A C_2-EQUIVARIANT ANALOG OF MAHOWALD'S THOM SPECTRUM THEOREM

MARK BEHRENS AND DYLAN WILSON

Abstract. We prove that the C_2-equivariant Eilenberg-MacLane spectrum associated with the constant Mackey functor $\underline{\mathbb{F}_2}$ is equivalent to a Thom spectrum over $\Omega^\rho S^{\rho+1}$.

1. Introduction

Let μ be the Möbius bundle over S^1, regarded as a virtual bundle of dimension 0. The mod 2 Moore spectrum is the Thom spectrum

$$M(2) \simeq (S^1)^\mu.$$

The classifying map for μ extends to a double loop map

$$\widetilde{\mu} : \Omega^2 S^3 \to BO.$$

Mahowald proved the following theorem [Mah77]:

Theorem 1.1 (Mahowald)**.** *There is an equivalence of spectra*

$$(\Omega^2 S^3)^{\widetilde{\mu}} \simeq H\mathbb{F}_2.$$

The bundle μ may also be regarded C_2-equivariant virtual bundle over S^1, by endowing both S^1 and the bundle with the trivial action. Since $B_{C_2}O$ is an equivariant infinite loop space [Ati68], the classifying map for μ extends to an Ω^ρ-loop map

$$\widetilde{\mu} : \Omega^\rho S^{\rho+1} \to B_{C_2}O.$$

Here, ρ is the regular representation of C_2. The purpose of this paper is to prove the following.

Theorem 1.2. *There is an equivalence of C_2-spectra*

$$(\Omega^\rho S^{\rho+1})^{\widetilde{\mu}} \simeq H\underline{\mathbb{F}_2}.$$

(Here, $\underline{\mathbb{F}_2}$ denotes the constant Mackey functor with value $\mathbb{F}_2$.)

Acknowledgements. Many tricks in this paper have been independently discovered by Doug Ravenel, and the first author's involvement in this project is an outgrowth of mathematical discussions with Agnés Beaudry, Prasit Bhattacharya, Dominic Culver, Doug Ravenel, and Zhouli Xu. The authors also benefited from valuable input from Mike Hill. The first author was supported by NSF grant DMS-1611786.

Date: August 16, 2017.

Conventions. Equivariant objects in this paper either live in Top^{C_2}, the category of C_2-spaces, or Sp^{C_2}, the category of genuine C_2-spectra. In both of these categories, the equivalences are those equivariant maps which induce equivalences on both C_2-fixed points and underlying fixed points. We let $\underline{H}$ denote the Eilenberg-Maclane spectrum $H\underline{\mathbb{F}_2}$, with underlying spectrum $H := H\mathbb{F}_2$. We use $\underline{H}_\star$ and $\pi_\star^{C_2}$ to denote $RO(C_2)$-graded homology and homotopy *groups* (i.e. *not* the Mackey functors) of C_2-equivariant spaces and spectra, and H_* and π_* to denote the ordinary homology and homotopy groups of non-equivariant spaces and spectra. We let σ denote the sign representation of C_2, and let $\rho = 1 + \sigma$ denote the regular representation. For a representation V, $S(V)$ denotes the unit sphere in V, and S^V denotes its one point compactification, and $|V|$ denotes its dimension.

2. Equivariant preliminaries

Euler class. Let a denote the Euler class in $\pi_{-\sigma}^{C_2}S$, given geometrically by the inclusion

$$S^0 \hookrightarrow S^\sigma.$$

There is a cofiber sequence

$$C_{2+} \to S^0 \hookrightarrow S^\sigma \tag{2.1}$$

so the cofiber of a is stably given by

$$Ca \simeq \Sigma^{1-\sigma} C_{2+}. \tag{2.2}$$

The transfer induces a map

$$u : S^{1-\sigma} \xrightarrow{tr} \Sigma^{1-\sigma} C_{2+} \simeq Ca$$

which serves as a Thom class for the representation σ:

$$u : S^1 \to Ca \wedge S^\sigma.$$

For $X \in \mathrm{Sp}^{C_2}$, we have

$$\pi_k^{C_2}(X) \cong \pi_k(X^{C_2}),$$
$$\pi_V^{C_2}(X \wedge Ca) \cong \pi_{|V|}(X^e).$$

