Plastic explosives:

A *C*₄ analog of the Kervaire invariant calculation

Conference on Equivariant, Chromatic and Motivic Homotopy Theory

Northwestern University

March 25, 2013

Mike Hill University of Virginia Mike Hopkins Harvard University Doug Ravenel University of Rochester

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What this talk is about



Mike Hill, myself and Mike Hopkins. Photo by Bill Browder, 2010.

In 2009 Mike Hill, Mike Hopkins and I proved the following.

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In 2009 Mike Hill, Mike Hopkins and I proved the following.

Theorem

The element $\theta_j \in \pi_{2^{j+1}-2}S^0$ associated with the Kervaire invariant does not exist for $j \ge 7$.

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Our method of proof involved a ring spectrum Ω with three properties:

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(i) Detection Theorem If θ_j exists it has a nontrivial image in $\pi_*\Omega$ under the unit map $S^0 \to \Omega$.

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- (i) Detection Theorem If θ_j exists it has a nontrivial image in $\pi_*\Omega$ under the unit map $S^0 \to \Omega$.
- (ii) Periodicity Theorem $\Sigma^{256}\Omega \simeq \Omega$, so $\pi_k\Omega$ depends only on the value of *k* modulo 256.

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The second two of these imply that $\pi_{2^{j+1}-2}\Omega = 0$ for $j \ge 7$,

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- (iii) Gap Theorem $\pi_{-2}\Omega = 0$.

The second two of these imply that $\pi_{2^{j+1}-2}\Omega = 0$ for $j \ge 7$, so (i) implies that θ_j does not exist for such *j*.

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The construction and study of the spectrum Ω involves equivariant stable homotopy theory.

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The construction and study of the spectrum Ω involves equivariant stable homotopy theory. The group *G* is question will always be a finite cyclic 2-group.

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Notational convention:

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The construction and study of the spectrum Ω involves equivariant stable homotopy theory. The group *G* is question will always be a finite cyclic 2-group.

Notational convention:

We will denote the cyclic group of order *n* by

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Notational convention:

We will denote the cyclic group of order n by

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when it acts on something

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The construction and study of the spectrum Ω involves equivariant stable homotopy theory. The group G is question will always be a finite cyclic 2-group.

Notational convention:

We will denote the cyclic group of order *n* by

- $\begin{cases} C_n & \text{when it acts on something} \\ \mathbf{Z}/n & \text{when it is the value of some functor.} \end{cases}$

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The construction and study of the spectrum Ω involves equivariant stable homotopy theory. The group *G* is question will always be a finite cyclic 2-group.

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When it appears as an index, we will abbreviate it by n.

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The complex cobordism spectrum MU has a C_2 -action defined in terms of complex conjugation.

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The complex cobordism spectrum MU has a C_2 -action defined in terms of complex conjugation. The resulting C_2 -equivariant spectrum is denoted by $MU_{\rm R}$. Here are some people who have studied it.

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Shoro Araki 1930–2005

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Nitu Kitchloo



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For any C_2 -spectrum X, there is a C_8 -action defined on its 4-fold smash power $X^{(4)}$.

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For any C_2 -spectrum X, there is a C_8 -action defined on its 4-fold smash power $X^{(4)}$. This C_8 -spectrum is denoted by $N_2^8 X$,

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For any C_2 -spectrum X, there is a C_8 -action defined on its 4-fold smash power $X^{(4)}$. This C_8 -spectrum is denoted by $N_2^8 X$, the norm N_2^8 being a functor from C_2 -spectra to C_8 -spectra.

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Inverting a certain element $D \in \pi_*^{C_8} N_2^8 M U_R$ gives a 256-periodic C_8 -spectrum $\widetilde{\Omega} = D^{-1} N_2^8 M U_R$.

For any C_2 -spectrum X, there is a C_8 -action defined on its 4-fold smash power $X^{(4)}$. This C_8 -spectrum is denoted by $N_2^8 X$, the norm N_2^8 being a functor from C_2 -spectra to C_8 -spectra.



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$$\Omega = \widetilde{\Omega}^{C_8}.$$

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We developed a new tool for studying such spectra called the slice spectral sequence.

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We developed a new tool for studying such spectra called the slice spectral sequence. I will say more about it later. It is indispensable for proving the Gap Theorem.

The homotopy groups of $\widetilde{\Omega}$ and Ω are inaccessibly complicated,

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What this talk is about (continued)

We developed a new tool for studying such spectra called the slice spectral sequence. I will say more about it later. It is indispensable for proving the Gap Theorem.

The homotopy groups of $\widetilde{\Omega}$ and Ω are inaccessibly complicated, at least for now.

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Analogous constructions with C_8 replaced by C_2 or C_4 are not so bad.

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The homotopy groups of $\widetilde{\Omega}$ and Ω are inaccessibly complicated, at least for now.

Analogous constructions with C_8 replaced by C_2 or C_4 are not so bad. This talk is about those cases.

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The C_2 case

In the C_2 case we start with $MU_{\mathbf{R}}$ itself,

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The C_2 case

In the C_2 case we start with $MU_{\rm R}$ itself, invert a 2-dimensional generator in its equivariant homotopy,

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In the C_2 case we start with MU_R itself, invert a 2-dimensional generator in its equivariant homotopy, and throw out some redundant higher dimensional generators.

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In the C_2 case we start with MU_R itself, invert a 2-dimensional generator in its equivariant homotopy, and throw out some redundant higher dimensional generators. The resulting spectrum is real K-theory K_R ,

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In the C_2 case we start with MU_R itself, invert a 2-dimensional generator in its equivariant homotopy, and throw out some redundant higher dimensional generators. The resulting spectrum is real K-theory K_R , meaning the complex K-theory spectrum K equipped with complex conjugation.

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Sir Michael Atiyah

It was originally studied by Atiyah in 1966 in a paper called *K*-theory and reality.

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Sir Michael Atiyah

It was originally studied by Atiyah in 1966 in a paper called *K*-theory and reality. Its C_2 fixed point set is the orthogonal K-theory spectrum *KO*.

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In the C_2 case we start with MU_R itself, invert a 2-dimensional generator in its equivariant homotopy, and throw out some redundant higher dimensional generators. The resulting spectrum is real K-theory K_R , meaning the complex K-theory spectrum K equipped with complex conjugation.



