2 1 2 34411 * * * 1←2 3 2 34411 * * * 1	3 1 1 2 34411 * * * 1←4 2 2 34411 * * * 1
21	5 4 1 1 * * * * 1←6 5 1 * * * * 1 3 6 1 1 * * * * 1←4 7 1 * * * * 1 3 1 2 34411 * * * 1←5 2 34411 * * * 1 1 2 1 1 * * * 24333←2 3 1 * * * 24333	3 2 2 34411 * * * 1←4 4 34411 * * * 1
20	9 1 1 * * * * 1←10 2 * * * * 1 5 5 1 * * * * 1←6 6 * * * * 1 3 1 1 * * * 24333←4 2 * * * 24333	10 1 1 * * * * 1←12 1 * * * * 1 4 1 1 * * * 24333←8 34411 * * * 1 3 4 34411 * * * 1←4 7 4 4 1 1 * * * 1
19	10 1 * * * * 1←12 * * * * 1 9 2 * * * * 1←10 4 4 1 1 * * * 1 5 6 * * * * 1←6 8 4 1 1 * * * 1 3 2 * * * 24333←4 3 6 2 34411 * * 1	11 1 * * * * 1←13 * * * * 1 7 34411 * * * 1←8 5 1 2 34411 * * 1 5 1 * * * 24333←11 4 4 1 1 * * * 1 1 2..........2 45333←2 4 2.........2 45333
18	11 * * * * 1←14 4 1 1 * * * 1 9 4 4 1 1 * * * 1←12 1 2 34411 * * 1 5 8 4 1 1 * * * 1←6 12 1 1 * * * 1 3 3 6 2 34411 * * 1←4 8 1 1 * * 24333 2...........2 45333	7 5 1 2 34411 * * 1←8 6 2 34411 * * 1 6 * * * 24333←13 1 2 34411 * * 1 3 2..........2 45333←4 4 2........2 45333
17	13 4 1 1 * * * 1←14 5 1 * * * 1 11 1 2 34411 * * 1←13 2 34411 * * 1 4 2.........2 45333 1 2.........2 35733←2 4 2.......2 35733	7 6 2 34411 * * 1←14 2 34411 * * 1 3 4 2........2 45333←4 6 2.......2 45333 2..........2 35733
16	17 1 1 * * * 1←18 2 * * * 1 13 5 1 * * * 1←14 6 * * * 1 11 1 1 * * 24333←12 2 * * 24333 6 2........2 45333 3 2........2 35733←4 4 2......2 35733	18 1 1 * * * 1←20 1 * * * 1 12 1 1 * * 24333←16 34411 * * 1 3 6 2.......2 45333 1 2.......2 3 3 6653←2 4 2.....2 3 3 6653

Stem 44 (filtrations ≧ 16)   Stem 45 (filtrations ≧ 16)

	35 2 * 1←36 4 4 1 1 1	
	31 6 * 1←32 8 4 1 1 1	
	30 1 24333←32 24333	33 34411 1←34 5 1 2 3 3
	29 2 24333←30 3 6 2 3 3	31 1 24333←33 24333
	23 2 35733←24 3 6653	27 2 45333←28 4 7 3 3 3
	20 3 3 6653←26 35733	21 3 3 6653←24 2 3577
	15 22 * 1←16 24 4 1 1 1	15 9 3 6653←18 12 9 3 3 3
	14 9 3 6653←38 * 1	13 23 4 4 1 1 1←14 25 1 2 3 3
	12 5 7 3577←16 9 3577	13 5 7 3577←17 9 3577
	9 13 2 3577←10 14 4 5 7 7	11 5 9 3577←25 3 6653
	8 3 59777	9 15 3 6653←10 20 9 3 3 3
	6 5 59777←8 7 13 5 7 7	9 3 59777
7	5 9 15 8 3 3 3←6 11 21 3 3 3	7 5 59777←8 7 14 5 7 7
	31 8 4 1 1 1←32 12 1 1 1	
	31 24333←33 6 2 3 3	33 5 1 2 3 3←34 6 2 3 3
	29 3 6 2 3 3←30 8 3 3 3	29 45333←30 5 7 3 3
	23 3 6653←27 6653	27 4 7 3 3 3←28 6653
	22 2 3577←24 4 5 7 7	23 2 3577←25 4 5 7 7
	16 12 9 3 3 3←40 4 1 1 1	17 12 9 3 3 3←18 13 11 3 3
	15 24 4 1 1 1←16 28 1 1 1	14 11 3577←16 13 5 7 7
	15 9 3577←25 3577	13 25 1 2 3 3←14 26 2 3 3
	13 11 3577←14 14 5 7 7	12 59777←26 3577
	11 59777	7 7 14 5 7 7←8 11 15 7 7
	7 7 13 5 7 7←9 17 7 7 7	6 9 11 7 7 7
6	5 9 11 7 7 7←6 10 13 11 7	5 9 14 5 7 7←6 11 13 11 7
	39 4 1 1 1←40 5 1 1	38 1 2 3 3←40 2 3 3
	31 12 1 1 1←32 13 1 1	29 9 3 3 3←30 10 5 3
	28 9 3 3 3←44 1 1 1	29 5 7 3 3←34 6 5 3
	23 4 5 7 7←26 7 7 7	17 13 11 3 3←18 14 13 3
	15 28 1 1 1←16 29 1 1	15 13 5 7 7←29 5 7 7
	14 13 5 7 7←28 5 7 7	13 26 2 3 3←14 28 3 3
	13 14 5 7 7←16 13 11 7	13 25 3 3 3←14 26 5 3
5	5 10 13 11 7←6 11 15 15	7 11 15 7 7
		39 2 3 3←42 3 3
	43 1 1 1←44 2 1	38 3 3 3←40 5 3
	39 5 1 1←40 6 1	33 6 5 3←34 7 7
	37 3 3 3←41 3 3	30 11 3 3←32 13 3
	31 13 1 1←32 14 1	29 10 5 3←30 11 7
	29 11 3 3←45 1 1	17 14 13 3←18 15 15
	25 7 7 7←33 7 7	15 30 1 1←16 31 1
	15 29 1 1←16 30 1	14 27 3 3←16 29 3
4	15 13 11 7←17 15 15	13 26 5 3←14 27 7
	43 2 1←44 3	
	39 6 1←40 7	39 5 3←41 7
	31 14 1←32 15	31 13 3←33 15
	30 13 3←46 1	29 11 7←45 3
3	15 30 1←16 31	15 29 3←17 31
2	31 15←47	

Stem 46 (filtrations ≤ 7)   Stem 47 (filtrations ≤ 7)