Said differently,

$$\pi_\star^{C_2} X \wedge Ca \cong \pi_* X^e[u^\pm]. \tag{2.3}$$

Tate square. We will let

$$X^h := F(EC_{2+}, X),$$
$$X^\Phi := X \wedge \widetilde{EC}_2$$

denote the homotopy completion and geometric localization of X, respectively. The fixed points of X^h are the homotopy fixed points of X, and the fixed points of X^Φ

are the geometric fixed points of X. X is recovered from these approximations by the pullback ("Tate square") [GM95]

$$\begin{array}{ccc} X & \longrightarrow & X^\Phi \\ \downarrow & & \downarrow \\ X^h & \longrightarrow & X^t \end{array}$$

where the spectrum X^t is the equivariant Tate spectrum

$$X^t := (X^h)^\Phi.$$

Note that a generalization of the argument establishing (2.2) yields an equivalence

$$\Sigma^{k\sigma-1}C(a^k) \simeq S(k\sigma)_+.$$

Taking a colimit, we see that we have

$$\begin{aligned} \operatorname*{hocolim}_k \Sigma^{k\sigma-1}C(a^k) &\simeq EC_{2+}, \\ \operatorname*{hocolim}_k S^{k\sigma} &\simeq \widetilde{EC}_2. \end{aligned}$$

It follows that homotopy completion and geometric localization can be reinterpreted as a-completion and a-localization:

$$\begin{aligned} X^h &\simeq X^\wedge_a, \\ X^\Phi &\simeq X[a^{-1}]. \end{aligned}$$

In this manner, the Tate square is equivalent to the "a-arithmetic square"

$$\begin{array}{ccc} X & \longrightarrow & X[a^{-1}] \\ \downarrow & & \downarrow \\ X^\wedge_a & \longrightarrow & X^\wedge_a[a^{-1}] \end{array}$$

Using (2.3), the a-Bockstein spectral sequence takes the form

$$\pi_*(X^e)[u^\pm, a] \Rightarrow \pi^{C_2}_\star(X^h).$$

The a-Bockstein spectral sequence can be regarded as an $RO(C_2)$-graded version of the homotopy fixed point spectral sequence (see [HM17, Lem. 4.8]).

The mod 2 Eilenberg-MacLane spectrum. We have [HK01]

$$\pi^{C_2}_\star \underline{H} = \mathbb{F}_2[a,u] \oplus \frac{\mathbb{F}_2[a,u]}{(a^\infty, u^\infty)}\{\theta\}$$

where

$$\begin{aligned} |u| &= 1-\sigma, \\ |\theta| &= 2\sigma - 2. \end{aligned}$$

The a-u divisible factor in $\pi_\star \underline{H}$ is best understood from the Tate square, using

$$\begin{aligned} \pi^{C_2}_\star \underline{H}^h &\cong \mathbb{F}_2[a, u^\pm], \\ \pi^{C_2}_\star \underline{H}^\Phi &\cong \mathbb{F}_2[a^\pm, u]. \end{aligned}$$

Actually, the second isomorphism lifts to an equivalence

$$\underline{H}^{\Phi C_2} \simeq H[a^{-1}u] := \bigvee_{i \geqslant 0} \Sigma^i H$$

so we have

$$\underline{H}^{\Phi}_{\star} X \cong H_*(X^{\Phi C_2})[a^{\pm}, u]$$

and, restricting the grading to trivial representations, we get

$$\underline{H}^{\Phi}_{*} X \cong H_*(X^{\Phi C_2})[a^{-1}u]. \tag{2.4}$$

By applying $\pi^{C_2}_V$ to the map

$$\underline{H} \wedge X \to \underline{H} \wedge X \wedge Ca$$

we get a homomorphism

$$\Phi^e : \underline{H}_V(X) \to H_{|V|}(X^e). \tag{2.5}$$

Taking geometric fixed points of a map

$$S^V \to \underline{H} \wedge X$$

gives a map

$$S^{V^{C_2}} \to \underline{H}^{\Phi C_2} \wedge X^{\Phi C_2}$$

Using (2.4) and passing to the quotient by the ideal generated by $a^{-1}u$, we get a homomorphism

$$\Phi^{C_2} : \underline{H}_V(X) \to H_{\left|V^{C_2}\right|}(X^{\Phi C_2}). \tag{2.6}$$

A useful lemma. Our main computational lemma is the following.

