NAME (print legibly): _____

Select your instructor: ◯ Prof. Harper (MWF 10 o'clock)
◯ Prof. Robles (MWF 9 o'clock)

No.	VALUE	SCORE
1	9	
2	10	
3	10	
4	12	
5	8	
6	6	
7	9	
8	12	
9	12	
10	12	
TOTAL	100	

Instructions

Read carefully.
You are responsible for this information.

Place your final answers (in a neat and simplified form) in the boxes provided.

You must show all your work (ie. each step in the solution/computation) to receive full credit.

You may use one 3-by-5 notecard 'cheat-sheet.'

No calculators.

The exam is 10 pages, and contains 10 problems. Make sure your exam is complete.

1a. Are the pairs of vectors below parallel, orthogonal or neither?

(i) $\langle -5, 15, 10\rangle$ and $\langle 3, -9, -6\rangle$

(ii) $\langle 1, 0, 7\rangle$ and $\langle 3, 6, 2\rangle$

(iii) $\langle \sin\theta, \cos\theta, e^{-\theta}\rangle$ and $\langle \sin\theta, \cos\theta, -e^{\theta}\rangle$

(i)	(ii)	(iii)

1b. Find a unit vector perpendicular to $\langle 0, 1, -1\rangle$ and $\langle 3, 2, 1\rangle$.

2. The planes $x - z = 4$ and $-2y + z = -3$ intersect in a line. Write down the symmetric equations of this line.

Answer

3. Compute the linearization of $f(x, y) = \sqrt{x^2 + y^2}$ at $(3, 4)$, and use it to approximate the number $\sqrt{(2.8)^2 + (4.1)^2}$.

Linearization
Approximation

4. Consider the hyperboloid of one-sheet $x^2 + 4y^2 - 9z^2 = 16$.

4(a). Find the tangent plane at $(5, 0, -1)$.

Answer

4(b). At which points of the hyperboloid is the tangent plane perpendicular to the line $L(t) = \langle -10, 10t, 1 + 9t \rangle$?

Answer

5. The table to the right lists some of the values for a function $f(x, y)$, and its derivatives, at the points $(2, 0)$ and $(0, \frac{\pi}{2})$.

	f	f_x	f_y
$(2, 0)$	10	5	-1
$(0, \frac{\pi}{2})$	-2	3	1

Use $f(x, y)$ to define a new function

$$g(\theta, \varphi) = f(\cos\theta + \sin\varphi, \tan(\theta\varphi)) \, .$$

Compute $\dfrac{\partial g}{\partial \theta}$ and $\dfrac{\partial g}{\partial \varphi}$ at $(0, \frac{\pi}{2})$.

$g_\theta(0, \frac{\pi}{2}) =$	$g_\varphi(0, \frac{\pi}{2}) =$

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6. Use the table above to find the maximum rate of change in $f(x, y)$ at $(2, 0)$ and the direction in which it occurs.

Max. rate =	*Direction* =

7. Suppose $f_x(-1,1) = 0 = f_y(-1,1)$. In each case, what can you say about $z = f(x,y)$?

(a) $f_{xx}(-1,1) = -100$, $f_{xy}(-1,1) = 5$, $f_{yy}(-1,1) = 0$

(b) $f_{xx}(-1,1) = 4$, $f_{xy}(-1,1) = -6$, $f_{yy}(0,2) = 9$

(c) $f_{xx}(-1,1) = -1$, $f_{xy}(-1,1) = 2$, $f_{yy}(-1,1) = -8$

(a)	(b)	(c)

8. Find the area of the largest rectangle that can be inscribed in the ellipse

$$a^2x^2 + b^2y^2 = 1\,.$$

Answer

9a. Given only the information that

$$f_{yx} = x^2 + \cos y\,,$$

determine f_{xy}.

Answer

9b. Compute the partial derivatives of $z = \tan(xy) + e^{xy^2} + \dfrac{\ln y}{x}$

$\dfrac{\partial z}{\partial x} =$
$\dfrac{\partial z}{\partial x} =$

10. Compute the volume enclosed by the paraboloid $z = x^2 + 3y^2$ and the planes $x = 0$, $y = 1$, $y = x$ and $z = 0$.

Volume =