

MATH 165

Midterm

Tuesday, March 20, 2007

Show all work (each step/computation) to receive full credit. No calculators. The exam contains 8 problems. Make sure it is complete.

No.	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	15	
6	15	
7	15	
8	15	
TOTAL	100	

NAME : _____

SECTION : _____

1. Solve the equation $y' + 2y = 1$, with $y(0) = 0$.

2. Solve the equation $4ty' = y^2 + ty^2$, with $y(1) = 1$.

3. Solve the initial value problem $y'' + y = 0$, where $y(\pi/2) = 2$ and $y'(\pi/2) = -4$.

4. A tank initially contains 120 liters of pure water. A mixture containing a concentration of γ g/liter of salt enters the tank at a rate of 2 liters/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t . Also find the limiting amount of salt in the tank as $t \rightarrow \infty$.

5. Solve the following system of equations:

$$\begin{cases} x + z = 2 \\ 2x - 3y + 5z = 4 \\ 3x + 2y - z = 4 \end{cases}$$

6. Find out whether the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}$ are linearly dependent or independent. Justify your answer.

7. Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^3$ be the transformation defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ x + y \\ x - y \end{bmatrix}$$

Prove that T is a linear transformation and compute its associated matrix. Is T one-to-one? Is T onto? Justify your answer.

8. Prove that the matrix

$$\begin{bmatrix} 1 & 4 & -3 \\ -2 & -7 & 6 \\ 1 & 7 & -2 \end{bmatrix}$$

is invertible and find its inverse.