Lemma 2.7. *Suppose that $X \in \mathrm{Sp}^{C_2}$ and suppose that $\{b_i\}$ is a set of elements of $H_\star(X)$ such that*

(1) *$\{\Phi^e(b_i)\}$ is a basis of $H_*(X^e)$, and*
(2) *$\{\Phi^{C_2}(b_i)\}$ is a basis of $H_*(X^{\Phi C_2})$.*

Then $H_\star(X)$ is free over $H_\star$, and $\{b_i\}$ is a basis.

Proof. The set $\{b_i\}$ corresponds to a map

$$\underline{H} \wedge \bigvee S^{|b_i|} \to \underline{H} \wedge X.$$

Assumption (1) implies this map is an equivalence upon applying Φ^e, while assumption (2) implies this map is an equivalence upon applying Φ^{C_2}. The result follows. □

3. Homology of ρ-loop spaces

We spell out some specific algebraic structure carried by the equivariant homology of a ρ-loop space. A more detailed and general study of this algebraic structure will appear in [Hil].

Products. Suppose $X = \Omega^\rho Y \in \mathrm{Top}^{C_2}$ is a ρ-loop space. Then X is in particular a 1-loop space, and is therefore an equivariant H-space with product

$$m : X \times X \to X.$$

However, the σ-loop space structure also endows X with a twisted product related to the transfer. Namely, let

$$S^\sigma \to S^\sigma / S^0 \approx C_{2+} \wedge S^1$$

be the pinch map. This gives rise to a twisted product

$$\widetilde{m} : N^\times \Omega Y \to \Omega^\sigma Y$$

where

$$N^\times Z := \mathrm{Map}(C_2, Z) = Z \underset{\overleftrightarrow{C_2}}{\times} Z$$

is the norm (with respect to Cartesian product). In particular, there is a map

$$\widetilde{m} : N^\times \Omega^2 Y \to X. \tag{3.1}$$

Upon applying fixed points to the map (3.1), we get an additive transfer

$$t : X^e \to X^{C_2}. \tag{3.2}$$

In homology, the H-space structure give rise to a product

$$m : \underline{H}_V X \otimes \underline{H}_W X \to \underline{H}_{V+W} X.$$

Using the equivariant commutative ring spectrum structure of $\underline{H}$ [Ull13], the twisted product $\widetilde{m}$ gives rise to a "norm map"(see [BH15, Thm. 7.2])

$$n : H_k X^e \to \underline{H}_{k\rho} X.$$

Dyer-Lashof operations. X has even more structure: X is an E_ρ-algebra [GM17]. Specifically, regard $S(\rho)$ as a $C_2 \times \Sigma_2$-space where C_2 acts on ρ and Σ_2 acts antipodally. Then the E_ρ-structure gives a map

$$S(\rho) \times_{\Sigma_2} X^{\times 2} \to X.$$

Note that $\underline{H}$ is itself an E_ρ-ring spectrum, because it is actually an equivariant commutative ring spectrum, so $\underline{H} \wedge X_+$ is an E_ρ-ring in $\underline{H}$-modules. Given $x \in \underline{H}_V(X)$, represented by a map

$$x : S^V \to \underline{H} \wedge X_+,$$

there is an induced composite

$$\begin{aligned} \underline{H} \wedge S(\rho)_+ \wedge_{\Sigma_2} S^{2V} &\xrightarrow{1\wedge 1 \wedge x \wedge x} \underline{H} \wedge S(\rho)_+ \wedge_{\Sigma_2} (\underline{H} \wedge X_+)^{\wedge 2} \\ &\to \underline{H} \wedge \underline{H} \wedge X_+ \\ &\to \underline{H} \wedge X_+ \end{aligned}$$

(where the unlabeled maps come from the E_ρ-ring and $\underline{H}$-module structure of $\underline{H} \wedge X_+$). Applying $\pi_\star^{C_2}$, we get a total power operation

$$\mathcal{P}(x) : \underline{\widetilde{H}}_\star(S(\rho)_+ \wedge_{\Sigma_2} S^{2V}) \to \underline{H}_\star X.$$

For the purposes of this paper we will be only concerned with the case of $V = k\rho - \sigma$.

