Homework 8, Math 471

1. Do Exercise 22 from Chapter 3 of Stein's book.

2. Show that

$$\|f+g\|_\infty \leq \|f\|_\infty + \|g\|_\infty$$

3. Let f be a bounded measurable function on $[0,1]$. Show that

$$\lim_{p\to\infty} \|f\|_p = \|f\|_\infty$$

4. Let $f \in \mathbf{L}^1$ and $g \in \mathbf{L}^\infty$. Show

$$\int |fg| \leq \|f\|_1 \cdot \|g\|_\infty$$

5. Prove that

$$\|f+g\|_1 \leq \|f\|_1 + \|g\|_1$$