MTH 165: Linear Algebra with Differential Equations

2nd Midterm
April 2, 2015

NAME (please print legibly): ________________________________

Your University ID Number: ________________________________

Indicate your instructor with a check in the box:

<table>
<thead>
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<th>Instructor</th>
<th>Time</th>
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<tr>
<td>Dummit</td>
<td>TR 16:50-18:05</td>
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<tr>
<td>Friedmann</td>
<td>MW 16:50-18:05</td>
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<tr>
<td>Petridis</td>
<td>MWF 10:25-11:15</td>
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<td>Rice</td>
<td>MW 14:00-15:15</td>
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• You have 75 minutes to work on this exam.

• No calculators, cell phones, other electronic devices, books, or notes are allowed during this exam.

• Show all your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

• You are responsible for checking that this exam has all 6 pages.

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1. (10 points)

(a) Let

\[
A = \begin{bmatrix} -1 & 3 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}.
\]

Find the determinant of the matrix \( C = ABA^2B^T \).

(b) Use the Wronskian to determine whether the functions \( f_1(x) = \cos(x) \), \( f_2(x) = \sin(x) \), and \( f_3(x) = x \) are linearly independent.
2. (10 points) Determine whether each given set $S$ is a subspace of the given vector space $V$. If so, give a proof; if not, explain why not.

(a) $V = \mathbb{R}^3$, and $S = \{(x, y, z) \in V \mid x^2 + y^2 + z^2 = 1\}$.

(b) $V = M_2(\mathbb{R})$, the set of $2 \times 2$ matrices, and $S = \{A \in V \mid \det(A) = 0\}$.

(c) $V = P_2(\mathbb{R})$, the set of polynomials of degree $\leq 2$, and $S = \{f \in V \mid f(2) = 2f(1)\}$.
3. (10 points) Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, and $v_4 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

(a) Do these vectors span $\mathbb{R}^3$? Explain why or why not.

(b) Are these vectors linearly independent? If so, justify why; if not, find an explicit linear dependence between them.
4. (10 points) Consider the matrix

\[
A = \begin{bmatrix}
1 & 2 & 1 & 2 \\
2 & 4 & 1 & 3 \\
2 & 4 & 0 & 0
\end{bmatrix}.
\]

(a) Find a basis for the row space of \( A \).

(b) Find a basis for the column space of \( A \).
5. (10 points) Answer the following about a $6 \times 17$ matrix $A$ (that is, a matrix with 6 rows and 17 columns) such that $\text{rank}(A) = 6$.

(a) $\text{rowspace}(A)$ is a ____-dimensional subspace of $\mathbb{R}^d$ with $d =$ ____

(b) $\text{colspace}(A)$ is a ____-dimensional subspace of $\mathbb{R}^d$ with $d =$ ____

(c) $\text{nullspace}(A)$ is a ____-dimensional subspace of $\mathbb{R}^d$ with $d =$ ____

(d) Are the rows of $A$ linearly independent? Explain why or why not.

(e) Are the columns of $A$ linearly independent? Explain why or why not.