We will need the following lemma.

Lemma 3.3. *We have the following identification of the C_2-fixed point space of the extended power:*

$$(S(\rho)_+ \wedge_{\Sigma_2} S^{2(k\rho-\sigma)})^{C_2} \approx S^{2k-1} \vee S^{2k}.$$

Proof. The extended power can be identified with the Thom complex of the equivariant vector bundle

$$S(\rho) \times_{\Sigma_2} \mathbb{R}^{2(k\rho-\sigma)} \to S(\rho)/\Sigma_2.$$

The fixed points is the Thom complex of the fixed point bundle. Thinking of $S(\rho)$ as the unit circle in $\mathbb{C}$, with C_2 acting by conjugation, the fixed points of the base are given by

$$[S(\rho)/\Sigma_2]^{C_2} = \{[1], [i]\}.$$

The bundle has fiber $\mathbb{R}^{2(k\rho-\sigma)}$ over $[1]$, and because Σ_2 acts with the antipodal action mixed with the interchange action, the fiber over $[i]$ is given by

$$\mathbb{R}^{\rho(k\rho-\sigma)} = \mathbb{R}^{(2k-1)\rho}.$$

The result follows. □

Proposition 3.4. *We have*

$$\widetilde{\underline{H}}_\star S(\rho)_+ \wedge_{\Sigma_2} S^{2(k\rho-\sigma)} \cong \underline{H}_\star\{e_{2k\rho-\sigma-1}, e_{2k\rho-\sigma}\}.$$

Proof. Theorem 2.15 of [Wil17] implies there is a cofiber sequence

$$S^{2k\rho-2\sigma} \to S(\rho)_+ \wedge_{\Sigma_2} S^{2(k\rho-\sigma)} \to S^{2k\rho-1}.$$

There are two possibilities for the long exact sequence in $\underline{H}_\star$: either (a) the connecting homomorphism sends $\iota_{2k\rho-1}$ to zero, or (b) the connecting homomorphism sends it to $\theta\iota_{2k\rho-2\sigma}$. Only possibility (b) is compatible with Lemma 3.3 from geometric fixed point considerations. The result follows. □

Thus we get a pair of Dyer-Lashof operations

$$Q^{k\rho} : \underline{H}_{k\rho-\sigma}X \to \underline{H}_{2k\rho-\sigma}X,$$
$$Q^{k\rho-1} : \underline{H}_{k\rho-\sigma}X \to \underline{H}_{2k\rho-\sigma-1}X$$

given by the formulas

$$Q^{k\rho}(x) := \mathcal{P}(x)(e_{2k\rho-\sigma}),$$
$$Q^{k\rho-1}(x) := \mathcal{P}(x)(e_{2k\rho-\sigma-1}).$$

Remark 3.5. If X is actually an equivariant infinite loop space, then $\underline{H}_\star X$ has an action by equivariant Dyer-Lashof operations [Wil17], and these operations agree with those defined in that paper.

Compatibility with fixed points. The compatibility of all this structure with the maps Φ^e and Φ^{C_2} of (2.5) and (2.6) is summarized as follows.

Products: Note that X^e is an E_2-algebra, and X^{C_2} is an E_1-algebra. The maps Φ^e and Φ^{C_2} are algebra homomorphisms.

Norms: The following diagram commutes:

$$\begin{array}{ccccc} & & H_k X^e & & \\ & \swarrow^{t} & \downarrow{\scriptstyle n} & \searrow^{\mathrm{Sq}} & \\ H_k X^{C_2} & \xleftarrow[\Phi^{C_2}]{} & \underline{H}_{k\rho} X & \xrightarrow[\Phi^e]{} & H_{2k} X^e \end{array}$$

Here t is the transfer (3.2) and Sq is the squaring map.