Filtration	Stem 46	Stem 47
11	27 2 * * 1←28 4 4 1 1 * 1 23 6 * * 1←24 8 4 1 1 * 1 22 1 * 24333←24 * 24333 21 2 * 24333←22 3 6 2 34411 1 15 14 * * 1←16 16 4 1 1 * 1 15 2...2 35733←16 3 6 2 2 45333 12 2..2 3 3 6653←18 2..2 35733 9 14 * 24333←10 15 6 2 34411 1 9 2 24333 6653←10 3 5 6 2 35733 6 1 24733 6653←8 24733 6653 5 6 24333 6653←6 7 5 6 2 35733 5 2 24733 6653←30 * * 1 2 2 4 5 5 5 3 6653 1 1 2 3 3 5 9 3577←2 2 4 3 5 9 3577	25 34411 * 1←26 5 1 2 34411 1 23 1 * 24333←25 * 24333 19 2...2 45333←20 4 2 2 2 45333 17 4 2..2 45333←18 5 8 1 1 24333 13 2..2 3 3 6653←14 3 5 6 2 45333 10 2 24333 6653←16 2 2 2 3 3 6653 9 12 2..2 45333←10 13 8 1 1 24333 7 14 2..2 45333←8 15 8 1 1 24333 7 1 24733 6653←9 24733 6653 3 6 2 3 5 5 3 6653←4 7 4 7 3 3 6653 3 2 4 5 5 5 3 6653←6 4 5 5 5 3 6653 1 2 2 3 3 5 9 3577←2 3 4 3 5 9 3577
10	23 8 4 1 1 * 1←24 12 1 1 * 1 23 * 24333←25 6 2 34411 1 21 3 6 2 34411 1←22 8 1 1 24333 17 2..2 35733←18 3 6 2 45333 15 16 4 1 1 * 1←16 20 1 1 * 1 15 3 6 2 2 45333←16 5 6 2 45333 14 2 2 2 3 3 6653←19 6 2 2 45333 11 24333 6653←12 3 6 3 3 6653 9 15 6 2 34411 1←10 20 1 1 24333 9 3 5 6 2 35733←10 5 6 3 3 6653 8 2 3 5 5 3 6653←32 4 1 1 * 1 7 24733 6653←10 4 7 3 3 6653 5 7 5 6 2 35733←6 9 6 3 3 6653 3 3 5 6 5 2 3577←5 7 6 5 2 3577 2 2 3 3 5 9 3577←4 4 3 5 9 3577 1 2 4 3 5 9 3577←2 3 6 5 9 3577	25 5 1 2 34411 1←26 6 2 34411 1 21 2..2 45333←22 4 2 2 45333 19 4 2 2 2 45333←20 6 2 2 45333 17 5 8 1 1 24333←18 7 13 1 * 1 15 2 2 2 3 3 6653←16 3 6 2 35733 13 3 5 6 2 45333←14 5 6 2 35733 12 24333 6653←17 5 6 2 45333 9 13 8 1 1 24333←10 15 13 1 * 1 9 2 3 5 5 3 6653←10 3 6 5 2 3577 5 5 4 7 3 3 6653←6 7 6 5 2 3577 5 4 5 5 5 3 6653←8 5 6 5 2 3577 4 3 5 6 5 2 3577 3 5 7 6 3 3 6653←4 7 7 12 45333 3 2 3 3 5 9 3577←4 7 3 5 7 3577
9	31 4 1 1 * 1←32 5 1 * 1 23 12 1 1 * 1←24 13 1 * 1 17 3 6 2 45333←18 6 2 35733 15 20 1 1 * 1←16 21 1 * 1 15 5 6 2 45333←21 6 2 45333 11 3 6 3 3 6653←12 6 5 2 3577 9 5 6 3 3 6653←10 7 12 45333 9 4 7 3 3 6653←36 1 1 * 1 5 9 6 3 3 6653←6 11 12 45333 3 6 11 3 3 6653←4 7 13 3 6653 3 6 3 5 7 3577←4 7 5 9 3577 3 4 3 5 9 3577←5 6 5 9 3577	30 1 2 34411 1←32 2 34411 1 21 4 2 2 45333←22 6 2 45333 17 7 13 1 * 1←18 10 2 45333 15 3 6 2 35733←16 6 3 3 6653 13 5 6 2 35733←19 6 2 35733 9 15 13 1 * 1←10 18 2 45333 9 3 6 5 2 3577←10 6 9 3 6653 7 5 6 5 2 3577←13 6 5 2 3577 5 9 5 5 3 6653←6 10 9 3 6653 3 5 3 5 9 3577←5 7 5 9 3577
8	35 1 1 * 1←36 2 * 1 31 5 1 * 1←32 6 * 1 29 1 1 24333←30 2 24333 17 6 2 35733←24 2 35733 15 21 1 * 1←16 22 * 1 13 5 5 3 6653←17 12 45333 11 6 5 2 3577←37 1 * 1 9 7 12 45333←10 13 2 3577 9 3 5 7 3577 7 13 3 3 6653←8 15 2 3577 7 3 5 9 3577←9 8 12 9 3 3 3 3 6 6 9 3577←4 7 14 3577 3 5 3 59777←5 7 59777	31 2 34411 1←34 34411 1 30 1 1 24333←32 1 24333 17 10 2 45333←18 12 45333 15 6 3 3 6653←22 3 3 6653 14 5 5 3 6653←16 9 3 6653 10 3 5 7 3577←12 5 9 3577 9 18 2 45333←10 20 45333 9 6 9 3 6653←10 8 12 9 3 3 3 8 3 5 9 3577 5 10 9 3 6653←6 12 12 9 3 3 3 4 5 3 59777 3 6 3 59777←4 7 14 4 5 7 7

Stem 46 (filtrations 8 to 11)   Stem 47 (filtrations 8 to 11)

	Stem 46 (filtrations 12 to 15)	Stem 47 (filtrations 12 to 15)
15	19 2 * * * 1←20 4 4 1 1 * * 1 15 6 * * * 1←16 8 4 1 1 * * 1 14 1 * * 24333←16 * * 24333 13 2 * * 24333←14 3 6 2 34411 * 1 10 2.......2 45333 7 2.......2 35733←8 3 6 2....2 45333 4 2......2 3 3 6653←10 2......2 35733 3 6 2......2 35733←4 7 6 2....2 45333 1 2....2 4 3 3 3 6653←2 3 5 6 2...2 35733	21 1 * * * 1 17 34411 * * 1←18 5 1 2 34411 * 1 15 1 * * 24333←17 * * 24333 11 2.......2 45333←12 4 2.....2 45333 7 13 4 4 1 1 * * 1←8 15 1 2 34411 * 1 5 2......2 3 3 6653←6 3 5 6 2...2 45333 3 4 2.....2 3 3 6653←4 5 5 6 2...2 45333 2.....2 4 3 3 3 6653←8 2.....2 3 3 6653
14	15 8 4 1 1 * * 1←16 12 1 1 * * 1 15 * * 24333←17 6 2 34411 * 1 13 3 6 2 34411 * 1←14 8 1 1 * 24333 12 2......2 45333 9 2......2 35733←10 3 6 2...2 45333 7 14 1 2 34411 * 1←8 15 2 34411 * 1 7 3 6 2....2 45333←8 5 6 2...2 45333 6 2.....2 3 3 6653←11 6 2....2 45333 5 5 6 2....2 45333←6 7 6 2...2 45333 3 2...2 4 3 3 3 6653←4 3 6 2..2 3 3 6653	22 * * * 1 17 5 1 2 34411 * 1←18 6 2 34411 * 1 13 2......2 45333←14 4 2....2 45333 11 4 2.....2 45333←12 6 2....2 45333 9 5 8 1 1 * 24333←10 8 2....2 45333 7 7 8 1 1 * 24333←8 10 2....2 45333 7 2.....2 3 3 6653←8 3 6 2...2 35733 5 3 5 6 2...2 45333←6 5 6 2...2 35733 4 2...2 4 3 3 3 6653←9 5 6 2...2 45333 3 5 5 6 2...2 45333←4 7 6 2...2 35733 1 2 2 1 2 24733 6653←2 3 6 2 24333 6653
13	23 4 1 1 * * 1←24 5 1 * * 1 15 12 1 1 * * 1←16 13 1 * * 1 9 18 1 1 * * 1←10 19 1 * * 1 9 3 6 2...2 45333←10 6 2...2 35733 7 14 1 1 * 24333←8 15 1 * 24333 7 5 6 2...2 45333←13 6 2...2 45333 5 7 6 2...2 45333←6 10 2...2 35733 3 3 6 2..2 3 3 6653←4 6 2 24333 6653 2 2 1 2 24733 6653←28 1 1 * * 1	24 4 1 1 * * 1 22 1 2 34411 * 1←24 2 34411 * 1 13 4 2....2 45333←14 6 2...2 45333 9 8 2....2 45333←10 10 2...2 45333 7 10 2....2 45333←8 12 2...2 45333 7 3 6 2...2 35733←8 6 2..2 3 3 6653 5 5 6 2...2 35733←11 6 2...2 35733 2 4 1 1 24733 6653
12	27 1 1 * * 1←28 2 * * 1 23 5 1 * * 1←24 6 * * 1 21 1 1 * 24333←22 2 * 24333 15 13 1 * * 1←16 14 * * 1 9 13 1 * 24333←10 14 * 24333 9 6 2...2 35733←16 2...2 35733 5 10 2...2 35733←6 12 2..2 35733 5 1 1 24733 6653←6 2 24733 6653 3 6 2 24333 6653←29 1 * * 1 1 1 2 4 5 5 5 3 6653←2 3 4 5 5 5 3 6653	23 2 34411 * 1←26 34411 * 1 22 1 1 * 24333←24 1 * 24333 17 2....2 45333←18 4 2..2 45333 9 10 2...2 45333←10 12 2..2 45333 7 12 2...2 45333←8 14 2..2 45333 7 6 2..2 3 3 6653←14 2..2 3 3 6653 6 1 1 24733 6653←8 1 24733 6653 1 2 2 4 5 5 5 3 6653←4 2 4 5 5 5 3 6653

25	1 2 1 1 * * * * * 1←2 3 1 * * * * * 1	
24	3 1 1 * * * * * 1←4 2 * * * * * 1	4 1 1 * * * * * 1
23	3 2 * * * * * 1←4 4 4 1 1 * * * * 1	5 1 * * * * * 1
22	3 4 4 1 1 * * * * 1	6 * * * * * 1
21	7 4 1 1 * * * * 1←8 5 1 * * * * 1 5 1 2 34411 * * * 1	8 4 1 1 * * * * 1 6 1 2 34411 * * * 1←8 2 34411 * * * 1 2 4 1 1 * * * 24333
20	11 1 1 * * * * 1←12 2 * * * * 1 7 5 1 * * * * 1←8 6 * * * * 1 6 2 34411 * * * 1 5 1 1 * * * 24333←6 2 * * * 24333	12 1 1 * * * * 1 7 2 34411 * * * 1←10 34411 * * * 1 6 1 1 * * * 24333←8 1 * * * 24333 1 2...........2 45333←2 4 2..........2 45333
19	11 2 * * * * 1←12 4 4 1 1 * * * 1 7 6 * * * * 1←8 8 4 1 1 * * * 1 6 1 * * * 24333←8 * * * 24333 5 2 * * * 24333←6 3 6 2 34411 * * 1 2............2 45333	13 1 * * * * 1 9 34411 * * * 1←10 5 1 2 34411 * * 1 7 1 * * * 24333←9 * * * 24333 5 7 4 4 1 1 * * * 1←6 9 1 2 34411 * * 1 3 2..........2 45333←4 4 2.........2 45333
18	7 8 4 1 1 * * * 1←8 12 1 1 * * * 1 7 * * * 24333←9 6 2 34411 * * 1 5 3 6 2 34411 * * 1←6 8 1 1 * * 24333 4 2..........2 45333 1 2..........2 35733←2 3 6 2.......2 45333	14 * * * * 1 9 5 1 2 34411 * * 1←10 6 2 34411 * * 1 5 9 1 2 34411 * * 1←6 10 2 34411 * * 1 5 2..........2 45333←6 4 2........2 45333 3 4 2.........2 45333←4 6 2........2 45333 2...........2 35733
17	15 4 1 1 * * * 1←16 5 1 * * * 1 7 12 1 1 * * * 1←8 13 1 * * * 1 5 8 1 1 * * 24333 3 2.........2 35733←4 4 2.......2 35733	16 4 1 1 * * * 1 14 1 2 34411 * * 1←16 2 34411 * * 1 5 10 2 34411 * * 1←6 12 34411 * * 1 5 4 2........2 45333←6 6 2.......2 45333 3 6 2........2 45333 1 2........2 3 3 6653←2 4 2......2 3 3 6653
16	19 1 1 * * * 1←20 2 * * * 1 15 5 1 * * * 1←16 6 * * * 1 13 1 1 * * 24333←14 2 * * 24333 7 13 1 * * * 1 3 4 2.......2 35733←4 6 2......2 35733 2........2 3 3 6653←8 2.......2 35733	20 1 1 * * * 1 15 2 34411 * * 1←18 34411 * * 1 14 1 1 * * 24333←16 1 * * 24333 7 14 1 * * * 1←8 15 * * * 1 5 10 1 * * 24333←6 11 * * 24333 5 6 2.......2 45333 3 2.......2 3 3 6653←4 4 2.....2 3 3 6653
	Stem 46 (filtrations ≥ 16)	Stem 47 (filtrations ≥ 16)

7	38 1 * 1←40 * 1 37 2 * 1←38 4 4 1 1 1 33 6 * 1←34 8 4 1 1 1 31 2 24333←32 3 6 2 3 3 28 2 45333←34 24333 25 2 35733←26 3 6653 17 22 * 1←18 24 4 1 1 1 14 5 7 3577←18 9 3577 11 22 24333←12 23 6 2 3 3 11 13 23577←12 14 4 5 7 7 10 3 59777←16 11 3577 8 5 59777 5 10 11 3577	39 1 * 1←41 * 1 35 34411 1←36 5 1 2 3 3 29 2 45333←30 4 7 3 3 3 26 2 35733←32 45333 23 3 3 6653←27 3 6653 15 5 7 3577←19 9 3577 13 25 4 4 1 1 1←14 27 1 2 3 3 11 20 45333←12 21 8 3 3 3 11 8 12 9 3 3 3←20 12 9 3 3 3 11 3 59777←13 14 4 5 7 7 9 5 59777←10 7 14 5 7 7 7 12 12 9 3 3 3←12 20 9 3 3 3 5 7 14 4 5 7 7←6 11 14 5 7 7
6	39 * 1←42 4 1 1 1 37 4 4 1 1 1←40 1 2 3 3 33 8 4 1 1 1←34 12 1 1 1 31 3 6 2 3 3←32 8 3 3 3 30 45333←35 6 2 3 3 27 35733←29 6653 17 24 4 1 1 1←18 28 1 1 1 15 11 3577←16 14 5 7 7 13 26 1 2 3 3←14 27 2 3 3 13 59777←17 13 5 7 7 11 14 4 5 7 7←27 3577 9 7 13 5 7 7 7 9 11 7 7 7←8 10 13 11 7	35 5 1 2 3 3←36 6 2 3 3 31 45333←32 5 7 3 3 29 4 7 3 3 3←30 6653 28 35733←33 8 3 3 3 25 23577←32 9 3 3 3 19 12 9 3 3 3←20 13 11 3 3 15 25 1 2 3 3←16 26 2 3 3 14 59777←18 13 5 7 7 11 21 8 3 3 3←12 24 6 5 3 11 20 9 3 3 3←12 21 11 3 3 10 7 13 5 7 7←12 17 7 7 7 9 18 3577←16 25 3 3 3 9 7 14 5 7 7←10 11 15 7 7 8 9 11 7 7 7
5	41 4 1 1 1←42 5 1 1 39 1 2 3 3←41 2 3 3 33 12 1 1 1←34 13 1 1 31 8 3 3 3←40 3 3 3 30 9 3 3 3←32 11 3 3 17 28 1 1 1←18 29 1 1 15 30 1 1 1←16 31 1 1 15 14 5 7 7←30 5 7 7 14 25 3 3 3←16 27 3 3 13 26 3 3 3←14 27 5 3 10 17 7 7 7 7 10 13 11 7←8 11 15 15	31 9 3 3 3←32 10 5 3 31 5 7 3 3←36 6 5 3 26 4 5 7 7←33 11 3 3 19 13 11 3 3←20 14 13 3 15 26 2 3 3←16 28 3 3 15 25 3 3 3←16 26 5 3 11 21 11 3 3←12 22 13 3 11 17 7 7 7←41 3 3 3 10 20 5 7 7←17 27 3 3 9 11 15 7 7
4	45 1 1 1←46 2 1 41 5 1 1←42 6 1 39 3 3 3←41 5 3 33 13 1 1←34 14 1 31 11 3 3←33 13 3 27 7 7 7←35 7 7 17 29 1 1←18 30 1 17 13 11 7←43 3 3 15 27 3 3←17 29 3 7 11 15 15	46 1 1 1←48 1 1 35 6 5 3←36 7 7 31 10 5 3←32 11 7 28 7 7 7←34 13 3 19 14 13 3←20 15 15 18 13 11 7←42 5 3 15 28 3 3←16 31 3 15 26 5 3←16 27 7 12 23 7 7←18 29 3 11 22 13 3←12 23 15
3	46 1 1←48 1 45 2 1←46 3 41 6 1←42 7 33 14 1←34 15 17 30 1←18 31	47 1 1←49 1 31 11 7←35 15 19 15 15←43 7 15 27 7←19 31
2	47 1←49	
1		
	Stem 48 (filtrations ≦ 7)	Stem 49 (filtrations ≦ 7)