Sir Michael Atiyah

It was originally studied by Atiyah in 1966 in a paper called *K*-theory and reality. Its C_2 fixed point set is the orthogonal K-theory spectrum *KO*. It is known to be 8-periodic.

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The C₂ case (continued)



The slice spectral sequence for $K_{\rm R}$ was the subject of Dan Dugger's thesis.

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The C₂ case (continued)



The slice spectral sequence for $K_{\rm R}$ was the subject of Dan Dugger's thesis. It gave a novel and elegant way to understand the 2-torsion in π_*KO , the subject of the real case of the Bott Periodicity Theorem. Mike Hill Mike Hopkins Doug Ravenel



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The C₂ case (continued)



The slice spectral sequence for $K_{\mathbf{R}}$ was the subject of Dan Dugger's thesis. It gave a novel and elegant way to understand the 2-torsion in π_*KO , the subject of the real case of the Bott Periodicity Theorem. I will describe it in detail later. Mike Hill Mike Hopkins Doug Ravenel



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The slice spectral sequence is an equivariant analog of the Postnikov filtration.

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The slice spectral sequence is an equivariant analog of the Postnikov filtration. In the latter we filter a spectrum X by its (n-1)-connected covers $\{P_nX\}$.

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The slice spectral sequence is an equivariant analog of the Postnikov filtration. In the latter we filter a spectrum *X* by its (n-1)-connected covers $\{P_nX\}$. The cofiber of the map $P_{n+1}X \rightarrow X$ is the spectrum obtained from *X* by killing all homotopy groups above dimension *n*.

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This collection of cofiber sequences leads to what might be called the Postnikov spectral sequence.

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This collection of cofiber sequences leads to what might be called the Postnikov spectral sequence. There is a good reason you have may not heard of it before:

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This collection of cofiber sequences leads to what might be called the Postnikov spectral sequence. There is a good reason you have may not heard of it before: it is useless.

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This collection of cofiber sequences leads to what might be called the Postnikov spectral sequence. There is a good reason you have may not heard of it before: it is useless. Its input and output are both π_*X .

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Nevertheless, note that P_n S, the category of (n - 1)-connected spectra,

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Nevertheless, note that P_n S, the category of (n-1)-connected

spectra, is the smallest subcategory of S (the category of all spectra), containing the set

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Nevertheless, note that P_n S, the category of (n - 1)-connected spectra, is the smallest subcategory of S (the category of all spectra), containing the set

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Nevertheless, note that P_n S, the category of (n - 1)-connected spectra, is the smallest subcategory of S (the category of all spectra), containing the set

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and closed under mapping cones, infinite wedges and retracts.

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Nevertheless, note that P_n S, the category of (n - 1)-connected spectra, is the smallest subcategory of S (the category of all spectra), containing the set

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and closed under mapping cones, infinite wedges and retracts. Hence the cofiber of a map between (n - 1)-connected spectra is again (n - 1)-connected, but the fiber of such a map need not be.

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Again, P_n S, the category of (n - 1)-connected spectra, is generated by the set

 $T_n = \{S^m \colon m \ge n\}$

We need an equivariant generalization of the set T_n .

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We need an equivariant generalization of the set T_n . For $G = C_2$ consider the following spectra for each integer *m*.

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 $G_+ \wedge S^m$, $S^{m\rho}$ and $S^{m\rho-1}$.

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Here $G_+ \wedge S^m$ is the wedge of two *m*-spheres that are interchanged by the generator $\gamma \in C_2$.

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Here $G_+ \wedge S^m$ is the wedge of two *m*-spheres that are interchanged by the generator $\gamma \in C_2$. $S^{m\rho}$ is the one point compactification of $m\rho$,

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Here $G_+ \wedge S^m$ is the wedge of two *m*-spheres that are interchanged by the generator $\gamma \in C_2$. $S^{m\rho}$ is the one point compactification of $m\rho$, where ρ denotes the regular representation of C_2 .

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 $G_+ \wedge S^m$, $S^{m\rho}$ and $S^{m\rho-1}$.

Here $G_+ \wedge S^m$ is the wedge of two *m*-spheres that are interchanged by the generator $\gamma \in C_2$. $S^{m\rho}$ is the one point compactification of $m\rho$, where ρ denotes the regular representation of C_2 . The latter is underlain by S^{2m} . Its desuspension is $S^{m\rho-1}$, underlain by S^{2m-1} .

We will call these spectra slice spheres.

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For $G = C_2$ the generalization of

$$T_n = \{S^m \colon m \ge n\}$$



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For $G = C_2$ the generalization of

$$T_n = \{S^m \colon m \ge n\}$$

is

 $T_n^G = \{G_+ \land S^m \colon m \ge n\} \cup \{S^{m\rho-\epsilon} \colon 2m-\epsilon \ge n, \epsilon = 0, 1\}.$



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For $G = C_2$ the generalization of

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is

$$T_n^{\boldsymbol{G}} = \{\boldsymbol{G}_+ \land \boldsymbol{S}^m \colon m \ge n\} \cup \{\boldsymbol{S}^{m\rho-\epsilon} \colon \boldsymbol{2m-\epsilon} \ge n, \, \epsilon = 0, 1\}$$

Let S^G denote the category of *G*-spectra.



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$$T_n^G = \{G_+ \land S^m \colon m \ge n\} \cup \{S^{m\rho - \epsilon} \colon 2m - \epsilon \ge n, \epsilon = 0, 1\}$$

Let S^G denote the category of *G*-spectra. Define $P_n S^G$ to be the subcategory generated by the elements of T_n^G ,

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$$T_n^G = \{G_+ \land S^m \colon m \ge n\} \cup \{S^{m\rho-\epsilon} \colon 2m-\epsilon \ge n, \epsilon = 0, 1\}$$

Let S^G denote the category of *G*-spectra. Define $P_n S^G$ to be the subcategory generated by the elements of T_n^G , i.e., by slice spheres of dimension $\geq n$.



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Let S^G denote the category of *G*-spectra. Define $P_n S^G$ to be the subcategory generated by the elements of T_n^G , i.e., by slice spheres of dimension $\geq n$.

This filtration of S^G leads to the slice spectral sequence.