Dyer-Lashof operations: The following diagrams commute, where $\epsilon = 0, 1$:

$$\begin{array}{ccc} \underline{H}_{k\rho-\sigma} X & \xrightarrow{\Phi^e} & H_{2k-1} X^e \\ {\scriptstyle Q^{k\rho-\epsilon}}\downarrow & & \downarrow{\scriptstyle Q^{2k-\epsilon}} \\ \underline{H}_{2k\rho-\sigma-\epsilon} X & \xrightarrow[\Phi^e]{} & H_{4k-1-\epsilon} X^e \end{array}$$

$$\begin{array}{ccc} \underline{H}_{k\rho-\sigma} X & \xrightarrow{\Phi^{C_2}} & H_k X^{C_2} \\ {\scriptstyle Q^{k\rho}}\downarrow & & \downarrow{\scriptstyle \mathrm{Sq}} \\ \underline{H}_{2k\rho-\sigma} X & \xrightarrow[\Phi^{C_2}]{} & H_{2k} X^{C_2} \end{array}$$

4. Homology of $\Omega^\rho S^{\rho+1}$

Theorem 4.1. *There is an* additive *isomorphism (of $\underline{H}_\star$-modules)*

$$\underline{H}_\star \Omega^\rho S^{\rho+1} \cong \underline{H}_\star \otimes E[t_0, t_1, \ldots] \otimes P[e_1, e_2, \ldots]$$

with

$$|t_i| = 2^i\rho - \sigma,$$
$$|e_i| = (2^i - 1)\rho.$$

Proof. Note that we have

$$H_* \Omega^2 S^3 = \mathbb{F}_2[x_1, x_2, \ldots]$$

with

$$|x_i| = 2^i - 1.$$

Here x_1 is the fundamental class ι_1, and

$$x_i := Q^{2^i} Q^{2^{i-1}} \cdots Q^2 x_1.$$

Define $t_0 \in \underline{H}_1 \Omega^\rho S^{\rho+1}$ to be the fundamental class, and define the other "generators" e_i and t_i by

$$e_i := n(x_i),$$
$$t_i := Q^{2^i \rho} Q^{2^{i-1}\rho} \cdots Q^\rho t_0.$$

Consider the product

$$t^{\underline{\epsilon}} e^{\underline{k}} := t_0^{\epsilon_0} t_1^{\epsilon_1} \cdots e_1^{k_1} e_2^{k_2} \cdots \in \underline{H}_\star(\Omega^\rho S^{\rho+1})$$

with $\epsilon_i \in \{0,1\}$ and $k_i \geqslant 0$. We compute

$$\Phi^e(t^{\underline{\epsilon}} e^{\underline{k}}) = x_1^{2k_1+\epsilon_0} x_2^{2k_2+\epsilon_1} \cdots .$$

Mapping out of the cofiber sequence (2.1) gives a fiber sequence

$$\Omega N^\times \Omega S^{\rho+1} \to \Omega^\rho S^{\rho+1} \to \Omega S^{\rho+1} \xrightarrow{\Delta} N^\times \Omega S^{\rho+1}.$$

Upon taking fixed points we get a fiber sequence

$$\Omega^2 S^3 \xrightarrow{t} (\Omega^\rho S^{\rho+1})^{C_2} \to \Omega S^2 \xrightarrow{\text{null}} \Omega S^3$$

In particular there is an equivalence

$$(\Omega^\rho S^{\rho+1})^{C_2} \simeq \Omega S^2 \times \Omega^2 S^3.$$

and we have

$$H_*(\Omega^\rho S^{\rho+1})^{C_2} \cong P[y] \otimes P[t(x_1), t(x_2), \ldots]$$

where y is the image of the fundamental class under the map

$$S^1 \to (\Omega^\rho S^{\rho+1})^{C_2}.$$

It follows that

$$\Phi^{C_2}(t^{\underline{\epsilon}} e^{\underline{k}}) = y^{\epsilon_0 + 2\epsilon_1 + 4\epsilon_2 + \cdots} t(x_1)^{k_1} t(x_2)^{k_2} \cdots .$$

