• No calculators are allowed on this exam.

• Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

• Label and circle your answers.

• Problems are not ordered according to difficulty. Look at all problems first and then start with the ones that seem easiest to you.
1. (10 points)

Determine whether each of the following sequences converges or diverges. If a sequence converges, compute its limit. If a sequence diverges, state whether it diverges to $+\infty$, $-\infty$, or neither.

(a) $\left\{\frac{3 + 5n^2}{n + n^2}\right\}$

(b) $\left\{\frac{2^n}{n!}\right\}$

(c) $\left\{\left(1 + \frac{2}{n}\right)^n\right\}$
2. (10 points)

(a) Compute the sum of each of the following series if possible. If it is not possible to compute a sum, state so and explain.

(i) \[ \sum_{n=1}^{\infty} \frac{3}{4^n} \]

(ii) \[ \sum_{n=1}^{\infty} \left[ \cos \left( \frac{\pi}{n} \right) - \cos \left( \frac{\pi}{n + 1} \right) \right] \]

(b) Use the **Test for Divergence** to determine if the series \[ \sum_{n=1}^{\infty} \frac{2n + 3}{3n + 2} \] diverges. If the test is not applicable, state so and explain.
3. (10 points)

(a) Use the **Integral Test** to determine whether each of the following series converges or diverges. If the test is not applicable, state so and explain.

\[
\text{(i) } \sum_{n=1}^{\infty} \frac{1}{n^2 + 1}
\]

\[
\text{(ii) } \sum_{n=2}^{\infty} \frac{1}{n \ln n}
\]

(b) The series \( \sum_{n=1}^{\infty} \frac{1}{n^3} \) converges by the Integral Test. How many terms of the series are needed to estimate the sum to within 0.005?
4. (10 points) Use either the Comparison Test or the Limit Comparison Test to determine whether each of the following series converges or diverges. If neither test is applicable, state so and explain.

(a) \( \sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2 + 1} \)

(b) \( \sum_{n=1}^{\infty} \frac{4 + 3^n}{2^n} \)

(c) \( \sum_{n=1}^{\infty} \frac{1 + n + n^2}{\sqrt{1 + n^2 + n^6}} \)
5. (10 points)

(a) Use the **Alternating Series Test** to determine whether each of the following series converges or diverges. If the test is not applicable, state so and explain.

(i) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \]

(ii) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \cos(\pi n)} \]

(b) The series \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} \] converges by the Alternating Series Test. How many terms of the series are needed to estimate the sum to within 0.001?
6. (10 points) Use either the **Ratio Test** or the **Root Test** to determine whether each of the following series converges absolutely or diverges. If neither test is conclusive, state so and explain.

(a) \( \sum_{n=1}^{\infty} \frac{n^2}{2^n} \)

(b) \( \sum_{n=1}^{\infty} \left( \frac{2n + 3}{3n + 2} \right)^n \)

(c) \( \sum_{n=1}^{\infty} \frac{n^n}{n!} \)
7. (20 points) Determine whether each of the following series converges absolutely, converges conditionally, or diverges.

(a) \[ \sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}} \]

(b) \[ \sum_{n=1}^{\infty} \frac{3^n n^2}{n!} \]

(c) \[ \sum_{n=1}^{\infty} n^2 e^{-n^3} \]
(d) $\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$

(e) $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 25}$
8. (10 points) Find the radius of convergence and interval of convergence of each of the following power series.

(a) \[ \sum_{n=0}^{\infty} \frac{x^n}{n!} \]

(b) \[ \sum_{n=0}^{\infty} \frac{x^n}{5^n} \]

(c) \[ \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n} \]
9. **(10 points)** Find a power series representation of each of the following functions by manipulating the representation \( \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \) centered at 0 with radius of convergence 1. State the radius of convergence in each case.

(a) \( \frac{1}{1 + 27x^3} \)

(b) \( \frac{1}{(1 - x)^2} \)

(c) \( \arctan x \)