

3/26/10

Description of congruence subgroups of S_n , i. e. units congruent to 1 mod S_n^k

- Each subgroup is normal in the next
- Each subquotient is $(\mathbb{Z}/p)^n$
- For $k > 0$, $H^k = \Lambda(n^2 \text{ gens})$.

• For $k \gg 0$ the group is cohomologically abelian.

My remarks

Consider the SES

$$0 \rightarrow N^0 \rightarrow M^0 \rightarrow N^1 \rightarrow 0$$

\parallel \parallel \parallel
 $\mathbb{Z}[x]$ $\mathbb{Z}[x]$ $\mathbb{Z}[x]/\langle p^2 \rangle$

The element $v_1^1/p_1 \in N^1$ is invariant since
 $\eta_R(v_1) = v_1 + p_1 t_1$ so $\eta_R(v_1^1) \equiv v_1^1 \pmod{p_1}$

There is a comm hom $\text{Ext}^0(N^1) \rightarrow \text{Ext}^1(N^0)$.

The images of $\frac{v_1^1}{p_1}$ is called $\bar{\alpha}_1$ and
represents a gen of $\text{Im } J$.