

# Special Algebraic Topology Seminar What is the telescope conjecture?



Doug Ravenel University of Rochester

October 28, 2023

What is the telescope conjecture?



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Smith-Toda complexe

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# Two developments in the early 70s

1. Morava K-theory

What is the telescope conjecture?



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It is related to height n formal group laws, and  $K(n)_*(K(n))$  is related to the Morava stabilizer group  $\mathbb{G}_n$ . It is a p-adic Lie group and the automorphism group of a height n formal group law over a suitable field of characteristic p.

# Two developments in the early 70s (continued)

2. Smith-Toda complexes

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for  $1 \le n \le 3$  and  $p \ge 2n + 1$ .



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for  $1 \le n \le 3$  and  $p \ge 2n + 1$ . We know that  $K(n)_* V(n - 1) \ne 0$  and  $w_n$  is a K(n)-equivalence. These lead to the construction of the  $v_n$ -periodic families aka Greek letter elements

$$\begin{aligned} &\alpha_t \in \pi_{t|v_1|-1}\mathbb{S} & \text{for } p \geq 3 \\ &\beta_t \in \pi_{t|v_2|-2p}\mathbb{S} & \text{for } p \geq 5 \\ &\gamma_t \in \pi_{t|v_3|-2p^2-2p+1}\mathbb{S} & \text{for } p \geq 7 \end{aligned}$$



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  - $\gamma_t \in \pi_{t|v_3|-2p^2-2p+1}\mathbb{S}$

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# The Adams-Novikov spectral sequence

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These are nicely displayed in the  $E_2$ -term the Adams-Novikov spectral sequence.

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Let Sp denote the category of spectra. Given a homology theory  $E_*$ , Bousfield constructed an endofunctor  $L_E: Sp \to Sp$  whose image category  $L_ESp$  is stable homotopy as seen through the eyes of E-theory.

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We are interested in the case E = K(n).

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We are interested in the case E = K(n).  $L_{K(n)}$ Sp is much easier to deal with than Sp itself. For example, we can compute  $\pi_* L_{K(2)} V(1)$ , but have no hope of computing  $\pi_* V(1)$ .



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Recall the cofiber sequence

$$\Sigma^{|v_n|}V(n-1) \xrightarrow{w_n} V(n-1) \longrightarrow V(n)$$

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$$V(n-1) \xrightarrow{w_n} \Sigma^{-|v_n|} V(n-1) \xrightarrow{w_n} \Sigma^{-2|v_n|} V(n-1) \xrightarrow{w_n} \dots$$

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We call this the  $v_n$ -periodic telescope  $w_n^{-1}V(n-1)$ , often denoted by T(n). The telescope conjecture says it is  $L_{K(n)}V(n-1)$ . The former is more closely related to the homotopy groups of spheres, while the latter is more computationally accessible.

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Can we generalize this to n > 3? Not exactly.

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#### Periodicity Theorem

Let X be a p-local type n, finite spectrum,

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 $w: \Sigma^d X \to X$ where  $K(n)_*w$  is an isomorphism.

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Again the telescope conjecture equates the geometrically appealing telescope  $w^{-1}X$  with the computationally accessible Bousfield localization  $L_{K(n)}X$ .

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to be true for n = 0 and n = 1.

When I stated the telescope conjecture in 1984, it was known

Morava K-theory Smith-Toda complex

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San Francisco earthquake of October 17, 1989

### **THANK YOU!**

What is the telescope conjecture?



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