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Let S^G denote the category of *G*-spectra. Define $P_n S^G$ to be the subcategory generated by the elements of T_n^G , i.e., by slice spheres of dimension $\geq n$.

This filtration of S^G leads to the slice spectral sequence. It maps to the classical one under the forgetful functor $S^G \to S$.

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Let S^G denote the category of *G*-spectra. Define $P_n S^G$ to be the subcategory generated by the elements of T_n^G , i.e., by slice spheres of dimension $\geq n$.

This filtration of S^G leads to the slice spectral sequence. It maps to the classical one under the forgetful functor $S^G \rightarrow S$. For a *G*-spectrum *X* it enables us to define *G*-analogs of connective covers.

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Let S^G denote the category of *G*-spectra. Define $P_n S^G$ to be the subcategory generated by the elements of T_n^G , i.e., by slice spheres of dimension $\geq n$.

This filtration of S^G leads to the slice spectral sequence. It maps to the classical one under the forgetful functor $S^G \to S$. For a *G*-spectrum *X* it enables us to define *G*-analogs of connective covers. The *n*th slice $P_n^n X$ is the cofiber of the map $P_{n+1}X \to P_n X$

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is

$$T_n^G = \{G_+ \land S^m \colon m \ge n\} \cup \{S^{m\rho-\epsilon} \colon 2m-\epsilon \ge n, \epsilon = 0, 1\}.$$

Let S^G denote the category of *G*-spectra. Define $P_n S^G$ to be the subcategory generated by the elements of T_n^G , i.e., by slice spheres of dimension $\geq n$.

This filtration of S^G leads to the slice spectral sequence. It maps to the classical one under the forgetful functor $S^G \to S$. For a *G*-spectrum *X* it enables us to define *G*-analogs of connective covers. The *n*th slice $P_n^n X$ is the cofiber of the map $P_{n+1}X \to P_n X$ just as in the classical case. Plastic explosives

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The slice spectral sequence is more interesting than the Postnikov spectral sequence for the following reason.



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The slice spectral sequence is more interesting than the Postnikov spectral sequence for the following reason. The fixed point set of an *n*-dimensional slice sphere need not be (n-1)-connected.

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The definitions above can be generalized to an arbitrary finite group *G*.

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The slice spectral sequence is more interesting than the Postnikov spectral sequence for the following reason. The fixed point set of an *n*-dimensional slice sphere need not be (n-1)-connected.

The definitions above can be generalized to an arbitrary finite group *G*. For each subgroup $H \subset G$, we define

$$G_+ \mathop{\wedge}_{H} S^{m
ho_H - \epsilon}$$

to be a slice sphere of dimension $m|H| - \epsilon$,

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to be a slice sphere of dimension $m|H| - \epsilon$, where ρ_H is the regular representation of H, m is any integer

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to be a slice sphere of dimension $m|H| - \epsilon$, where ρ_H is the regular representation of H, m is any integer and ϵ is 0 or 1. Then we define

$$T_n^G = \left\{ G_+ \underset{H}{\wedge} S^{m_{\rho_H} - \epsilon} \colon m|H| - \epsilon \ge n, \ H \subset G, \ \epsilon = 0, 1 \right\},$$

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the set of slice spheres of dimension $\geq n$.

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We use the resulting filtration of S^G to define

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We use the resulting filtration of S^G to define "connective covers" $P_n X$,

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We use the resulting filtration of S^G to define "connective covers" P_nX , "Postnikov sections" P^nX

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Dugger's slice spectral sequence

We use the resulting filtration of S^G to define "connective covers" P_nX , "Postnikov sections" P^nX and slices P_n^nX as before.

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We use the resulting filtration of S^G to define "connective covers" P_nX , "Postnikov sections" P^nX and slices P_n^nX as before.

Determining the slices of a G-spectrum X is not easy in general.

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Dugger's slice spectral sequence

We use the resulting filtration of S^G to define "connective covers" P_nX , "Postnikov sections" P^nX and slices P_n^nX as before.

Determining the slices of a G-spectrum X is not easy in general. The main technical computation of HHR is the identification of these slices for the spectra of interest in the paper.

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Determining the slices of a *G*-spectrum X is not easy in general. The main technical computation of HHR is the identification of these slices for the spectra of interest in the paper. These spectra are all relatives of $MU_{\rm R}$.

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Determining the slices of a *G*-spectrum *X* is not easy in general. The main technical computation of HHR is the identification of these slices for the spectra of interest in the paper. These spectra are all relatives of $MU_{\rm R}$. In each case the *n*th slice is contractible for odd *n*,

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$$P_n^n X = W_n \wedge H\mathbf{Z}$$

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$$P_n^n X = W_n \wedge H \underline{Z}$$

where W_n is a wedge of *n*-dimensional slice spheres

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$$P_n^n X = W_n \wedge H \mathbf{Z}$$

where W_n is a wedge of *n*-dimensional slice spheres and $H\mathbf{Z}$ is the integer Eilenberg-Mac Lane spectrum with trivial *G*-action.

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If you know the homotopy groups of the fixed point sets of the slices of X,

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Dugger's slice spectral sequence

If you know the homotopy groups of the fixed point sets of the slices of X, then you can use the slice spectral sequence to learn the same about X itself.

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Dugger's slice spectral sequence

If you know the homotopy groups of the fixed point sets of the slices of X, then you can use the slice spectral sequence to learn the same about X itself.

The best way to keep track of this information is to use Mackey functors.

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If you know the homotopy groups of the fixed point sets of the slices of X, then you can use the slice spectral sequence to learn the same about X itself.

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Here is another such slogan.

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Here is another such slogan. Indecomposable finite G-sets are the equivariant analog of points in ordinary homotopy theory.

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I will explain what a Mackey functor is shortly.

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I will explain what a Mackey functor is shortly. For the moment suffice it to say that they form an abelian category.

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Here is another such slogan. Indecomposable finite G-sets are the equivariant analog of points in ordinary homotopy theory.

I will explain what a Mackey functor is shortly. For the moment suffice it to say that they form an abelian category. This means one can have a spectral sequence of Mackey functors.

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I will explain what a Mackey functor is shortly. For the moment suffice it to say that they form an abelian category. This means one can have a spectral sequence of Mackey functors.

God help us!

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Fix a group G.

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Fix a group *G*. Assume for simplicity that it is finite and abelian.

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Fix a group *G*. Assume for simplicity that it is finite and abelian. The examples of interest to us are finite cyclic 2-groups.

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Fix a group *G*. Assume for simplicity that it is finite and abelian. The examples of interest to us are finite cyclic 2-groups.

Formally a Mackey functor \underline{M} assigns an abelian group to every finite G-set

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Dugger's slice spectral sequence

Fix a group *G*. Assume for simplicity that it is finite and abelian. The examples of interest to us are finite cyclic 2-groups.

Formally a Mackey functor \underline{M} assigns an abelian group to every finite G-set and is additive on disjoint unions,

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Fix a group *G*. Assume for simplicity that it is finite and abelian. The examples of interest to us are finite cyclic 2-groups.

Formally a Mackey functor \underline{M} assigns an abelian group to every finite G-set and is additive on disjoint unions,

 $\underline{M}(A \amalg B) = \underline{M}(A) \oplus \underline{M}(B).$

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Formally a Mackey functor \underline{M} assigns an abelian group to every finite G-set and is additive on disjoint unions,

 $\underline{M}(A \amalg B) = \underline{M}(A) \oplus \underline{M}(B).$

Hence <u>*M*</u> is determined by its values on G/H for subgroups *H*.

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Formally a Mackey functor \underline{M} assigns an abelian group to every finite G-set and is additive on disjoint unions,

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Hence <u>*M*</u> is determined by its values on G/H for subgroups *H*. It is both covariant and contravariant.

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Hence \underline{M} is determined by its values on G/H for subgroups H. It is both covariant and contravariant. Given subgroups

 $K \subset H \subset G$

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Formally a Mackey functor \underline{M} assigns an abelian group to every finite G-set and is additive on disjoint unions,

 $\underline{M}(A \amalg B) = \underline{M}(A) \oplus \underline{M}(B).$

Hence \underline{M} is determined by its values on G/H for subgroups H. It is both covariant and contravariant. Given subgroups

 $K \subset H \subset G$

we get maps

$$\frac{\underline{M}(G/H)}{\operatorname{Res}_{\kappa}^{H}(\)}\operatorname{Tr}_{\kappa}^{H}$$
$$\underline{M}(G/K),$$

called restrictions and transfers,

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Fix a group *G*. Assume for simplicity that it is finite and abelian. The examples of interest to us are finite cyclic 2-groups.

Formally a Mackey functor \underline{M} assigns an abelian group to every finite G-set and is additive on disjoint unions,

 $\underline{M}(A \amalg B) = \underline{M}(A) \oplus \underline{M}(B).$

Hence \underline{M} is determined by its values on G/H for subgroups H. It is both covariant and contravariant. Given subgroups

 $K \subset H \subset G$

we get maps

$$\underline{M}(G/H)$$

$$\operatorname{Res}^{H}_{\kappa} \bigwedge \operatorname{Tr}^{H}_{\kappa}$$

$$\underline{M}(G/K),$$

called restrictions and transfers, with certain properties.

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Rather than spell out these properties, we give two instructive examples.

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Rather than spell out these properties, we give two instructive examples.

Example 1. Let **Z***G* denote the integral group ring of *G*,

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Rather than spell out these properties, we give two instructive examples.

Example 1. Let ZG denote the integral group ring of G, and let M be ZG-module.

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Rather than spell out these properties, we give two instructive examples.

Example 1. Let ZG denote the integral group ring of G, and let M be ZG-module. Associated with it is the fixed point Mackey functor \underline{M} defined by

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Rather than spell out these properties, we give two instructive examples.

Example 1. Let ZG denote the integral group ring of G, and let M be ZG-module. Associated with it is the fixed point Mackey functor \underline{M} defined by

 $\underline{M}(G/H) = M^{H}$, the fixed point set of *H* in *M*.

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 $\underline{M}(G/H) = M^{H}$, the fixed point set of *H* in *M*.

Then the restriction map

 $\begin{array}{c}
M^{H} \\
\operatorname{Res}_{\kappa}^{H} \downarrow \\
M^{K}
\end{array}$

is obvious:

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Then the restriction map

is obvious: an element of M that is fixed by H is also fixed by the smaller subgroup K.

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 $\underline{M}(G/H) = M^{H}$, the fixed point set of *H* in *M*.

Then the restriction map

is obvious: an element of M that is fixed by H is also fixed by the smaller subgroup K. In this example the restriction map is one-to-one, but in general it need not be.

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The transfer map

M^H ∱Tr[⊬] M^K

is defined by



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The transfer map

M^H ∱Tr[⊬] M^K

is defined by

$$\operatorname{Res}_{K}^{H}(\operatorname{Tr}_{K}^{H}(x)) = \sum_{\gamma \in H/K} \gamma(x) \in \underline{M}(G/K).$$

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The transfer map

∱⊤ Μ^κ

 M^{H}

is defined by

$$\operatorname{Res}_{\operatorname{K}}^{\operatorname{H}}(\operatorname{Tr}_{\operatorname{K}}^{\operatorname{H}}(x)) = \sum_{\gamma \in \operatorname{H}/\operatorname{K}} \gamma(x) \in \underline{M}(G/\operatorname{K}).$$

Note that the sum on the right is fixed by *H*, so it is in the image of the monomorphism Res_{K}^{H} .

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The transfer map

M^H ↑ Tr *M^κ*

is defined by

$$\operatorname{Res}_{K}^{H}(\operatorname{Tr}_{K}^{H}(x)) = \sum_{\gamma \in H/K} \gamma(x) \in \underline{M}(G/K).$$

Note that the sum on the right is fixed by *H*, so it is in the image of the monomorphism Res_{κ}^{H} . Thus $Tr_{\kappa}^{H}(x) \in \underline{M}(G/H)$ is determined by its image in $\underline{M}(G/K)$.

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Example 2. Let *X* by a *G*-spectrum and $n \in \mathbf{Z}$.

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Example 2. Let X by a G-spectrum and $n \in \mathbb{Z}$. Then its *n*th equivariant homotopy group is the Mackey functor defined by

$$\underline{\pi}_n X(G/H) = \pi_n X^H,$$



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Example 2. Let X by a G-spectrum and $n \in \mathbb{Z}$. Then its *n*th equivariant homotopy group is the Mackey functor defined by

 $\underline{\pi}_n X(G/H) = \pi_n X^H,$

the *n*th ordinary homotopy group of the fixed point spectrum X^{H} .

