CHAPTER 6

Bousfield localization

Bousfield localization, first introduced by Bousfield in [**Bou75**], is one of the most useful constructions in model category theory. Briefly, one starts with a model category $\mathcal{M}$ and enlarges the class of weak equivalences to form a new model category $\mathcal{M}'$ with the same underlying category as $\mathcal{M}$. The cofibrations and trivial fibrations of $\mathcal{M}'$ are the same as those of $\mathcal{M}$. Recall that trivial fibrations are by definition maps having the right lifting property with respect to cofibrations, and fibrations are by definition maps having the right lifting property with respect to trivial cofibrations. Since there are more weak equivalences in $\mathcal{M}'$ than in $\mathcal{M}$, there are **more trivial cofibrations** and hence **fewer fibrations**. There are also fewer fibrant objects, and fibrant replacement tends to be more interesting (or drastic) in $\mathcal{M}'$ than in $\mathcal{M}$. As indicated in Remark 4.1.7, the hard part of showing that the new model structure exists is verifying that it satisfies the factorization axiom **MC5**.

This is our third way of modifying a model structure without changing the underlying category. Our first is the enlargement procedure of Theorem 5.1.32, which left the class $\mathcal{W}$ of weak equivalences unchanged while enlarging the class $\mathcal{C}$ of cofibrations. Our second applies to a functor category $\mathcal{M}^J$. We can replace its projective model structure by the one induced from the similar one on $\mathcal{M}^K$ for a subcategory K of J as in Theorem 5.2.18. It has both more weak equivalences and fibrations than the projective model structure, as explained in Remark 5.2.20.

In each case there is a Quillen adjunction in which both functors are the identity, The left adjoint has the original model category as its domain in the first two cases and as its codomain in the third case.

The following table indicates how the classes of weak equivalences $\mathcal{W}$, cofibrations $\mathcal{C}$ and fibrations $\mathcal{F}$ in the new model structure on $\mathcal{M}$ or $\mathcal{M}^J$ compare with those in the original one.

Construction	$\mathcal{W}$	$\mathcal{C}$	$\mathcal{F}$	Identity functor from new to original model category
Enlargement as in Theorem 5.1.32	Same	More	Less	Right adjoint
Induction as in Theorem 5.2.18	More	Less	More	Left adjoint
Bousfield localization	More	Same	Less	Right adjoint