# 2nd University of Rochester Mathematical Olympiad

Saturday, March 29, 2008

**Show all work (each step/computation) to receive full credit. You may use back pages if necessary. No calculators. The test contains 4 problems. Make sure it is complete.**

| No. | VALUE | SCORE |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| TOTAL | 40 | |

**REGISTRATION NO.** : ________________________________________

**1.** Let $A$ be a point on the curve $y = x^3$, different from the origin. The tangent line to the curve at point $A$ intersects the curve at another point $B$ ($\neq A$). Prove that the ratio of the slopes for the tangent lines to the curve at points $A$ and $B$ is constant (i.e. doesn't depend on point $A$).

**2.** Let $a, b, c$ be three integers with $a > 0$, such that the equation $ax^2 + bx + c = 0$ has two distinct roots in the open interval $(0, 2)$. Prove that $a \geq 2$, $b \leq -3$, and $c \geq 1$.

**3.** Prove that the equation

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{2008}$$

has finitely many solutions in the set of positive integers.

**4.** Let $ABCD$ be a convex quadrilateral satisfying $AB = BC = CD = 1$. Find the maximum area of such a quadrilateral and describe the shape for which this area is attained.