

Y. Zou on the positive complete model category structure on Sp^G .

Goal:

$$I = \left\{ G_H \wedge_H S^{-V} \wedge i_{n+1} : n \geq 0, H \subseteq G, V^H \neq 0 \right\}$$

$$i_n: S^{n-1} \rightarrow D^n$$

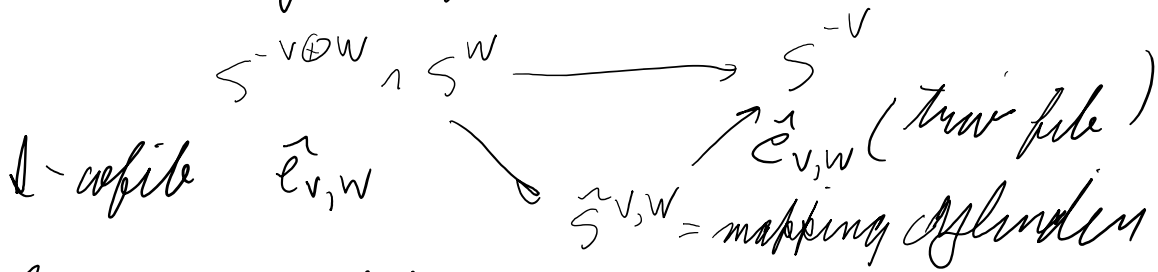
$$j_n: I^n \rightarrow I^{n+1}$$

= generating set of cofibrations

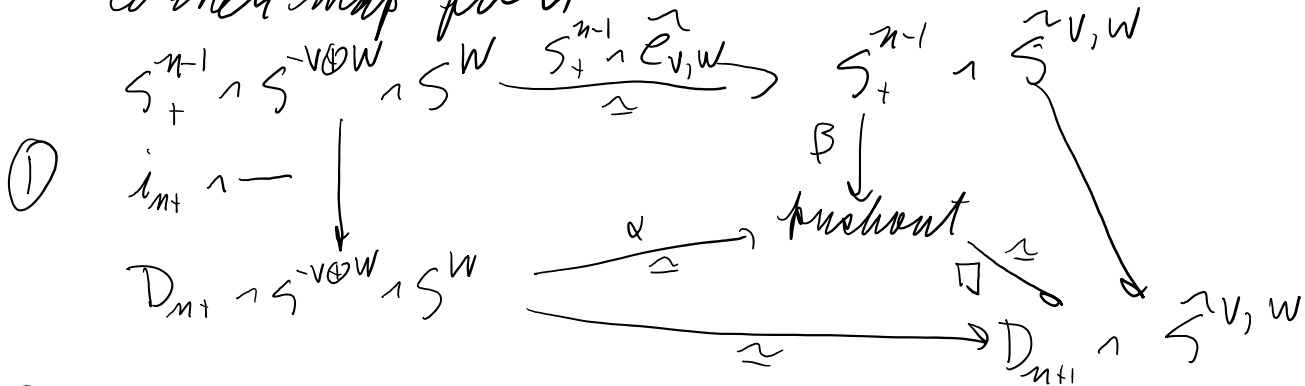
$$J = \left\{ G_H \wedge_H S^{-V} \wedge j_{n+1} : \text{same} \right\} \cup \left\{ G_H \wedge_H i_{n+1} \square \tilde{e}_{V,W} \right\}$$

= generating set of trivial cofibrations

\square denotes pushout corner map
 $W = \text{any rep of } H$



Corner map fib



Main Theorem I and J above define a cofibrantly generated model category structure on Sp^G .