	Stem 48	Stem 49
10	31 * * 1←34 4 1 1 * 1 29 4 4 1 1 * 1←32 1 2 34411 1 25 8 4 1 1 * 1←26 12 1 1 * 1 23 3 6 2 34411 1←24 8 1 1 24333 22 2..2 45333←27 6 2 34411 1 19 2..2 35733←20 3 6 2 45333 17 16 4 1 1 * 1←18 20 1 1 * 1 17 3 6 2 2 45333←18 5 6 2 45333 13 24333 6653←14 3 6 3 3 6653 11 3 5 6 2 35733←12 5 6 3 3 6653 10 2 3 5 5 3 6653←12 4 7 3 3 6653 7 13 6 2 2 45333←8 15 6 2 45333 3 5 5 6 5 2 3577←9 5 6 5 2 3577	27 5 1 2 34411 1←28 6 2 34411 1 23 2..2 45333←24 4 2 2 45333 21 4 2 2 2 45333←22 6 2 2 45333 20 2..2 35733←25 8 1 1 24333 19 5 8 1 1 24333←20 7 13 1 * 1 17 2 2 2 3 3 6653←18 3 6 2 35733 15 3 5 6 2 45333←16 5 6 2 35733 14 24333 6653←19 5 6 2 45333 11 13 8 1 1 24333←12 15 13 1 * 1 11 2 3 5 5 3 6653←12 3 6 5 2 3577 9 15 8 1 1 24333←10 17 13 1 * 1 7 4 5 5 5 3 6653←10 5 6 5 2 3577 5 7 4 7 3 3 6653←6 11 5 5 3 6653 2 3 5 3 5 9 3577←8 9 5 5 3 6653 1 2 3 5 3 59777←2 4 6 3 59777
9	33 4 1 1 * 1←34 5 1 * 1 31 1 2 34411 1←33 2 34411 1 25 12 1 1 * 1←26 13 1 * 1 23 8 1 1 24333←32 1 1 24333 19 3 6 2 45333←20 6 2 35733 17 20 1 1 * 1←18 21 1 * 1 13 3 6 3 3 6653←14 6 5 2 3577 11 20 1 1 24333←12 21 1 24333 11 5 6 3 3 6653←12 7 12 45333 11 4 7 3 3 6653←16 5 5 3 6653 7 9 6 3 3 6653←8 11 12 45333 6 9 5 5 3 6653←8 15 3 3 6653 5 10 5 5 3 6653←6 11 9 3 6653 4 5 3 5 9 3577 3 6 3 5 9 3577←4 7 8 12 9 3 3 3 3 3 6 5 9 3577←4 7 7 9 3577 2 3 5 3 59777	23 4 2 2 45333←24 6 2 45333 21 6 2 2 45333←27 13 1 * 1 19 7 13 1 * 1←20 10 2 45333 17 3 6 2 35733←18 6 3 3 6653 15 5 6 2 35733←21 6 2 35733 11 15 13 1 * 1←12 18 2 45333 11 3 6 5 2 3577←12 6 9 3 6653 9 17 13 1 * 1←10 20 2 45333 7 9 5 5 3 6653←8 10 9 3 6653 5 5 3 5 9 3577←17 5 5 3 6653 4 6 3 5 9 3577←9 11 12 45333 2 4 5 3 59777←4 8 3 59777
8	37 1 1 * 1←38 2 * 1 33 5 1 * 1←34 6 * 1 31 1 1 24333←32 2 24333 25 13 1 * 1←33 1 24333 17 21 1 * 1←18 22 * 1 15 5 5 3 6653←17 9 3 6653 11 21 1 24333←12 22 24333 11 7 12 45333←12 13 2 3577 11 3 5 7 3577←13 5 9 3577 9 3 5 9 3577←19 12 45333 7 11 12 45333←9 17 3 6653 4 6 3 59777	38 1 1 * 1←40 1 * 1 23 6 2 45333←30 2 45333 19 10 2 45333←20 12 45333 17 6 3 3 6653←24 3 3 6653 13 26 1 * 1←14 27 * 1 12 3 5 7 3577←16 5 7 3577 11 22 1 24333←12 23 24333 11 18 2 45333←12 20 45333 11 6 9 3 6653←12 8 12 9 3 3 3 10 3 5 9 3577←18 9 3 6653 7 10 9 3 6653←8 12 12 9 3 3 3 6 7 5 9 3577←10 17 3 6653 4 7 3 59777←6 7 14 4 5 7 7 3 6 5 59777←4 7 7 13 5 7 7

Stem 48 (filtrations 8 to 10)   Stem 49 (filtrations 8 to 10)

