

MTH 165 Spring 2007
Workshop 3
Week starting February 5th
Qualitative behavior of differential equations

This workshop is a bit different than the others in that it concentrates on differential equations but not on differential equations with linear properties. There is little computation in these exercises, but that doesn't mean they are easy. If you can get these pictures solidly in your head you will have a framework for understanding the technical calculations we'll learn for differential equations. Discussion is the best way to understand. Please participate.

A deeper exploration of non-linear equations (dynamical systems), their qualitative behavior and their solutions is the subject of Math 263 and many upper level and graduate physics and engineering courses.

Goals:

- (1) Match equations, direction fields, solutions and properties.
 - (2) Learn to use xFunctions and/or Dfield
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Problem 1. Let $x(t)$ be the number of rabbits at time t and $y(t)$ be the number of wolves. Suppose that the net growth rate of rabbits (i.e. the combined birth and death rate) of rabbits, left to themselves with plenty of grass is 0.5. The net growth rate of wolves, with no rabbits to eat, is -0.5. The probability of a wolf eating a rabbit is directly proportional (proportionality constant a) to the number of wolves times the number of rabbits and each of these events decreases the number of rabbits and increases the number of wolves. Write two equations modeling this phenomenon and plot it using xFunctions. I suggest using a value of $a = 0.2$ for the first try. **xFunctions**

This is known as the predator – prey model – you can google it. This plot is called a phase plane plot and is slightly different from a direction field plots below. What is the difference? (Hint: What happened to the t axis?)

Problem 2. Match the transparencies containing the direction fields and the graphs of solution curves with each other and with the equations and descriptions below. You can record your answers using the worksheet on the next page. Use **xFunctions** to plot any of the direction fields which aren't obvious to you.

- (1) $y' = y$
 (2) $y' = y + \cos(t)$
 (3) $y' = \sqrt{y}$
 (4) $y' = y^2$
 (5) $y' = 0.1 \frac{t}{y}$
 (6) $y' = \frac{\cos t}{1 + t^2}$
 (7) $y' = \sqrt{y} + \log(-y)$
 (8) $y' = y^2 - y$
 (9) $y' = y^{2/3}$
 (10) $y' = \frac{y}{t}$

Qualitative behavior:

- (I) The solutions are time invariant – that is if $y(t)$ is a solution so is $y(t - a)$ for any a .
- (II) Shifting a solution curve to the right any amount creates a new solution curve.
- (III) Shifting a solution curve by a multiple of 2π creates a new solution (solutions are periodic).
- (IV) Shifting a solution curve up or down creates a new solution curve.
- (V) Solution can be solved solely using integration techniques.
- (VI) There are one or more constant solutions to the ODE. Write a note about what happens to solutions close to these constant solutions?
- (VII) OMIT! If $y(t)$ is a solution then $z(t) = \frac{1}{c}y(ct)$ is a new solution.

Find portions of the direction field where $\frac{\partial f}{\partial y} > 0$, or $\frac{\partial f}{\partial y} < 0$ or $\frac{\partial f}{\partial y}$ is infinity or undefined.

Which points in the $t - y$ plane have unique solutions passing through them? Where are there no solutions? which points have more than one solution?

- (i) All points in the $t - y$ plane.
- (ii) Points where $y > 0$ and t is arbitrary.
- (iii) Points where $y \neq 0$ and t is arbitrary.
- (iv) Points where $t \neq 0$ and y is arbitrary.

In these examples above are there any examples where the solution curves cross “transversally” i.e. at a positive angle? Are there any examples where the solution curves cross tangentially? What does this imply about using these differential equations to predict the future?

Comments:

Problem 3.

Find as many solutions as you can for each of the 10 ODE’s above. How can you tell if you have found all of the solutions for that equation?

Worksheet:

Equation	Behavior	Exist.?Uniq.	Dir Field	Sol. Curve
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Notes: