NAME (please print legibly): ________________________________
Your University ID Number: ________________________________

Instructions:

1. Indicate your instructor with a check in the appropriate box:

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Time</th>
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</thead>
<tbody>
<tr>
<td>Herman</td>
<td>MWF 10:25</td>
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<tr>
<td>Lubkin</td>
<td>MW 2:00</td>
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<tr>
<td>Madhu</td>
<td>TR 2:00</td>
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<tr>
<td>McTague</td>
<td>MWF 9:00</td>
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<tr>
<td>Rivera-Letelier</td>
<td>TR 3:25</td>
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</tbody>
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2. Read the notes below:

- The presence of any electronic or calculating device at this exam is strictly forbidden, including (but not limited to) calculators, cell phones, and iPods.
- Notes of any kind are strictly forbidden.
- Show work and justify all answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You are responsible for checking that this exam has all 10 pages.

3. Read the following Academic Honesty Statement and sign:

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ____________________________________________
<table>
<thead>
<tr>
<th>QUESTION</th>
<th>VALUE</th>
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<td>TOTAL</td>
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1. **(10 points)** Compute the determinant of the following matrix:

\[
A = \begin{bmatrix}
3 & 3 & 2 & 1 \\
3 & 0 & 0 & -3 \\
-1 & -1 & 0 & 0 \\
-3 & -4 & 2 & 2 \\
\end{bmatrix}
\]
2. (10 points)

(a) Suppose $A$ and $B$ are $4 \times 4$ matrices with $\det(A) = -3$ and $\det(B) = 2$. Evaluate $\det(2B^2(AB^{-1})^T)$.

(b) Let $C$ be a $3 \times 3$ matrix such that $C^T = -C$ (i.e. $C$ is skew-symmetric). Show that necessarily $\det(C) = 0$. 

3. **(10 points)** For the following subsets $S$ of vector spaces $V$, determine whether or not $S$ is a subspace of $V$. Justify your answer.

(a) Let $V = P_3$ be the set of polynomials of degree at most 3, and

$$S = \{ p \in P_3 | p(5) = 0 \}.$$ 

(b) Let $V = \mathbb{R}^4$, and $S = \{ (-t, 2t - 1, t, 4t) | t \in \mathbb{R} \}$. 

(c) Let $V = M_2(\mathbb{R})$ be the set of $2 \times 2$ matrices with real entries, and

$$S = \{ A \in M_2(\mathbb{R}) | A \text{ is not invertible} \}.$$
4. (10 points) Consider the vectors:

\[ v_1 = (2,1,2,1,3) \]
\[ v_2 = (4,2,4,2,6) \]
\[ v_3 = (1,1,0,0,2) \]
\[ v_4 = (3,2,2,1,5) \].

(a) Do they span \( \mathbb{R}^5 \)?

(b) Find a subset of \( \{v_1, v_2, v_3, v_4\} \) which is a basis for \( V = \text{span}(v_1, v_2, v_3, v_4) \).
(c) Does the point \((0, 1, -2, -1, 1)\) lie on \(V = \text{span}(v_1, v_2, v_3, v_4)\)? Justify your answer.

(d) Are \(v_1, v_2\) linearly independent? [Justify your answer.]

(e) Are \(v_2, v_4\) linearly independent? [Justify your answer.]
5. **(10 points)** Consider the following matrix,

\[
M = \begin{bmatrix}
1 & 2 & -1 & 3 \\
3 & 6 & -3 & 5 \\
1 & 2 & -1 & -1 \\
5 & 10 & -5 & 7 \\
1 & 2 & 9 & 3 \\
\end{bmatrix}
\]

(a) Using elementary row operations, find a row-echelon matrix that is row-equivalent to \( M \).

(b) Find a basis for the columnspace of \( M \).
(c) Find a basis for the nullspace of $M$.

(d) Compute the rank of the matrix $M$.

(e) Compute the nullity of the matrix $M$. 
6. (10 points) Denote by $M_2(\mathbb{R})$ the space of $2 \times 2$ matrices and set 

$$A_0 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

(a) Prove that the map $T: M_2(\mathbb{R}) \to M_2(\mathbb{R})$ defined by $T(A) = AA_0 - A_0A$ is linear.

(b) Find bases of the kernel and of the range of the map $T$. 