	Stem 48	Stem 49
15	22 1 * * * 1←24 * * * 1 21 2 * * * 1←22 4 4 1 1 * * 1 17 6 * * * 1←18 8 4 1 1 * * 1 15 2 * * 24333←16 3 6 2 34411 * 1 12 2.......2 45333←18 * * 24333 9 2.......2 35733←10 3 6 2....2 45333 6 2......2 3 3 6653 5 6 2......2 35733←6 7 6 2....2 45333 3 2....2 4 3 3 3 6653←4 3 5 6 2...2 35733	23 1 * * * 1←25 * * * 1 19 34411 * * 1←20 5 1 2 34411 * 1 13 2.......2 45333←14 4 2.....2 45333 10 2.......2 35733←16 2......2 45333 7 2......2 3 3 6653←8 3 5 6 2...2 45333 4 2....2 4 3 3 3 6653←10 2.....2 3 3 6653 3 6 2.....2 3 3 6653←4 7 5 6 2...2 45333 1 2 2 2 1 2 24733 6653←2 3 4 1 1 24733 6653 1 1 * 24733 6653
14	23 * * * 1←26 4 1 1 * * 1 21 4 4 1 1 * * 1←24 1 2 34411 * 1 17 8 4 1 1 * * 1←18 12 1 1 * * 1 15 3 6 2 34411 * 1←16 8 1 1 * 24333 14 2......2 45333←19 6 2 34411 * 1 11 2......2 35733←12 3 6 2...2 45333 9 3 6 2....2 45333←10 5 6 2...2 45333 5 7 6 2....2 45333←6 9 6 2...2 45333 5 2...2 4 3 3 3 6653←6 3 6 2..2 3 3 6653 3 3 5 6 2...2 35733←4 5 6 2..2 3 3 6653 2 2 2 1 2 24733 6653←4 4 1 1 24733 6653 1 * 24733 6653	19 5 1 2 34411 * 1←20 6 2 34411 * 1 15 2......2 45333←16 4 2....2 45333 13 4 2.....2 45333←14 6 2....2 45333 12 2......2 35733←17 8 1 1 * 24333 9 2.....2 3 3 6653←10 3 6 2...2 35733 7 3 5 6 2...2 45333←8 5 6 2...2 35733 6 2...2 4 3 3 3 6653←11 5 6 2...2 45333 5 5 5 6 2...2 45333←6 7 6 2...2 35733 3 2 2 1 2 24733 6653←4 3 6 2 24333 6653 2 * 24733 6653
13	25 4 1 1 * * 1←26 5 1 * * 1 23 1 2 34411 * 1←25 2 34411 * 1 17 12 1 1 * * 1←18 13 1 * * 1 15 8 1 1 * 24333←24 1 1 * 24333 11 3 6 2...2 45333←12 6 2...2 35733 5 9 6 2...2 45333←6 12 2...2 35733 5 3 6 2..2 3 3 6653←6 6 2 24333 6653 3 5 6 2..2 3 3 6653←4 8 2 24333 6653 3 4 1 1 24733 6653←8 1 1 24733 6653 1 1 2 2 4 5 5 5 3 6653←2 3 2 4 5 5 5 3 6653	15 4 2....2 45333←16 6 2...2 45333 13 6 2....2 45333←20 2....2 45333 9 3 6 2...2 35733←10 6 2..2 3 3 6653 7 5 6 2...2 35733←13 6 2...2 35733 5 7 6 2...2 35733←6 10 2..2 3 3 6653 3 3 6 2 24333 6653←4 5 2 24733 6653 1 2 2 2 4 5 5 5 3 6653←4 2 2 4 5 5 5 3 6653
12	29 1 1 * * 1←30 2 * * 1 25 5 1 * * 1←26 6 * * 1 23 1 1 * 24333←24 2 * 24333 18 2....2 45333←25 1 * 24333 17 13 1 * * 1←18 14 * * 1 7 10 2...2 35733←8 12 2..2 35733 7 1 1 24733 6653←8 2 24733 6653 5 6 2 24333 6653←9 1 24733 6653 2 2 2 4 5 5 5 3 6653	30 1 1 * * 1←32 1 * * 1 19 2....2 45333←20 4 2..2 45333 15 6 2...2 45333←22 2...2 45333 9 6 2..2 3 3 6653←16 2..2 3 3 6653 5 10 2..2 3 3 6653←6 12 2 2 2 3 3 6653 3 6 1 24733 6653←4 7 24733 6653 3 5 2 24733 6653←4 8 2 3 5 5 3 6653 3 2 2 4 5 5 5 3 6653←6 2 4 5 5 5 3 6653 1 2 3 3 5 6 5 2 3577←2 3 5 5 6 5 2 3577
11	30 1 * * 1←32 * * 1 29 2 * * 1←30 4 4 1 1 * 1 25 6 * * 1←26 8 4 1 1 * 1 23 2 * 24333←24 3 6 2 34411 1 20 2...2 45333←26 * 24333 17 14 * * 1←18 16 4 1 1 * 1 17 2...2 35733←18 3 6 2 2 45333 11 2 24333 6653←12 3 5 6 2 35733 7 12 2..2 35733←8 13 6 2 2 45333 7 2 24733 6653←10 24733 6653 2 3 3 5 6 5 2 3577←4 5 5 6 5 2 3577	31 1 * * 1←33 * * 1 27 34411 * 1←28 5 1 2 34411 1 21 2...2 45333←22 4 2 2 2 45333 19 4 2..2 45333←20 5 8 1 1 24333 18 2...2 35733←24 2..2 45333 15 2..2 3 3 6653←16 3 5 6 2 45333 12 2 24333 6653←18 2 2 2 3 3 6653 11 12 2..2 45333←12 13 8 1 1 24333 9 14 2..2 45333←10 15 8 1 1 24333 5 2 4 5 5 5 3 6653←8 4 5 5 5 3 6653 2 4 3 5 6 5 2 3577

Stem 48 (filtrations 11 to 15) Stem 49 (filtrations 11 to 15)

25		2 4 1 1 * * * * * 1
24	5 1 1 * * * * * 1←6 2 * * * * * 1	6 1 1 * * * * * 1←8 1 * * * * * 1 1 2 34411 * * * * 1
23	6 1 * * * * * 1←8 * * * * * 1 5 2 * * * * * 1←6 4 4 1 1 * * * * 1 2 34411 * * * * 1	7 1 * * * * * 1←9 * * * * * 1 3 34411 * * * * 1←4 5 1 2 34411 * * * 1 1 1 * * * * 24333
22	7 * * * * * 1←10 4 1 1 * * * * 1 5 4 4 1 1 * * * * 1←8 1 2 34411 * * * 1 1 * * * * 24333	3 5 1 2 34411 * * * 1←4 6 2 34411 * * * 1 2 * * * * 24333
21	9 4 1 1 * * * * 1←10 5 1 * * * * 1 7 1 2 34411 * * * 1←9 2 34411 * * * 1 3 4 1 1 * * * 24333←4 5 1 * * * 24333	3 6 2 34411 * * * 1 1 2............2 45333←2 4 2...........2 45333
20	13 1 1 * * * * 1←14 2 * * * * 1 9 5 1 * * * * 1←10 6 * * * * 1 7 1 1 * * * 24333←8 2 * * * 24333 3 5 1 * * * 24333←4 6 * * * 24333 2............2 45333←9 1 * * * 24333	14 1 1 * * * * 1←16 1 * * * * 1 8 1 1 * * * 24333 5 10 1 * * * * 1←6 11 * * * * 1 3 6 1 * * * 24333←4 7 * * * 24333 3 2...........2 45333←4 4 2..........2 45333
19	14 1 * * * * 1←16 * * * * 1 13 2 * * * * 1←14 4 4 1 1 * * * 1 9 6 * * * * 1←10 8 4 1 1 * * * 1 7 2 * * * 24333←8 3 6 2 34411 * * 1 4 2..........2 45333←10 * * * 24333 3 6 * * * 24333←4 7 6 2 34411 * * 1 1 2..........2 35733←2 3 6 2........2 45333	15 1 * * * * 1←17 * * * * 1 11 34411 * * * 1←12 5 1 2 34411 * * 1 5 9 4 4 1 1 * * * 1←6 11 1 2 34411 * * 1 5 2..........2 45333←6 4 2.........2 45333 3 4 2.........2 45333←4 5 8 1 1 * * 24333 2...........2 35733←8 2..........2 45333
18	15 * * * * 1←18 4 1 1 * * * 1 13 4 4 1 1 * * * 1←16 1 2 34411 * * 1 9 8 4 1 1 * * * 1←10 12 1 1 * * * 1 7 3 6 2 34411 * * 1←8 8 1 1 * * 24333 6 2.........2 45333←11 6 2 34411 * * 1 5 10 1 2 34411 * * 1←6 11 2 34411 * * 1 3 2.........2 35733←4 3 6 2.......2 45333	11 5 1 2 34411 * * 1←12 6 2 34411 * * 1 7 9 1 2 34411 * * 1←8 10 2 34411 * * 1 7 2.........2 45333←8 4 2........2 45333 5 4 2.........2 45333←6 6 2........2 45333 4 2.........2 35733←9 8 1 1 * * 24333 3 5 8 1 1 * * 24333←4 7 13 1 * * * 1 1 2.........2 3 3 6653←2 3 6 2.......2 35733
17	17 4 1 1 * * * 1←18 5 1 * * * 1 15 1 2 34411 * * 1←17 2 34411 * * 1 9 12 1 1 * * * 1←10 13 1 * * * 1 7 14 1 1 * * * 1←8 15 1 * * * 1 7 8 1 1 * * 24333←16 1 1 * * 24333 5 10 1 1 * * 24333←6 11 1 * * 24333 3 3 6 2.......2 45333←4 6 2.......2 35733 2.........2 3 3 6653	7 10 2 34411 * * 1←8 12 34411 * * 1 7 4 2........2 45333←8 6 2.......2 45333 5 6 2........2 45333←11 13 1 * * * 1 3 2........2 3 3 6653←4 4 2......2 3 3 6653
16	21 1 1 * * * 1←22 2 * * * 1 17 5 1 * * * 1←18 6 * * * 1 15 1 1 * * 24333←16 2 * * 24333 9 13 1 * * * 1←17 1 * * 24333 3 6 2.......2 35733 1 2.....2 4 3 3 3 6653←2 4 2...2 4 3 3 3 6653	22 1 1 * * * 1←24 1 * * * 1 7 12 34411 * * 1←8 15 4 4 1 1 * * 1 7 6 2.......2 45333←14 2.......2 45333 3 4 2......2 3 3 6653←4 6 2.....2 3 3 6653 2......2 4 3 3 3 6653←8 2......2 3 3 6653