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 $\underline{\pi}_n X(G/H) = \pi_n X^H,$

the *n*th ordinary homotopy group of the fixed point spectrum X^H . The restriction map for $K \subset H \subset G$ is induced by the evident map

X^H ↓ X^K

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Example 2. Let X by a G-spectrum and $n \in \mathbb{Z}$. Then its *n*th equivariant homotopy group is the Mackey functor defined by

 $\underline{\pi}_n X(G/H) = \pi_n X^H,$

the *n*th ordinary homotopy group of the fixed point spectrum X^H . The restriction map for $K \subset H \subset G$ is induced by the evident map

X^H

γ χκ

The induced homomorphism need not be 1-1.

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 $\underline{\pi}_n X(G/H) = \pi_n X^H,$

the *n*th ordinary homotopy group of the fixed point spectrum X^H . The restriction map for $K \subset H \subset G$ is induced by the evident map

ХН

γ χκ

The induced homomorphism need not be 1-1. In the stable category there is a transfer map

X^H ↑ X^K **Plastic explosives**

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What is a Mackey functor? (continued)

Example 2. Let X by a G-spectrum and $n \in \mathbb{Z}$. Then its *n*th equivariant homotopy group is the Mackey functor defined by

 $\underline{\pi}_n X(G/H) = \pi_n X^H,$

the *n*th ordinary homotopy group of the fixed point spectrum X^H . The restriction map for $K \subset H \subset G$ is induced by the evident map

X^H

YK

The induced homomorphism need not be 1-1. In the stable category there is a transfer map

that induces the desired map of ordinary homotopy groups.

YK

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The category of Mackey functors is abelian, with kernels and cokernels defined in the obvious way.

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The category of Mackey functors is abelian, with kernels and cokernels defined in the obvious way. In the first example above we defined a functor to it from the category of ZG-modules.

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The category of Mackey functors is abelian, with kernels and cokernels defined in the obvious way. In the first example above we defined a functor to it from the category of ZG-modules. This functor is not exact.

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The category of Mackey functors is abelian, with kernels and cokernels defined in the obvious way. In the first example above we defined a functor to it from the category of **Z***G*-modules. This functor is not exact. Given a module map $\phi: M \to N$, the kernel and cokernel of the Mackey functor map

$$\underline{M} \xrightarrow{\underline{\phi}} \underline{N}$$

need not be fixed point Mackey functors.

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$$\underline{M} \xrightarrow{\underline{\phi}} \underline{N}$$

need not be fixed point Mackey functors. This is actually a good thing.

An example is cokernel of the map

$$\underline{ZG} \xrightarrow{1+\gamma} \underline{ZG}$$
 where $G = C_2$ with generator γ .

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need not be fixed point Mackey functors. This is actually a good thing.

An example is cokernel of the map

ZG
$$\xrightarrow{1+\gamma}$$
 ZG where $G = C_2$ with generator γ .

Consider this a homework problem.

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We will denote Mackey functors \underline{M} for $G = C_2$ and $G = C_4$ by diagrams

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$$\frac{\underline{M}(G/G)}{\operatorname{Res}_{1}^{2}} \int \operatorname{Tr}_{1}^{2}$$
$$\underline{M}(G/e)$$

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We will denote Mackey functors \underline{M} for $G = C_2$ and $G = C_4$ by diagrams





and





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We will denote Mackey functors \underline{M} for $G = C_2$ and $G = C_4$ by diagrams



and later abbreviate certain ones by hieroglyphic symbols.



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A G-CW spectrum is built out of "cells" of the form

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A G-CW spectrum is built out of "cells" of the form

$$G_+ \underset{H}{\wedge} D^n$$
 for a subgroup $H \subset G$.

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A G-CW spectrum is built out of "cells" of the form

 $G_+ \underset{H}{\wedge} D^n$ for a subgroup $H \subset G$.

Its boundary is a wedge of |G/H| copies of S^{n-1} which are permuted by *G* and fixed by *H*.

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 $G_+ \underset{H}{\wedge} D^n$ for a subgroup $H \subset G$.

Its boundary is a wedge of |G/H| copies of S^{n-1} which are permuted by *G* and fixed by *H*. Attaching maps are equivariant.

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Its boundary is a wedge of |G/H| copies of S^{n-1} which are permuted by *G* and fixed by *H*. Attaching maps are equivariant.

Such a spectrum X has a cellular chain complex $C_*(X)$ of **Z***G*-modules.

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Such a spectrum *X* has a cellular chain complex $C_*(X)$ of **Z***G*-modules. A cell of the above form gives an additive summand of $C_n(X)$ of the form **Z***G*/*H*.

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 $G_+ \underset{H}{\wedge} D^n$ for a subgroup $H \subset G$.

Its boundary is a wedge of |G/H| copies of S^{n-1} which are permuted by G and fixed by H. Attaching maps are equivariant.

Such a spectrum *X* has a cellular chain complex $C_*(X)$ of **Z***G*-modules. A cell of the above form gives an additive summand of $C_n(X)$ of the form **Z***G*/*H*.

The homology of this chain complex is the underlying homology of X,

$$H^u_*X = \pi^u_*(X \wedge H\mathbf{Z})$$

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A G-CW spectrum is built out of "cells" of the form

 $G_+ \underset{H}{\wedge} D^n$ for a subgroup $H \subset G$.

Its boundary is a wedge of |G/H| copies of S^{n-1} which are permuted by G and fixed by H. Attaching maps are equivariant.

Such a spectrum *X* has a cellular chain complex $C_*(X)$ of **Z***G*-modules. A cell of the above form gives an additive summand of $C_n(X)$ of the form **Z***G*/*H*.

The homology of this chain complex is the underlying homology of X,

$$H^u_*X = \pi^u_*(X \wedge H\mathbf{Z})$$

which is a **Z***G*-module.

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Applying the fixed point functor to C_*X gives us a chain complex \underline{C}_*X of fixed point Mackey functors.