Thus the set

$$\{t^{\underline{\epsilon}} e^{\underline{k}}\} \subset \underline{H}_\star X$$

satisfies the hypotheses of Lemma 2.7, and the result follows. □

5. The equivariant Mahowald theorem

In order to prove Theorem 1.2 we will need to establish a Thom isomorphism

$$\underline{H}_\star(\Omega^\rho S^{\rho+1})^{\tilde{\mu}} \cong \underline{H}_\star \Omega^\rho S^{\rho+1}.$$

We will do so in two steps. Recall that an E_0-algebra is just a spectrum X equipped with a map $S^0 \to X$. Let $\mathrm{Free}^*_{E_\rho} : \mathrm{Alg}_{E_0}(\mathrm{Sp}^{C_2}) \to \mathrm{Alg}_{E_\rho}(\mathrm{Sp}^{C_2})$ denote a homotopical left adjoint to the forgetful functor. An explicit model for this functor is the homotopy pushout of E_ρ-algebras:

$$\begin{array}{ccc} \mathrm{Free}_{E_\rho}(S^0) & \longrightarrow & \mathrm{Free}_{E_\rho}(X) \\ \downarrow & & \downarrow \\ S^0 & \longrightarrow & \mathrm{Free}^*_{E_\rho}(X) \end{array}$$

We will need the following theorem.

Theorem 5.1. *Let $f : X \to B_{C_2}O$ classify a virtual bundle of dimension zero and denote by $\tilde{f} : \Omega^\rho\Sigma^\rho X \to B_{C_2}O$ the associated Ω^ρ-map. Then there is a canonical equivalence of E_ρ-algebras in* Sp^{C_2}

$$\mathrm{Free}^*_{E_\rho}(X^f) \cong (\Omega^\rho\Sigma^\rho X)^{\tilde{f}}.$$

Proof. Combine the equivariant approximation theorem [GM17, RS00] with Theorem IX.7.1 and Remark X.6.4 of [LMSM86]. □

Remark 5.2. The non-equivariant version of Theorem 5.1 was first observed by Mark Mahowald, and then proven by Lewis. A nice modern account in the non-equivariant setting via universal properties can be found in [AB14].

Proposition 5.3. *There is a Thom isomorphism*

$$\underline{H}_\star(\Omega^\rho S^{\rho+1})^{\tilde{\mu}} \cong \underline{H}_\star\Omega^\rho S^{\rho+1}.$$

Proof. Let $\mathrm{Free}^*_{E_\rho,\underline{H}} : \mathrm{Alg}_{E_0}(\mathrm{Mod}_{\underline{H}}) \to \mathrm{Alg}_{E_\rho}(\mathrm{Mod}_{\underline{H}})$ denote a homotopical left adjoint to the forgetful functor. Along with the previous theorem, we will need two facts:

1. $\underline{H} \wedge (-) : \mathrm{Sp}^{C_2} \to \mathrm{Mod}_{\underline{H}}$ is symmetric monoidal.
2. There is a Thom isomorphism $\underline{H} \wedge (S^1)^\mu \cong \underline{H} \wedge S^1_+$.

The proposition is now proved by the following string of equivalences:

$$\begin{aligned}
\underline{H} \wedge \left(\Omega^\rho\Sigma^\rho S^1\right)^{\tilde{\mu}} &\cong \underline{H} \wedge \mathrm{Free}^*_{E_\rho}\left((S^1)^\mu\right) && \text{by Theorem 5.1} \\
&\cong \mathrm{Free}^*_{E_\rho,\underline{H}}\left(\underline{H} \wedge (S^1)^\mu\right) && \text{by (1)} \\
&\cong \mathrm{Free}^*_{E_\rho,\underline{H}}\left(\underline{H} \wedge S^1_+\right) && \text{by (2)} \\
&\cong \underline{H} \wedge \mathrm{Free}^*_{E_\rho}\left(S^1_+\right) && \text{by (1)} \\
&\cong \underline{H} \wedge \Omega^\rho\Sigma^\rho S^1_+.
\end{aligned}$$