Stem 48 (filtrations $\geq$ 16) Stem 49 (filtrations $\geq$ 16)

	Stem 50 (filtrations ≤ 6)	Stem 51 (filtrations ≤ 6)
6	39 4 4 1 1 1←44 4 1 1 1 35 8 4 1 1 1←36 12 1 1 1 35 24333←37 6 2 3 3 33 3 6 2 3 3←34 8 3 3 3 29 35733←33 5 7 3 3 26 23577←28 4 5 7 7 19 24 4 1 1 1←20 28 1 1 1 17 11 3577←18 14 5 7 7 15 59777←19 13 5 7 7 13 23 6 2 3 3←14 28 3 3 3 11 21 5 7 3 3←13 24 6 5 3 11 7 13 5 7 7←13 17 7 7 7 10 18 3577←12 20 5 7 7 9 18 4 5 7 7←10 20 7 7 7 9 9 11 7 7 7←10 10 13 11 7 5 9 15 7 7 7	37 5 1 2 3 3←38 6 2 3 3 33 45333←34 5 7 3 3 31 4 7 3 3 3←32 6653 30 35733←35 8 3 3 3 27 23577←29 4 5 7 7 21 12 9 3 3 3←22 13 11 3 3 18 11 3577←20 13 5 7 7 13 21 8 3 3 3←14 24 6 5 3 13 20 9 3 3 3←14 21 11 3 3 11 22 9 3 3 3←12 23 11 3 3 11 18 3577←13 20 5 7 7 11 7 14 5 7 7←12 11 15 7 7 10 19 3577←12 21 5 7 7 10 9 11 7 7 7←14 17 7 7 7 7 11 14 5 7 7←8 13 13 11 7 6 9 15 7 7 7
5	43 4 1 1 1←44 5 1 1 41 1 2 3 3←48 1 1 1 35 12 1 1 1←36 13 1 1 28 3577←32 5 7 7 27 4 5 7 7←30 7 7 7 19 28 1 1 1←20 29 1 1 17 14 5 7 7←20 13 11 7 14 27 3 3 3←16 29 3 3 11 22 11 3 3←12 23 13 3 11 20 5 7 7←14 23 7 7 10 19 7 7 7←12 21 11 7 9 10 13 11 7←10 11 15 15	42 1 2 3 3←44 2 3 3 33 9 3 3 3←34 10 5 3 31 6653←38 6 5 3 29 3577←33 5 7 7 21 13 11 3 3←22 14 13 3 17 25 3 3 3←18 26 5 3 13 21 11 3 3←14 22 13 3 11 21 5 7 7←13 25 7 7 11 19 7 7 7←13 21 11 7 11 11 15 7 7←21 13 11 7 7 13 13 11 7
4	47 1 1 1←48 2 1 43 5 1 1←44 6 1 42 2 3 3←49 1 1 35 13 1 1←36 14 1 31 5 7 7←37 7 7 29 7 7 7←33 11 7 19 29 1 1←20 30 1 19 13 11 7←21 15 15 15 29 3 3←17 31 3 13 23 7 7←17 27 7 11 21 11 7←13 23 15 9 11 15 15	43 2 3 3←46 3 3 42 3 3 3←44 5 3 37 6 5 3←38 7 7 34 11 3 3←36 13 3 33 10 5 3←34 11 7 21 14 13 3←22 15 15 18 27 3 3←20 29 3 17 26 5 3←18 27 7 13 22 13 3←14 23 15
3	47 2 1←48 3 44 3 3←50 1 43 6 1←44 7 35 14 1←36 15 19 30 1←20 31	45 3 3←49 3 43 5 3←45 7 35 13 3←37 15 19 29 3←21 31
2	47 3←51	
1		

Stem 50 (filtrations ≤ 6) Stem 51 (filtrations ≤ 6)

9	35 4 1 1 * 1←36 5 1 * 1 33 1 2 34411 1←40 1 1 * 1 27 12 1 1 * 1←28 13 1 * 1 21 3 6 2 45333←22 6 2 35733 19 20 1 1 * 1←20 21 1 * 1 15 3 6 3 3 6653←16 6 5 23577 13 26 1 1 * 1←14 27 1 * 1 13 5 6 3 3 6653←14 7 12 45333 13 4 7 3 3 6653←19 6 3 3 6653 11 22 1 1 24333←12 23 1 24333 9 15 6 2 45333←10 18 2 35733 4 7 3 5 9 3577←6 7 8 12 9 3 3 3 3 4 5 3 59777←5 8 3 59777	34 1 2 34411 1←36 2 34411 1 25 4 2 2 45333←26 6 2 45333 23 6 2 2 45333←29 13 1 * 1 21 7 13 1 * 1←22 10 2 45333 19 3 6 2 35733←20 6 3 3 6653 13 15 13 1 * 1←14 18 2 45333 13 3 6 5 23577←14 6 9 3 6653 11 17 13 1 * 1←12 20 2 45333 11 5 6 5 23577←17 6 5 23577 9 9 5 5 3 6653←10 10 9 3 6653 5 9 3 5 7 3577 3 4 6 3 59777←4 5 10 11 3577
8	39 1 1 * 1←40 2 * 1 35 5 1 * 1←36 6 * 1 34 2 34411 1←41 1 * 1 33 1 1 24333←34 2 24333 19 21 1 * 1←20 22 * 1 15 6 5 23577←21 12 45333 13 21 1 24333←14 22 24333 13 7 12 45333←14 13 23577 13 3 5 7 3577←17 5 7 3577 11 3 5 9 3577←13 8 12 9 3 3 3 9 18 2 35733←10 20 35733 9 15 3 3 6653←10 17 23577 5 7 8 12 9 3 3 3←6 12 11 3577	35 2 34411 1←38 34411 1 34 1 1 24333←36 1 24333 25 6 2 45333←32 2 45333 21 10 2 45333←22 12 45333 18 5 5 3 6653←20 9 3 6653 14 3 5 7 3577←16 5 9 3577 13 18 2 45333←14 20 45333 13 6 9 3 6653←14 8 12 9 3 3 3 12 3 5 9 3577←18 5 7 3577 11 20 2 45333←12 22 45333 10 11 12 45333←12 17 3 6653 9 10 9 3 6653←10 12 12 9 3 3 3 3 5 10 11 3577
7	39 2 * 1←40 4 4 1 1 1 36 34411 1←42 * 1 35 6 * 1←36 8 4 1 1 1 34 1 24333←36 24333 33 2 24333←34 3 6 2 3 3 27 2 35733←28 3 6653 19 22 * 1←20 24 4 1 1 1 14 5 9 3577←20 9 3577 13 22 24333←14 23 6 2 3 3 13 13 23577←14 14 4 5 7 7 12 3 59777←16 59777 10 19 35733←12 21 5 7 3 3 10 5 59777←12 7 13 5 7 7 9 17 23577←10 18 4 5 7 7 3 6 9 11 7 7 7←4 7 11 15 7 7	37 34411 1←38 5 1 2 3 3 35 1 24333←37 24333 31 2 45333←32 4 7 3 3 3 28 2 35733←34 45333 25 3 3 6653←28 23577 19 9 3 6653←22 12 9 3 3 3 15 5 9 3577←21 9 3577 13 20 45333←14 21 8 3 3 3 13 3 59777←17 59777 11 22 45333←12 23 8 3 3 3 11 17 3 6653←14 20 9 3 3 3 11 5 59777←12 7 14 5 7 7 9 18 23577←10 19 4 5 7 7 9 12 12 9 3 3 3←12 18 3577 7 7 14 4 5 7 7←8 11 14 5 7 7 4 6 9 11 7 7 7

