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ESHT lecture 2

Read JFA - Quote

Recall def of orthogonal spectra

Define the category $\mathcal{T}^G$ and its Bredon model structure.

Generating sets

$$I = \{ G_+ \wedge_H (S_+^{n-1} \to D_+^n) : n \geq 0,\ H \leq G \}$$

$$J = \{ G_+ \wedge_H (I_+^n \to I_+^{n+1}) : \quad " \quad \}$$

Change of group adjunction

$$\mathcal{T}^H \underset{i_H^G}{\overset{G_+ \wedge_H (-)}{\rightleftarrows}} \mathcal{T}^G$$

It is a Quillen adjunction, i.e.
left adjoint preserves (trivial) cofibs
right " fibs

Define the MM category $\mathcal{J}_G$. It is enriched over $\mathcal{T}^G$ and is an SMC.

e.g. $\mathcal{J}_G(~~U,~~ V, W) \wedge \mathcal{J}_G(U, V) \xrightarrow{\circ_{U,V,W}} \mathcal{J}_G(U, W)$

is a G-map

$Sp = [\mathcal{J}, \mathcal{T}]$ (2)

Def $Sp^G = [\mathcal{J}_G, \mathcal{T}^G]$ G-spectra with G-maps

and $Sp_G = [\mathcal{J}_G, \mathcal{T}_G]$ G-spectra with cont maps

Morphism objects (in $\mathcal{T}^G$) are ends

$$Sp^G(X,Y) = \int_{\mathcal{J}_G} \mathcal{T}^G(X_V, Y_V) \quad \text{in } \mathcal{T}$$

$$Sp_G(X,Y) = \int_{\mathcal{J}_G} \mathcal{T}_G(X_V, Y_V) \quad \text{in } \mathcal{T}^G$$

Define Quillen ring, tensors and cotensors.

~~Tauto~~ Yoneda spectra S^{-V}

Tautological presentation

$$X \cong \int^{\mathcal{J}_G} S^{-V} \wedge X_V$$

Smash product

$$X \wedge Y = \int^{\mathcal{J}_G \times \mathcal{J}_G} S^{-V' \oplus W''} \wedge X_{V'} \wedge Y_{V''}$$

$$(X \wedge Y)_W = \int^{\mathcal{J}_G \times \mathcal{J}_G} \mathcal{J}_G(V' \oplus V'', W) \wedge X_{V'} \wedge Y_{V''}$$

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Smashing ~~with~~ a a spectrum X with a space K (tensor) is the same as smashing (Day conv) with the suspension spectrum $S^{-0} \wedge K$, which is defined by $(S^{-0} \wedge K)_V = S^V \wedge K$ for all V.

Closed SMC structure in Sp_G

Sp_G has an internal hom $F_G(-,-)$ with

$$Sp_G(X \wedge Y, Z) \cong Sp_G(X, F_G(Y, Z))$$

The morphism spectrum $F_G(Y, Z)$ is given by

$$F_G(Y, Z)_V = Sp_G(S^{-V} \wedge \cancel{\#} Y, Z)$$

In fact

$$F_G(S^{-0}, X) = X$$

$$F_G(S^{-V} \wedge K, \cancel{\#} X)_W = T_G(K, X_{V \oplus W})$$

SIDEBAR (OPTIONAL) 4 What are ends + coends?

Given a functor

$$J^{op} \times J \xrightarrow{H} C$$

where J is small, for each morphism $X \xrightarrow{f} Y$ in J we have a diagram in C

$$\begin{array}{ccc} H(X,X) & \xleftarrow{f^*} & H(Y,X) \\ \downarrow f_* & & \downarrow f_* \\ H(X,Y) & \xleftarrow{f^*} & H(Y,Y) \end{array}$$

assume C is cocomplete and consider

① $$\coprod_{f \in \mathrm{Arr}\, J} H(Y,X) \underset{\varphi_*}{\overset{\varphi^*}{\rightrightarrows}} \coprod_{z \in \mathrm{Ob}\, J} H(z,z)$$

The coend $\int^{z \in J} H(z,z)$ is the coequalizer of ①

For C complete we have

$$\int_{z \in J} H(z,z) \dashrightarrow \prod H(z,z) \underset{\phi_*}{\overset{\phi^*}{\rightrightarrows}} \prod H(X,Y)$$

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We will define 8 different model structures on Sp^G, starting with the projective one, to be defined below. It can be modified in any combination of 3 different ways:

positivized, stabilized and enlarged.

We need all 3 to prove the KI theorem!

Define projective structure on M^J (page 6 of 10/15 notes)

Dis: discuss its CG structure

Outline the 3 modifications