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Applying the fixed point functor to C_*X gives us a chain complex \underline{C}_*X of fixed point Mackey functors. Its homology is a graded Mackey functor.

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Applying the fixed point functor to C_*X gives us a chain complex \underline{C}_*X of fixed point Mackey functors. Its homology is a graded Mackey functor. It may not be a graded fixed point Mackey functor,

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It turns out that $H_*(\underline{C}_*X)(G/K)$ is the latter group.

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Slices again

Recall that for the G-spectra we are interested in,

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Slices again

Recall that for the *G*-spectra we are interested in, each slice has the form $W \land H\mathbf{Z}$, where *W* is a wedge of slice spheres.

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 $H_*(\underline{C}_*W) = \underline{\pi}_*W \wedge H\underline{Z},$

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For example suppose $G = C_2$ and $W = S^{m_{\rho}}$.

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For example suppose $G = C_2$ and $W = S^{m_{\rho}}$. Then we find that

 $C_i W = \left\{$

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For example suppose $G = C_2$ and $W = S^{m_{\rho}}$. Then we find that

$$C_i W = \begin{cases} \mathbf{Z} = \mathbf{Z} G / (1 - \gamma) & \text{for } i = m \end{cases}$$

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For example suppose $G = C_2$ and $W = S^{m\rho}$. Then we find that

$$C_i W = \begin{cases} \mathbf{Z} = \mathbf{Z}G/(1-\gamma) & \text{for } i = m \\ \mathbf{Z}G & \text{for } |m| < |i| \le |2m| \text{ and } mi > 0 \end{cases}$$

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There is a unique $\mathbf{Z}G$ -linear boundary operator giving the required homology,

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$$H_*(C_*W) = H_*S^{2m}$$

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For example suppose $G = C_2$ and $W = S^{m_{\rho}}$. Then we find that

$$C_i W = \begin{cases} \mathbf{Z} = \mathbf{Z}G/(1-\gamma) & \text{for } i = m \\ \mathbf{Z}G & \text{for } |m| < |i| \le |2m| \text{ and } mi > 0 \\ 0 & \text{otherwise.} \end{cases}$$

There is a unique $\mathbf{Z}G$ -linear boundary operator giving the required homology,

$$H_*(C_*W)=H_*S^{2m},$$

and it is easy to work out the graded Mackey functor $H_*(\underline{C}_*W)$.

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Recall that

$$\pi_* MU = \mathbf{Z}[x_1, x_2, \dots]$$
 where $x_i \in \pi_{2i}$

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Recall that

$$\pi_*MU = \mathbf{Z}[x_1, x_2, \dots]$$
 where $x_i \in \pi_{2i}$.

Let $G = C_2$. In the *G*-spectrum $MU_{\mathbf{R}}$, the maps $x_i : S^{2i} \to MU$ for i > 0

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Recall that

$$\pi_*MU = \mathbf{Z}[x_1, x_2, ...]$$
 where $x_i \in \pi_{2i}$.

Let $G = C_2$. In the *G*-spectrum $MU_{\mathbf{R}}$, the maps $x_i : S^{2i} \to MU$ for i > 0 get replaced by maps $\overline{x}_i : S^{i\rho} \to MU_{\mathbf{R}}$.

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Recall that

 $\pi_*MU = \mathbf{Z}[x_1, x_2, ...]$ where $x_i \in \pi_{2i}$.

Let $G = C_2$. In the *G*-spectrum $MU_{\mathbf{R}}$, the maps $x_i : S^{2i} \to MU$ for i > 0 get replaced by maps $\overline{x}_i : S^{i\rho} \to MU_{\mathbf{R}}$. They represent elements in the RO(G)-graded homotopy of $MU_{\mathbf{R}}$.

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It is known that if we invert \overline{x}_1 and kill suitably chosen generators \overline{x}_i for i > 1,

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It is known that if we invert \overline{x}_1 and kill suitably chosen generators \overline{x}_i for i > 1, we get Atiyah's spectrum $K_{\mathbf{R}}$, which is underlain by the classical complex K-theory spectrum K.

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 $\pi_* MU = \mathbf{Z}[x_1, x_2, \dots]$ where $x_i \in \pi_{2i}$.

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It is known that if we invert \overline{x}_1 and kill suitably chosen generators \overline{x}_i for i > 1, we get Atiyah's spectrum $K_{\mathbf{R}}$, which is underlain by the classical complex K-theory spectrum K. $K_{\mathbf{R}}$ is known to be 8-periodic and to have KO as its fixed point set.

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 $\pi_* MU = \mathbf{Z}[x_1, x_2, \dots]$ where $x_i \in \pi_{2i}$.

Let $G = C_2$. In the *G*-spectrum $MU_{\mathbf{R}}$, the maps $x_i : S^{2i} \to MU$ for i > 0 get replaced by maps $\overline{x}_i : S^{i\rho} \to MU_{\mathbf{R}}$. They represent elements in the RO(G)-graded homotopy of $MU_{\mathbf{R}}$.

It is known that if we invert \overline{x}_1 and kill suitably chosen generators \overline{x}_i for i > 1, we get Atiyah's spectrum $K_{\mathbf{R}}$, which is underlain by the classical complex K-theory spectrum K. $K_{\mathbf{R}}$ is known to be 8-periodic and to have KO as its fixed point set.

We also know that its 2mth slice is $S^{m\rho} \wedge H\underline{Z}$ for each integer m.

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Recall that

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We also know that its 2*m*th slice is $S^{m\rho} \wedge H\mathbf{Z}$ for each integer *m*. The oddly indexed slices are contractible.

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We also know that its 2mth slice is $S^{m\rho} \wedge H\mathbb{Z}$ for each integer m. The oddly indexed slices are contractible. This enables us to compute the \underline{E}_2 -term of the slice spectral sequence converging to $\underline{\pi}_* K_{\mathbf{R}}$.

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We start with the C_4 -spectrum $N_2^4 M U_{\mathbf{R}}$.