Stem 50 (filtrations 7 to 9) — Stem 51 (filtrations 7 to 9)

12	31 1 1 * * 1←32 2 * * 1 27 5 1 * * 1←28 6 * * 1 26 2 34411 * 1←33 1 * * 1 25 1 1 * 24333←26 2 * 24333 19 13 1 * * 1←20 14 * * 1 9 1 1 24733 6653←10 2 24733 6653 7 12 2...2 35733←8 14 2..2 35733 7 6 2 24333 6653←14 2 24333 6653 5 8 2 24333 6653←6 10 24333 6653 1 2 4 3 5 6 5 23577←4 4 3 5 6 5 23577	27 2 34411 * 1←30 34411 * 1 26 1 1 * 24333←28 1 * 24333 21 2....2 45333←22 4 2..2 45333 17 6 2...2 45333←24 2...2 45333 11 22 1 * * 1←12 23 * * 1 10 1 1 24733 6653←12 1 24733 6653 9 18 1 * 24333←10 19 * 24333 7 10 2..2 3 3 6653←8 12 2 2 2 3 3 6653 5 5 2 24733 6653←6 8 2 3 5 5 3 6653 5 2 2 4 5 5 5 3 6653←8 2 4 5 5 5 3 6653 2 2 4 3 5 6 5 23577
11	31 2 * * 1←32 4 4 1 1 * 1 28 34411 * 1←34 * * 1 27 6 * * 1←28 8 4 1 1 * 1 26 1 * 24333←28 * 24333 25 2 * 24333←26 3 6 2 34411 1 19 14 * * 1←20 16 4 1 1 * 1 19 2...2 35733←20 3 6 2 2 45333 13 2 24333 6653←14 3 5 6 2 35733 10 1 24733 6653←12 24733 6653 9 2 24733 6653←16 24333 6653 7 14 2..2 35733←8 15 6 2 2 45333 5 10 24333 6653←6 11 5 6 2 35733 3 5 4 5 5 5 3 6653←4 7 5 6 5 23577 3 4 3 5 6 5 23577←6 5 5 6 5 23577 1 2 3 5 3 5 9 3577←2 4 6 3 5 9 3577	29 34411 * 1←30 5 1 2 34411 1 27 1 * 24333←29 * 24333 23 2...2 45333←24 4 2 2 2 45333 21 4 2..2 45333←22 5 8 1 1 24333 20 2...2 35733←26 2..2 45333 17 2..2 3 3 6653←18 3 5 6 2 45333 11 21 4 4 1 1 * 1←12 23 1 2 34411 1 11 1 24733 6653←13 24733 6653 7 12 2 2 2 3 3 6653←8 13 5 6 2 45333 7 2 4 5 5 5 3 6653←10 4 5 5 5 3 6653 5 8 2 3 5 5 3 6653←6 9 4 7 3 3 6653 3 3 5 5 6 5 23577←5 7 5 6 5 23577 2 2 3 5 3 5 9 3577←4 4 5 3 5 9 3577 1 2 4 5 3 5 9 3577←2 4 7 3 5 9 3577 1 2 2 3 5 3 59777←2 3 4 5 3 59777
10	31 4 4 1 1 * 1←36 4 1 1 * 1 27 8 4 1 1 * 1←28 12 1 1 * 1 27 * 24333←29 6 2 34411 1 25 3 6 2 34411 1←26 8 1 1 24333 21 2..2 35733←22 3 6 2 45333 19 16 4 1 1 * 1←20 20 1 1 * 1 19 3 6 2 2 45333←20 5 6 2 45333 15 24333 6653←16 3 6 3 3 6653 13 3 5 6 2 35733←14 5 6 3 3 6653 12 2 3 5 5 3 6653←17 5 6 2 35733 11 22 1 2 34411 1←12 23 2 34411 1 11 24733 6653←14 4 7 3 3 6653 9 13 6 2 2 45333←10 15 6 2 45333 5 5 5 6 5 23577 2 4 5 3 5 9 3577←4 8 3 5 9 3577 2 2 3 5 3 59777←4 4 5 3 59777 1 2 4 5 3 59777←2 4 7 3 59777	29 5 1 2 34411 1←30 6 2 34411 1 25 2..2 45333←26 4 2 2 45333 23 4 2 2 2 45333←24 6 2 2 45333 22 2..2 35733←27 8 1 1 24333 21 5 8 1 1 24333←22 7 13 1 * 1 19 2 2 2 3 3 6653←20 3 6 2 35733 17 3 5 6 2 45333←18 5 6 2 35733 13 13 8 1 1 24333←14 15 13 1 * 1 13 2 3 5 5 3 6653←14 3 6 5 23577 11 15 8 1 1 24333←12 17 13 1 * 1 9 4 5 5 5 3 6653←12 5 6 5 23577 7 13 5 6 2 45333←8 15 6 2 35733 5 9 4 7 3 3 6653←6 11 6 5 23577 3 6 9 5 5 3 6653←4 7 11 12 45333 3 4 5 3 5 9 3577←5 8 3 5 9 3577 3 2 3 5 3 59777←4 4 6 3 59777
	Stem 50 (filtrations 10 to 12)	Stem 51 (filtrations 10 to 12)

