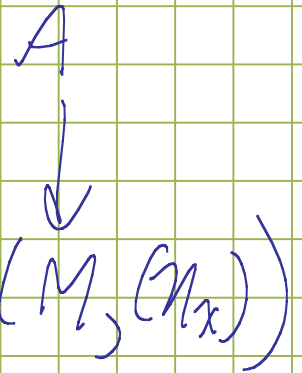


TAF-2-11-11

Beeson

Thm (Classification of ab vltres / \bar{F}_p)

1) The natural map $\left\{ \begin{array}{l} \text{quasi-isogeny classes} \\ \text{of simple ab-var}/\bar{F}_p \end{array} \right\}$



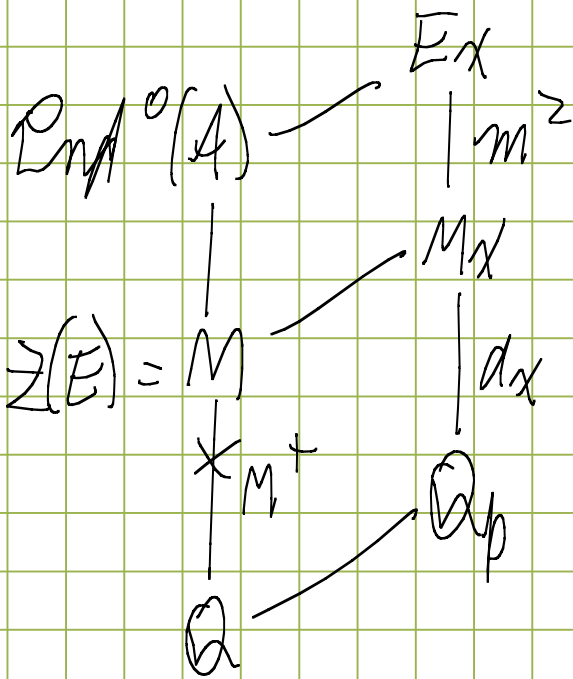
$\left\{ \begin{array}{l} \text{minimal } p\text{-adic types} \end{array} \right\}$

$f_x = \text{residual gp}$

$$\dim A(x) = n_x f_x d_x m$$

defines $n_x \in \mathbb{Q}$

$$A(p) = \prod_{x/p} A(x)$$



A p -adic type $(M, (\eta_x))$ is

1) CM field M

2) (η_x) $\eta_x \in \mathcal{O}$ $\forall x/p$ in M

A morphism of p -adic types

$$(M, (\eta_x)) \longrightarrow (M', (\eta'_{x'}))$$

is an embedding $M \hookrightarrow M'$

and $\forall x'/x$ a prime of M we have

$$\eta'_{x'} = e_{x'/x} \eta_x$$

minimal means not the target of a nontrivial morphism

B-linear ab-norms

We want A/\bar{F}_p s.t. $A(p)$ contains a
set of n 1-dim FGL, slope $A(p) = 1/n$

$\text{End}(A)_p \cong \text{End}(A(p)) =$ unique maximal orders
in a central div alg / \mathbb{Q}_p
of invariant $1/n$.