□

Proof of Theorem 1.2. The Thom class is represented by a map

$$(\Omega^\rho S^{\rho+1})^{\tilde{\mu}} \to \underline{H}.$$

We wish to show this map is an isomorphism on $\underline{H}_\star$. The homology of $\underline{H}$ is the C_2-equivariant Steenrod algebra, computed in [HK01] to be

$$\underline{H}_\star\underline{H} = \underline{H}_\star[\tau_0, \tau_1, \cdots, \xi_1, \xi_2, \cdots]/(\tau_i^2 = (u + a\tau_0)\xi_{i+1} + a\tau_{i+1})$$

with

$$|\tau_i| = 2^i\rho - \sigma,$$
$$|\xi_i| = (2^i - 1)\rho.$$

It suffices to show it is surjective, since the two homologies are abstractly isomorphic and of finite type. Observe that the composite

$$M(2) \simeq (S^1)^\mu \to (\Omega^\rho S^{\rho+1})^{\tilde{\mu}} \to \underline{H}$$

hits τ_0. Everything is hit then, by [Wil17, Thm. 5.4]. □

References

[AB14] O. Antolín-Camarena and T. Barthel, *A simple universal property of Thom ring spectra*, ArXiv e-prints (2014).

[Ati68] M. F. Atiyah, *Bott periodicity and the index of elliptic operators*, Quart. J. Math. Oxford Ser. (2) **19** (1968), 113–140. MR 0228000

[BH15] Andrew J. Blumberg and Michael A. Hill, *Operadic multiplications in equivariant spectra, norms, and transfers*, Adv. Math. **285** (2015), 658–708. MR 3406512

[GM95] J. P. C. Greenlees and J. P. May, *Generalized Tate cohomology*, Mem. Amer. Math. Soc. **113** (1995), no. 543, viii+178. MR 1230773

[GM17] Bertrand Guillou and J. P. May, *Equivariant iterated loop space theory and permutative G-categories*, arXiv:1207.3459, 2017.

[Hil] Michael A. Hill, *On algebras over equivariant little disks*, In preparation.

[HK01] Po Hu and Igor Kriz, *Real-oriented homotopy theory and an analogue of the Adams-Novikov spectral sequence*, Topology **40** (2001), no. 2, 317–399. MR 1808224

[HM17] Michael A. Hill and Lennart Meier, *The C_2-spectrum* $\mathrm{Tmf}_1(3)$ *and its invertible modules*, Algebr. Geom. Topol. **17** (2017), no. 4, 1953–2011.

[LMSM86] L. G. Lewis, Jr., J. P. May, M. Steinberger, and J. E. McClure, *Equivariant stable homotopy theory*, Lecture Notes in Mathematics, vol. 1213, Springer-Verlag, Berlin, 1986, With contributions by J. E. McClure. MR 866482

[Mah77] Mark Mahowald, *A new infinite family in* ${}_2\pi_*{}^s$, Topology **16** (1977), no. 3, 249–256. MR 0445498

[May96] J. P. May, *Equivariant homotopy and cohomology theory*, CBMS Regional Conference Series in Mathematics, vol. 91, Published for the Conference Board of the Mathematical Sciences, Washington, DC; by the American Mathematical Society, Providence, RI, 1996, With contributions by M. Cole, G. Comezaña, S. Costenoble, A. D. Elmendorf, J. P. C. Greenlees, L. G. Lewis, Jr., R. J. Piacenza, G. Triantafillou, and S. Waner. MR 1413302

[RS00] Colin Rourke and Brian Sanderson, *Equivariant configuration spaces*, J. London Math. Soc. (2) **62** (2000), no. 2, 544–552. MR 1783643

[Ull13] John Ullman, *Tambara functors and commutative ring spectra*, arXiv:1304.4912v2, 2013.

[Wil17] Dylan Wilson, *Power operations for* $\mathrm{H}\underline{\mathbb{F}}_2$ *and a cellular construction of* $\mathrm{BP}\mathbf{R}$, arXiv:1611.06958v2, 2017.