17	19 4 1 1 * * * 1←20 5 1 * * * 1 17 1 2 34411 * * 1←24 1 1 * * * 1 11 12 1 1 * * * 1←12 13 1 * * * 1 5 3 6 2.......2 45333←6 6 2.......2 35733 3 5 6 2.......2 45333 1 2......24333 6653←2 4 2....24333 6653	18 1 2 34411 * * 1←20 2 34411 * * 1 9 4 2........2 45333←10 6 2.......2 45333 7 6 2........2 45333←13 13 1 * * * 1 5 7 13 1 * * * 1←6 10 2.......2 45333 3 3 6 2.......2 35733←4 6 2......2 3 3 6653 2.......24333 6653
16	23 1 1 * * * 1←24 2 * * * 1 19 5 1 * * * 1←20 6 * * * 1 18 2 34411 * * 1←25 1 * * * 1 17 1 1 * * 24333←18 2 * * 24333 5 6 2.......2 35733 3 2.....24333 6653←4 4 2...24333 6653 1 1 1 * 24733 6653←2 2 * 24733 6653	19 2 34411 * * 1←22 34411 * * 1 18 1 1 * * 24333←20 1 * * 24333 9 6 2.......2 45333←16 2.......2 45333 5 10 2.......2 45333←6 12 2......2 45333 3 6 2......2 3 3 6653 2 1 1 * 24733 6653←4 1 * 24733 6653 1 2 1 * 24733 6653←2 3 * 24733 6653 1 1 2 * 24733 6653←2 4 2 2 1 2 24733 6653
15	23 2 * * * 1←24 4 4 1 1 * * 1 20 34411 * * 1←26 * * * 1 19 6 * * * 1←20 8 4 1 1 * * 1 18 1 * * 24333←20 * * 24333 17 2 * * 24333←18 3 6 2 34411 * 1 11 2.......2 35733←12 3 6 2....2 45333 5 2....24333 6653←6 3 5 6 2...2 35733 3 4 2...24333 6653←4 5 5 6 2...2 35733 2 1 * 24733 6653←4 * 24733 6653 1 2 * 24733 6653←8 2...24333 6653	21 34411 * * 1←22 5 1 2 34411 * 1 19 1 * * 24333←21 * * 24333 15 2.......2 45333←16 4 2.....2 45333 12 2.......2 35733←18 2......2 45333 9 15 4 4 1 1 * * 1←10 17 1 2 34411 * 1 9 2......2 3 3 6653←10 3 5 6 2...2 45333 6 2....24333 6653 5 6 2.....2 3 3 6653←6 7 5 6 2...2 45333 3 1 * 24733 6653←5 * 24733 6653
14	23 4 4 1 1 * * 1←28 4 1 1 * * 1 19 8 4 1 1 * * 1←20 12 1 1 * * 1 19 * * 24333←21 6 2 34411 * 1 17 3 6 2 34411 * 1←18 8 1 1 * 24333 13 2......2 35733←14 3 6 2...2 45333 11 3 6 2....2 45333←12 5 6 2...2 45333 7 2...24333 6653←8 3 6 2..2 3 3 6653 5 3 5 6 2...2 35733←6 5 6 2..2 3 3 6653 4 2 2 1 2 24733 6653←9 5 6 2...2 35733 3 5 5 6 2...2 35733←4 7 6 2..2 3 3 6653 3 * 24733 6653←6 4 1 1 24733 6653 1 1 2 2 2 4 5 5 5 3 6653←2 3 2 2 4 5 5 5 3 6653	21 5 1 2 34411 * 1←22 6 2 34411 * 1 17 2......2 45333←18 4 2....2 45333 15 4 2.....2 45333←16 6 2....2 45333 14 2......2 35733←19 8 1 1 * 24333 11 2.....2 3 3 6653←12 3 6 2...2 35733 9 17 1 2 34411 * 1←10 18 2 34411 * 1 9 3 5 6 2...2 45333←10 5 6 2...2 35733 5 7 5 6 2...2 45333←6 9 6 2...2 35733 5 2 2 1 2 24733 6653←6 3 6 2 24333 6653 3 3 4 1 1 24733 6653←4 5 6 2 24333 6653 1 2..2 4 5 5 5 3 6653←4 2 2 2 4 5 5 5 3 6653
13	27 4 1 1 * * 1←28 5 1 * * 1 25 1 2 34411 * 1←32 1 1 * * 1 19 12 1 1 * * 1←20 13 1 * * 1 13 3 6 2...2 45333←14 6 2...2 35733 7 9 6 2...2 45333←8 12 2...2 35733 7 3 6 2..2 3 3 6653←8 6 2 24333 6653 5 5 6 2..2 3 3 6653←6 8 2 24333 6653 5 4 1 1 24733 6653←11 6 2..2 3 3 6653 3 6 1 1 24733 6653←4 7 1 24733 6653 2 2 2 2 4 5 5 5 3 6653	26 1 2 34411 * 1←28 2 34411 * 1 17 4 2....2 45333←18 6 2...2 45333 15 6 2....2 45333←22 2....2 45333 11 3 6 2...2 35733←12 6 2..2 3 3 6653 9 18 2 34411 * 1←10 20 34411 * 1 5 9 6 2...2 35733←6 12 2..2 3 3 6653 5 3 6 2 24333 6653←6 5 2 24733 6653 3 5 6 2 24333 6653←4 7 2 24733 6653 3 2 2 2 4 5 5 5 3 6653←6 2 2 4 5 5 5 3 6653 1 1 2 4 3 5 6 5 23577←2 3 4 3 5 6 5 23577

Stem 50 (filtrations 13 to 17)

Stem 51 (filtrations 13 to 17)

27		1 1 * * * * * * * 1
26	1 * * * * * * * 1	2 * * * * * * * 1 1 1 1 2 34411 * * * * 1←2 2 2 34411 * * * * 1
25	3 4 1 1 * * * * * * 1←4 5 1 * * * * * * 1 1 1 2 34411 * * * * 1←8 1 1 * * * * * * 1	4 4 1 1 * * * * * * 1 2 1 2 34411 * * * * 1←4 2 34411 * * * * 1 1 2 2 34411 * * * * 1←2 4 34411 * * * * 1
24	7 1 1 * * * * * * 1←8 2 * * * * * * 1 3 5 1 * * * * * * 1←4 6 * * * * * * 1 2 2 34411 * * * * 1←9 1 * * * * * * 1 1 1 1 * * * * 24333←2 2 * * * * 24333	3 6 1 * * * * * * 1←4 7 * * * * * * 1 3 2 34411 * * * * 1←6 34411 * * * * 1 2 1 1 * * * * 24333←4 1 * * * * 24333 1 2 1 * * * * 24333←2 3 * * * * 24333
23	7 2 * * * * * 1←8 4 4 1 1 * * * * 1 4 34411 * * * * 1←10 * * * * * 1 3 6 * * * * * 1←4 8 4 1 1 * * * * 1 2 1 * * * * 24333←4 * * * * 24333 1 2 * * * * 24333←2 3 6 2 34411 * * * 1	5 34411 * * * * 1←6 5 1 2 34411 * * * 1 3 5 4 4 1 1 * * * * 1←4 7 1 2 34411 * * * 1 3 1 * * * * 24333←5 * * * * 24333
22	7 4 4 1 1 * * * * 1←12 4 1 1 * * * * 1 3 6 1 2 34411 * * * 1←4 7 2 34411 * * * 1 3 * * * * 24333←5 6 2 34411 * * * 1	5 5 1 2 34411 * * * 1←6 6 2 34411 * * * 1 1 2.............2 45333←2 4 2...........2 45333
21	11 4 1 1 * * * * 1←12 5 1 * * * * 1 9 1 2 34411 * * * 1←16 1 1 * * * * 1 5 10 1 1 * * * * 1←6 11 1 * * * * 1 3 6 1 1 * * * 24333←4 7 1 * * * 24333 2.............2 45333	10 1 2 34411 * * * 1←12 2 34411 * * * 1 3 2............2 45333←4 4 2..........2 45333
20	15 1 1 * * * * 1←16 2 * * * * 1 11 5 1 * * * * 1←12 6 * * * * 1 10 2 34411 * * * 1←17 1 * * * * 1 9 1 1 * * * 24333←10 2 * * * 24333 5 5 1 * * * 24333←6 6 * * * 24333 4 2...........2 45333 1 2...........2 35733←2 4 2.........2 35733	11 2 34411 * * * 1←14 34411 * * * 1 10 1 1 * * * 24333←12 1 * * * 24333 5 2...........2 45333←6 4 2.........2 45333 3 4 2..........2 45333←4 6 2.........2 45333 2............2 35733←8 2..........2 45333
19	15 2 * * * * 1←16 4 4 1 1 * * * 1 12 34411 * * * 1←18 * * * * 1 11 6 * * * * 1←12 8 4 1 1 * * * 1 10 1 * * * 24333←12 * * * 24333 9 2 * * * 24333←10 3 6 2 34411 * * 1 6 2..........2 45333 5 6 * * * 24333←6 7 6 2 34411 * * 1 3 2..........2 35733←4 3 6 2........2 45333	13 34411 * * * 1←14 5 1 2 34411 * * 1 11 1 * * * 24333←13 * * * 24333 7 2..........2 45333←8 4 2.........2 45333 5 4 2.........2 45333←6 5 8 1 1 * * 24333 4 2..........2 35733←10 2.........2 45333 3 6 2.........2 45333←4 7 8 1 1 * * 24333 1 2.........2 3 3 6653←2 3 5 6 2.......2 45333
18	15 4 4 1 1 * * * 1←20 4 1 1 * * * 1 11 8 4 1 1 * * * 1←12 12 1 1 * * * 1 11 * * * 24333←13 6 2 34411 * * 1 9 3 6 2 34411 * * 1←10 8 1 1 * * 24333 5 7 6 2 34411 * * 1←6 12 1 1 * * 24333 5 2..........2 35733←6 3 6 2.......2 45333 3 3 6 2........2 45333←4 5 6 2.......2 45333 2.........2 3 3 6653	13 5 1 2 34411 * * 1←14 6 2 34411 * * 1 9 2.........2 45333←10 4 2........2 45333 7 4 2.........2 45333←8 6 2........2 45333 6 2.........2 35733←11 8 1 1 * * 24333 5 5 8 1 1 * * 24333←6 7 13 1 * * * 1 3 2.........2 3 3 6653←4 3 6 2.......2 35733
	Stem 50 (filtrations ≧ 18)	Stem 51 (filtrations ≧ 18)

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