MATH 549 4-21-10

Note Title 4/21/2010

Extra meetings

Tomorrow 3:30 here

Friday 4/30 4:00 Hylan 101

Toward the detection theorem

Structure of $BP_*BP = BP_*[t_1, t_2, \ldots]$

$BP_* = \mathbb{Z}_{(p)}[v_1, v_2, \ldots]$

$|v_n| = |t_n| = 2(p^n-1)$

3 facts about this structure

1) $\Delta(t_1) = t_1 \otimes 1 + 1 \otimes t_1$

2) $\eta_R(v_1) = v_1 + pt_1$

3) $\eta_R(v_2) \equiv v_2 + v_1 t_1^p - v_1^p t_1 \mod p$

How to construct some elements in

$\mathcal{E}xt = \mathcal{E}xt_{BP_*(BP)}(BP_*, BP_*)$

a) Consider the short exact seq

$$\begin{array}{ccccccccc} & & & & p^{-1}v_1^i & \longmapsto & v_1^i/p & & \\ 0 & \to & BP_* & \to & p^{-1}BP_* & \longrightarrow & BP_*/p^\infty & \to & 0 \\ & & & & \| & & & & \\ & & & & BP_* \otimes \mathbb{Q} & & & & \\ 0 & \to & \mathbb{Z}_{(p)} & \to & \mathbb{Q} & \longrightarrow & \mathbb{Q}/\mathbb{Z}_{(p)} & \to & 0 \\ & & & & & & \| & & \\ & & & & & & \varinjlim_{i\to\infty} \mathbb{Z}/p^i & & \end{array}$$

In BP_*/p^∞ we have $\frac{v_1^i}{p}$, which is invariant, i.e. $\eta_L\left(\frac{v_1^i}{p}\right) = \eta_R\left(\frac{v_1^i}{p}\right)$

$$\eta_R\left(\frac{v_1^i}{p}\right) = \frac{(v_1 + pt_1)^i}{p} = \frac{v_1^i}{p} = \eta_R\left(\frac{v_1^i}{p}\right)$$

This means it is in $\mathrm{Ext}^0(BP_*/p^\infty)$

$$0 \to BP_* \to p^{-1}BP_* \to BP_*/p^\infty \to 0$$

We get a long exact seq of Ext gps

$$
\begin{tikzcd}
 & \mathbb{Z}_{(p)} \arrow[d, phantom, "\parallel"] & \mathbb{Q} \arrow[d, phantom, "\parallel"] & \mathbb{Q}/\mathbb{Z}_{(p)} \oplus \text{other stuff} \arrow[d, phantom, "\parallel"] \\
0 \arrow[r] & \mathrm{Ext}^0 \arrow[r] & \mathrm{Ext}^0(p^{-1}BP_*) \arrow[r] & \mathrm{Ext}^0(BP_*/p^\infty) \arrow[dll, out=-90, in=90] \\
 & \mathrm{Ext}^1 \arrow[r] \arrow[d, phantom, "\parallel"] & \mathrm{Ext}^1(p^{-1}BP_*) \arrow[r] \arrow[d, phantom, "\parallel"] & \cdots \\
 & \text{other stuff} & 0 &
\end{tikzcd}
$$

$\frac{v_1^{i}}{p}$

α_i

for $i > 0$ $\quad \alpha_i \in \mathrm{Ext}^{1,2i}$ $\quad$ for $p = 2$

There is another SES

$$0 \to BP_*/p^\infty \longrightarrow v_1^{-1}BP_*/p^\infty \longrightarrow BP_*/(p^\infty, v_1^\infty) \to 0$$

$$\frac{v_2^{\,i}}{2\,v_1^{\,j}} \qquad i,j>0$$

$$\eta_R(v_2) = v_2 + v_1 t_1^2 - v_1^2 t_1 \mod 2$$

$\frac{v_2^{\,i}}{2v_1^{\,j}}$ is invariant for certain i,j

This leads (via LES of Ext gps) to

$$\beta_{i/j} \in \mathrm{Ext}^{2,\,6i-2j}$$

$$\beta_{2^j/2^j} \in \mathrm{Ext}^{2,\,6\cdot 2^j - 2\cdot 2^j} = \mathrm{Ext}^{2,\,2^{j+2}}$$

This elements maps (under ρ) to h_{j+1}^2

and is related to θ_{j+1}

We need to show this element this maps nontrivially to a similar gp for the spectrum Ω.

Toward the slice theorem

The slice theorem describes the layers of the slice tower for $MU^{(g/2)}$ $\quad g = G = C_{2^n}$

The slice tower is an equiv analog of the classical Postnikov tower.

$X \longrightarrow P^n X$ is the map obtained by killing all htpy gps above dim n.

Its fiber $P_n X$ is the n-connected cover of X.

In the equiv analog we replace spheres by the following : $H \subset G$ ρ_H = reg rep of H

$m \in \mathbb{Z}$ $\hat{S}(m\rho_H) = G_+ \wedge_H S^{m\rho_H}$

= wedge of g/h copies of S^{mh}

and $\Sigma^{-1}\hat{S}(m\rho_H)$. These are called slice cells. We get a tower

$$\cdots \longrightarrow P_G^n X \longrightarrow P_G^{n-1} X \longrightarrow \cdots$$

The n-th layer (n-th slice) is the fiber of the map shown above

$$[\hat{S}(m\rho_h), {}^{G}P^{n}X] = 0 \quad \text{if } mh > n$$

$$[\Sigma^{-1}\hat{S}(m\rho_h), {}^{G}P^{n}X] = 0 \quad \text{if } mh-1 > n$$

The Slice Theorem identifies

$${}^{G}P^{n}_{n} MU^{(g/2)}$$

It is: contractible for n odd

wedge of $\hat{S}(m\rho_h) \wedge H\underline{\mathbb{Z}}$ for

$mh = n$

n even. The trivial subgp of G

never occurs.