Math 143 Spring 2005
Practice Final Exam

Time allowed: 3 hours

NAME:

STUDENT ID NUMBER:

INSTRUCTIONS

• Read carefully and understand these instructions. They apply to all problems unless otherwise specified in a problem itself.

• No calculators/notes/textbooks are allowed in this exam.

• Show all your work. You may use back pages if necessary. You will not receive any credit for a correct final answer if there is no work shown supporting your answer.

• Any trigonometric, exponential or logarithm functions in your answers that can be simplified without the use of a calculator must be simplified in order to get full credit.

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1. (10 points) Consider the following infinite series:

\[ \sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^3} \]

(a) Is this series convergent or is it divergent?

(b) Does this series converge absolutely or not?
2. (10 points) Consider the following infinite series:

\[ \sum_{n=2}^{\infty} \frac{1}{n \ln n} \]

Does this series converge or does it diverge?
3. **(15 points)** Determine the interval and radius of convergence of the following power series. You have to also determine converge/divergence at the endpoints of the interval of convergence.

\[ \sum_{n=1}^{\infty} \frac{(-2)^n (x + 3)^n}{\sqrt{n}} \]
4. (10 points) Find the power series expansion of $f(x) = \ln(x)$ around $x = 2$. Note: You have to determine the formula for the general term of the series.
5. (10 points) Evaluate the following integral using a power series:

\[ \int \frac{x}{1 - x^8} \, dx \]
1. (10 points)

(a) Compute the arclength along the curve \( y = \frac{x^2}{2} - \frac{\ln(x)}{4} \) between \( x = 2 \) and \( x = 4 \).
2. (15 points)
(a) Determine Cartesian coordinates for the point whose polar coordinates are \((-2, \pi/6)\).

(a) Identify the curve whose polar equation is \(r = 3 \sin(\theta)\).
3. (10 points) Determine a polar equation for the curve given by the following cartesian equation:

\[ x + y^2 = 0 \]
4. (10 points) Find the points on the following curve where the tangent line is vertical or horizontal:

\[ r = \sin(\theta) + \cos(\theta) \]