Proof Will use Kan recognition theorem, which requires 4 conditions

- 1) Smallness of I and J , technical but easy

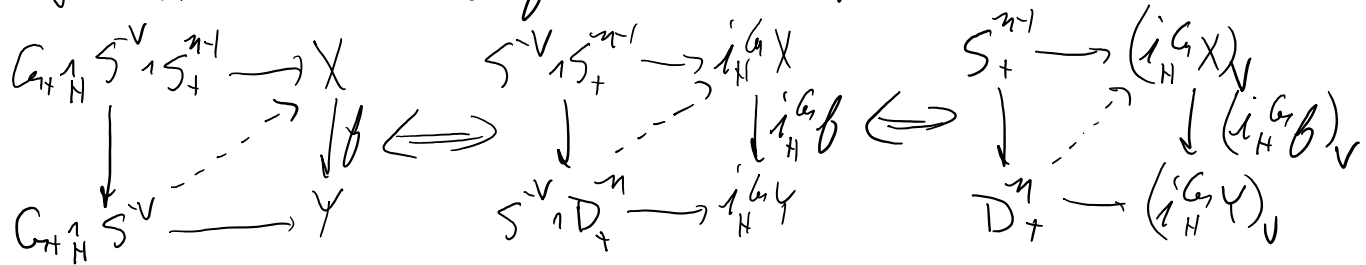
- (2) Each elt of f is stable equiv
 - (3) Every map with RLP for d is weak equiv and has RLP for f
 - (4) A weak equiv and d -cofibr is also a f -cofibr
- A weak equiv with RLP for f has it for d .

We need to show these 4 conditions are met

1) For each H-rep V with $V^H \neq 0$, $S^V \triangleright i_{n+}^m$ and corner map of \mathbb{D} are all small. $\tilde{e}_{v,w}$ is in the saturated class generated by d , so it is small relative to d . Pushout is colimit which preserves smallness.

2) Need to show f -cofibrs are d -cofibrs and weak equivs. For the noncorner maps in f , this is easy. We need to show $\tilde{e}_{v,w}$ is an d -cofibr. This is true objectwise. Same goes for $i_{n+} \sqcap \tilde{e}_{v,w}$. The map $\tilde{e}_{v,w}$ is a weak equiv since $S^V \triangleright e_w$ is one. α in \mathbb{D} is weak equiv so corner map is.

3) Let $f: X \rightarrow Y$ have RLP for d . Need to show it has it for d and is a weak equiv. RLP for $d \Rightarrow$ struct equiv



so $(i_H^{G_+} b)_V$ is a trivial fibration, so f is weak equiv. Showing RLP for f is harder.

Prop $f: X \rightarrow Y$ has RLP for f iff $\forall H \in \mathcal{G}$ and rep V of H with $V^H \neq 0$,

$(i_H^G X)_V \longrightarrow (i_H^G Y)_V$ is a fibration and for each H -rep W the diagram

$$\begin{array}{ccc} (i_H^G X)_V & \xrightarrow{(i_H^G f)_V} & (i_H^G Y)_V \\ \downarrow & \searrow^{\Omega^W(i_H^G f)_{V \oplus W}} & \downarrow \\ \Omega^W(i_H^G X)_{V \oplus W} & \xrightarrow{\cong} & \Omega^W(i_H^G Y)_{V \oplus W} \end{array}$$

is homotopy Cartesian
i.e. upper left space is pull back.

(Proof omitted)

The map $(i_H^G f)_V$ is a trivial fibration hence a fibration

Both horizontal maps above are weak eqivs.

This concludes (3).

For (4) we need

Prop If $f: X \rightarrow Y$ is a weak eqiv with RLP for f then it has RLP for \downarrow .

Proof The RLP for $f \Rightarrow (i_H^G f)_V$ is fibration and

we have a hty Cartesian diagram

$$\begin{array}{ccc} (i_H^G X)_V & \xrightarrow{\cong} & (i_H^G Y)_V \\ \downarrow & & \downarrow \\ \text{hocolim}_W \Omega^W(i_H^G X)_{V \oplus W} & \xrightarrow{\cong} & \text{hocolim}_W \Omega^W(i_H^G Y)_{V \oplus W} \end{array}$$

Hence f is trivial fibration

QED.