Name: 

Student ID Number: 

Circle your professor: Bridy Kleene

Academic honesty statement:
With my signature, I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ___________________________ Date: ____________

- Justify your answers.
- No calculators are allowed on this exam, but you are allowed one sheet of paper with writing on both sides.

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a) Determine whether \((p \land q) \lor \neg p \rightarrow q\) is a tautology.

b) Determine whether the compound proposition \((p \rightarrow r) \land (q \rightarrow r)\) is logically equivalent to \((p \lor q) \rightarrow r\).
2. (30 points) Determine the truth value of the following quantified statements, if the domain of discourse is the real numbers. Briefly explain your answers.

a) $\forall x \exists y (xy \geq 0)$.

b) $\exists y \forall x (xy \geq 0)$.

c) $\exists x \exists y (x - y = 1 \land x + y = \pi)$

d) $\forall x \forall y \exists z (x \geq y \lor y \geq x)$

e) $\forall x ((x > 0) \rightarrow \exists y (y > 0 \land x < y))$

f) $\forall x \exists y (xy = 1)$.
3. (20 points) a) Prove the theorem “if $5n - 1$ is even, then $n$ is odd”. (The domain of discourse is the integers.)

b) Determine whether the following argument is valid: "If it is Monday, then Smithers goes to Math 150", "If it is not Monday, Then Smithers stays home or takes a stroll", "Smithers does not take a stroll or it is Monday", "Smithers does not go to Math 150". Therefore "Smithers stays home"
4. (15 points)

a) Write the decimal number 314 in octal (base 8) notation.

b) Compute $500 \mod 7$.

c) Compute gcd(105, 308).
5. **(25 points)** Let $A = \{0, 1, 4, 7, 10\}$ and $B = \{-2, 0, 4\}$. Let $E$ be the set of all even integers.

a) Compute the set $A - B$.

b) Compute the set $(A \cup B) \cap E$.

c) Compute the power set $\mathcal{P}(B)$.

d) Let the function $f : B \to E$ be given by $f(b) = b^2$. Is $f$ injective? Explain your answer.

e) Is $f$ surjective? Explain your answer.
6. (20 points)

a) Let \( f(n) = \frac{12n^3 - 12}{23n^3} + 9\sqrt{n}\log(n!) + n^2\log(n^7) \). What is the smallest integer \( k \) such that \( f(n) \) is \( \mathcal{O}(n^k) \)?

b) Use the definition of big-O notation to show that \( 8x \) is \( \mathcal{O}(x^2) \). (That is, find constants \( C \) and \( k \) that fit the definition, and show that your constants work.)