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We start with the C_4 -spectrum $N_2^4 M U_R$. Its underlying homotopy is

 $\pi_*^u N_2^4 M U_{\mathbf{R}} = \pi_* M U \land M U = \mathbf{Z}[x_i, y_i : i > 0]$ where $x_i, y_i \in \pi_{2i}$

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 $\pi_*^{u} N_2^{d} M U_{\mathbf{R}} = \pi_* M U \land M U = \mathbf{Z}[x_i, y_i : i > 0]$ where $x_i, y_i \in \pi_{2i}$

The action of a generator $\gamma \in \mathbf{G} = \mathbf{C}_{4}$ is

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 $\pi_*^{u} N_2^{4} M U_{\mathbf{R}} = \pi_* M U \land M U = \mathbf{Z}[x_i, y_i : i > 0] \text{ where } x_i, y_i \in \pi_{2i}$

The action of a generator $\gamma \in G = C_4$ is

$$\gamma(x_i) = y_i$$
 and $\gamma(y_i) = (-1)^i x_i$.

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We start with the C_4 -spectrum $N_2^4 M U_{\rm R}$. Its underlying homotopy is

 $\pi_*^U N_2^0 M U_{\mathbf{B}} = \pi_* M U \wedge M U = \mathbf{Z}[x_i, y_i : i > 0]$ where $x_i, y_i \in \pi_{2i}$

The action of a generator $\gamma \in G = C_4$ is

$$\gamma(x_i) = y_i$$
 and $\gamma(y_i) = (-1)^i x_i$.

Then we invert

$$D = (x_1y_1)^2 \left(-5(x_1^4 + y_1^4) + 19(x_1y_1)^2 + 20x_1y_1(x_1^2 - y_1^2) \right)$$

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and kill suitably chosen x_i and y_i for i > 1.

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and kill suitably chosen x_i and y_i for i > 1. We denote the resulting C_4 -spectrum by $K_{\rm H}$.

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and kill suitably chosen x_i and y_i for i > 1. We denote the resulting C_4 -spectrum by $K_{\rm H}$. It is 32-periodic.

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 $\pi_*^u N_2^4 M U_{\mathbf{R}} = \pi_* M U \land M U = \mathbf{Z}[x_i, y_i : i > 0]$ where $x_i, y_i \in \pi_{2i}$

The action of a generator $\gamma \in G = C_4$ is

$$\gamma(x_i) = y_i$$
 and $\gamma(y_i) = (-1)^i x_i$.

Then we invert

$$D = (x_1y_1)^2 \left(-5(x_1^4 + y_1^4) + 19(x_1y_1)^2 + 20x_1y_1(x_1^2 - y_1^2) \right)$$

and kill suitably chosen x_i and y_i for i > 1. We denote the resulting C_4 -spectrum by $K_{\rm H}$. It is 32-periodic. It has a connective version $k_{\rm H}$ that we get without inverting D.

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The C_4 -spectrum K_H is known to be equivalent to $TMF_1(5)$,

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The C_4 -spectrum K_H is known to be equivalent to $TMF_1(5)$, which has been studied by Behrens and Ormsby.





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It is defined as follows.

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The C_4 -spectrum K_H is known to be equivalent to $TMF_1(5)$, which has been studied by Behrens and Ormsby.



It is defined as follows. The spectrum *TMF* is derived from the moduli stack of elliptic curves \mathcal{M} .

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The C_4 -spectrum K_H is known to be equivalent to $TMF_1(5)$, which has been studied by Behrens and Ormsby.



It is defined as follows. The spectrum *TMF* is derived from the moduli stack of elliptic curves \mathcal{M} . Roughly speaking a point on \mathcal{M} corresponds to an elliptic curve *C*.

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It is defined as follows. The spectrum *TMF* is derived from the moduli stack of elliptic curves \mathcal{M} . Roughly speaking a point on \mathcal{M} corresponds to an elliptic curve *C*.

One can consider the moduli stack of $\mathcal{M}_1(5)$ for which each point is an elliptic curve *C* equipped with a point *P* of order 5.

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It is defined as follows. The spectrum *TMF* is derived from the moduli stack of elliptic curves \mathcal{M} . Roughly speaking a point on \mathcal{M} corresponds to an elliptic curve *C*.

One can consider the moduli stack of $\mathcal{M}_1(5)$ for which each point is an elliptic curve *C* equipped with a point *P* of order 5. The group $C_4 = (\mathbf{Z}/5)^{\times}$ acts on it by sending each *P* to appropriate powers.

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It is defined as follows. The spectrum *TMF* is derived from the moduli stack of elliptic curves \mathcal{M} . Roughly speaking a point on \mathcal{M} corresponds to an elliptic curve *C*.

One can consider the moduli stack of $\mathcal{M}_1(5)$ for which each point is an elliptic curve *C* equipped with a point *P* of order 5. The group $C_4 = (\mathbf{Z}/5)^{\times}$ acts on it by sending each *P* to appropriate powers. The orbit stack $\mathcal{M}_0(5)$ classifies elliptic curves equipped with subgroups of order 5.

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There are corresponding spectra $TMF_1(5)$ and $TMF_0(5)$.

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There are corresponding spectra $TMF_1(5)$ and $TMF_0(5)$. The forgetful maps of stacks

$$\mathcal{M}_1(5) \longrightarrow \mathcal{M}_0(5) \longrightarrow \mathcal{M}_1(5) \longrightarrow \mathcal{M}_1(5) / C_4$$

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$$\mathcal{M}_{1}(5) \longrightarrow \mathcal{M}_{0}(5) \longrightarrow \mathcal{M}_{0}(5) \longrightarrow \mathcal{M}_{1}(5)/C_{4}$$

lead to maps of spectra

$$TMF_{1}(5) \longleftarrow TMF_{0}(5) \longleftarrow TMF$$

$$\prod_{II} TMF_{1}(5)^{C_{4}}$$

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There are corresponding spectra $TMF_1(5)$ and $TMF_0(5)$. The forgetful maps of stacks

$$\mathcal{M}_{1}(5) \longrightarrow \mathcal{M}_{0}(5) \longrightarrow \mathcal{M}_{1}(5) / C_{4}$$

lead to maps of spectra

$$TMF_1(5) \leftarrow TMF_0(5) \leftarrow TMF$$

 $\parallel TMF_1(5)^{C_4}$

The C_4 -spectrum $TMF_1(5)$ is equivariantly equivalent to our $K_{\rm H}$.

