Today’s Topics: Radius of convergence, power series as functions.

Solve as much of, as many of, the following problems as you can. You are not necessarily expected to solve all of the problems, but you are expected to think about and (most importantly) discuss all of them.

1. Find the radius of convergence and the interval of convergence for each power series, and then identify for which $x$ the series converges absolutely and for which $x$ the series converges conditionally.

   (a) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$.

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n^3}$.
(c) \[ \sum_{n=1}^{\infty} \frac{(x + 1)^n}{n!} \]

(d) \[ \sum_{n=1}^{\infty} \frac{(x - 2)^n}{n^n} \]
(e) \[ \sum_{n=1}^{\infty} \frac{(x - 2)^n}{3^n} \]
2. Find a power series representation for functions

(a) \[ \frac{1}{2 + x^3} \]

(b) \[ \ln(5 - x) \]
3. Compute the integral as a power series

(a) \[ \int_{0}^{3} \frac{1}{1 - x^8} \, dx \]

(b) \[ \int_{1/3}^{1/2} \frac{\ln(1 - x)}{x} \, dx \]