$$\text{End}^0(A) = E$$



$$d < 0$$

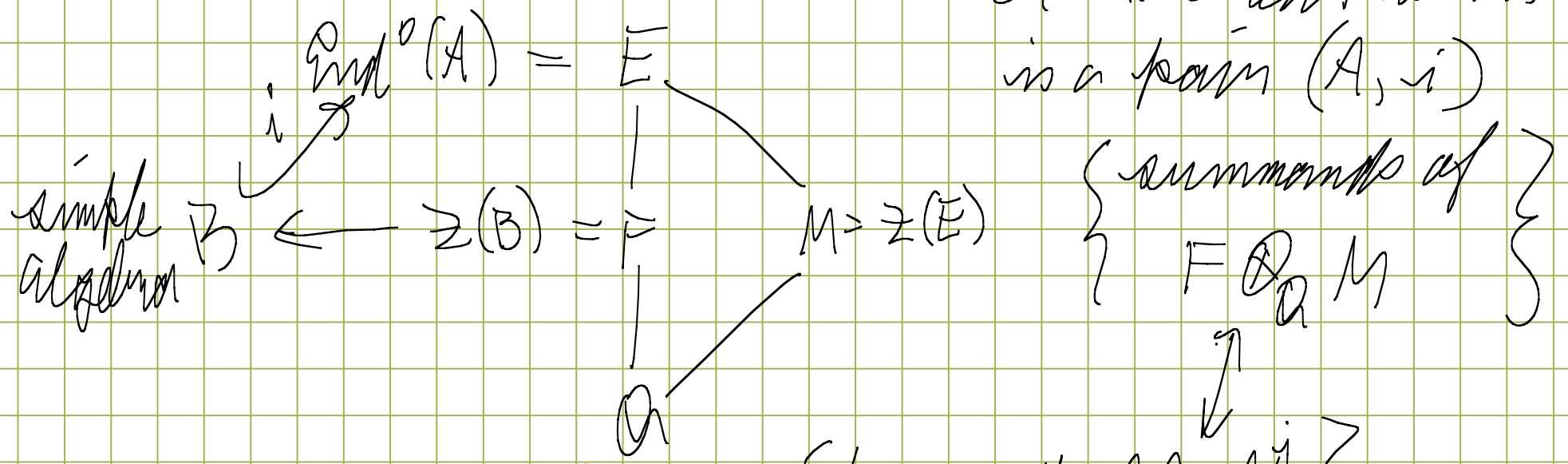
$$M = \mathbb{Q}(\sqrt{d}) \quad p = \chi(d)$$



$$\mathbb{Q} \quad \eta_x = \frac{1}{n} \quad , \quad \eta_{d(x)} = 1 - \frac{1}{n}$$

In this setup $\exists!$ quasi-isog class of ab var
of dim n with CM by F ($\mathcal{O}_F \hookrightarrow \text{End}^0(A)$)
s.t. $A(x)$ has slope $1/n$. $(M, \langle \cdot, \cdot \rangle) / F$, $F \subseteq M$.

A B -linear ab var
is a pair (A, i)



$\{(A, i) \text{ s.t. } A \cong A_0^i\}$
isotypical

WHY DO WE CARE?

Tate modules

$l \neq \text{char } K = p$

$$T_l(A) := \begin{cases} \varprojlim A[l^n] \simeq \mathbb{Z}_l^{2g} & g = \dim A \\ \varprojlim T_l(A \otimes_K \bar{K}) & \text{if } A \text{ defined over separably closed } K = \bar{K} \end{cases}$$

$$V_l(A) = T_l(A) \otimes_{\mathbb{Z}} \mathbb{Q}$$

$$T^p(A) = \prod_{l \neq p} T_l(A)$$

Lemma $l \neq p = \text{char}(k)$

$$\text{Hom}(A, B) \hookrightarrow \text{Hom}_{\mathbb{Z}_l}(T_l A, T_l B)$$

Corry For $\text{char}(k) \neq l$, f, g over prime field.

$$\text{Hom}_k(A, B) \otimes \mathbb{Z}_l \cong \text{Hom}_{\mathbb{Z}_l}(T_l(A), T_l(B))$$

$G_{\mathbb{Z}_l}$

Basis $g_{\mathbb{Z}_l}$

Level structures V a vector space/ \mathbb{Q} of $\dim \geq g$.

$L \subseteq V$ a \mathbb{Z} -lattice n

$\dim(A) = g$

$$L^{\text{up}} := \prod_{l \neq p} L \otimes \mathbb{Z}_l \xrightarrow{\cong} T^p(A)$$

choice of iso is called an uniformization of A

$$V^p := L^p \otimes \mathbb{Q} \xrightarrow{\cong} V^p(A) \quad \text{rational uniformization}$$

$\text{Aut}(V^p)$ acts on set of all rational uniformizations.
 $[V]_{K^p}$

Let K^p be a subgroup of $\text{Aut}(V^p)$. A K^p -level rational structure $[V]_{K^p}$ is the K^p -orbit of a rational uniformization η .