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lead to maps of spectra

$$TMF_1(5) \leftarrow TMF_0(5) \leftarrow TMF$$

 $\parallel TMF_1(5)^{C_4}$

The C_4 -spectrum $TMF_1(5)$ is equivariantly equivalent to our $K_{\rm H}$. This makes the fixed point spectrum $K_{\rm H}^{C_4}$ a TMF-module.

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There are corresponding spectra $TMF_1(5)$ and $TMF_0(5)$. The forgetful maps of stacks

$$\mathcal{M}_{1}(5) \longrightarrow \mathcal{M}_{0}(5) \longrightarrow \mathcal{M}_{1}(5) / C_{4}$$

lead to maps of spectra

$$TMF_1(5) \leftarrow TMF_0(5) \leftarrow TMF$$

 $\parallel TMF_1(5)^{C_4}$

The C_4 -spectrum $TMF_1(5)$ is equivariantly equivalent to our $K_{\rm H}$. This makes the fixed point spectrum $K_{\rm H}^{C_4}$ a TMF-module. This is helpful for understanding its homotopy.

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Let $G = C_4$ and $G \supset G' = C_2$.

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Let $G = C_4$ and $G \supset G' = C_2$. Denote their regular representations by ρ_4 and ρ_2 .

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Let $G = C_4$ and $G \supset G' = C_2$. Denote their regular representations by ρ_4 and ρ_2 .

To describe the slices of $k_{\rm H}$ and $K_{\rm H}$, let

$$W_{m,n} = \begin{cases} S^{m_{\rho_4}} & \text{for } m = n \\ G_+ \underset{G'}{\wedge} S^{(m+n)\rho_2} & \text{for } m < n. \end{cases}$$

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$$W_{m,n} = \left\{ egin{array}{cc} S^{m_{
ho_4}} & ext{for } m = n \ G_+ & \bigwedge_{G'} S^{(m+n)
ho_2} & ext{for } m < n. \end{array}
ight.$$

Then we have

$$P_n^n k_{\mathbf{H}} = \begin{cases} \left(\bigvee_{0 \le m \le n/4} W_{m,n/2-m} \right) \land H \underline{\mathbf{Z}} & \text{for } n \text{ even and } n \ge 0 \\ * & \text{otherwise} \end{cases}$$

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Dugger's slice spectral sequence

Let $G = C_4$ and $G \supset G' = C_2$. Denote their regular representations by ρ_4 and ρ_2 .

To describe the slices of $k_{\rm H}$ and $K_{\rm H}$, let

$$W_{m,n} = \begin{cases} S^{m_{\rho_4}} & \text{for } m = n \\ G_+ \underset{G'}{\wedge} S^{(m+n)_{\rho_2}} & \text{for } m < n. \end{cases}$$

Then we have

$$P_n^n k_{\mathbf{H}} = \begin{cases} \left(\bigvee_{0 \le m \le n/4} W_{m,n/2-m} \right) \land H \underline{\mathbf{Z}} & \text{for } n \text{ even and } n \ge 0 \\ * & \text{otherwise} \end{cases}$$

and

$$P_n^n \mathcal{K}_{\mathbf{H}} = \begin{cases} \left(\bigvee_{m \le n/4} W_{m,n/2-m} \right) \land H \underline{\mathbf{Z}} & \text{for } n \text{ even} \\ * & \text{otherwise.} \end{cases}$$

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$$P_n^n \mathcal{K}_{\mathsf{H}} = \begin{cases} \left(\bigvee_{m \le n/4} W_{m,n/2-m} \right) \land H \underline{\mathsf{Z}} & \text{for } n \text{ even} \\ * & \text{otherwise.} \end{cases}$$

The latter slices have infinitely many summands.

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The C₄ case (continued)

These slices are uncomfortably large.

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The C₄ case (continued)

These slices are uncomfortably large.

Fortunately there is a remedy.

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The C₄ case (continued)

These slices are uncomfortably large.

Fortunately there is a remedy. Consider $k_{\rm H}$ as a C_2 -spectrum via the forgetful functor.

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The C₄ case (continued)

These slices are uncomfortably large.

Fortunately there is a remedy. Consider $k_{\rm H}$ as a C_2 -spectrum via the forgetful functor. Here is its slice spectral sequence.



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The C₄ case The spectra K_H and k_H The reduced E4-term

These slices are uncomfortably large.

Fortunately there is a remedy. Consider k_H as а C_2 -spectrum via the forgetful functor. Here is its slice spectral sequence.

The C_4 case (continued)



The differentials and exotic transfers above have maximal rank.

1.33

The C₄ case (continued)

These slices are uncomfortably large.

Fortunately there is a remedy. Consider $k_{\rm H}$ as a C_2 -spectrum via the forgetful functor. Here is its slice spectral sequence.



The differentials and exotic transfers above have maximal rank. This pattern is easy to understand.

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The C₄ case (continued)

These slices are uncomfortably large.

Fortunately there is a remedy. Consider $k_{\rm H}$ as a C_2 -spectrum via the forgetful functor. Here is its slice spectral sequence.



The differentials and exotic transfers above have maximal rank. This pattern is easy to understand. There is a way to remove most of these elements and their transfers from the picture.

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The reduced \underline{E}_4 -term

Here is the resulting reduced \underline{E}_4 -term for $k_{\rm H}$.

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The reduced \underline{E}_4 -term (continued) And here it is for $K_{\rm H}$.

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The Hopkins poster again



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The reduced \underline{E}_4 -term (continued)



This is the resulting $\underline{\underline{E}}_{14} = \underline{\underline{E}}_{\infty}$ -term.

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The reduced \underline{E}_4 -term (continued)



This is the resulting $\underline{\underline{E}}_{14} = \underline{\underline{E}}_{\infty}$ -term.



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The reduced E_4 -term (continued)



This is the resulting $\underline{\underline{E}}_{14} = \underline{\underline{E}}_{\infty}$ -term.



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The reduced E_4 -term (continued)



This is the resulting $\underline{\underline{E}}_{14} = \underline{\underline{E}}_{\infty}$ -term.



The exotic Mackey functor extensions in the first and third quadrants

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The reduced E_4 -term (continued)



This is the resulting $\underline{\underline{E}}_{14} = \underline{\underline{E}}_{\infty}$ -term.



The exotic Mackey functor extensions in the first and third quadrants lead to the Mackey functors shown in violet in the second and fourth quadrants.

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