Math 246 Practice Problems For The First Midterm

1. Section 1.2, problem number 9.

2. True or false and explain: $\{\leftrightarrow, \neg\}$ is complete. (If true, prove it, if false, find a counterexample.)

3. Recall the definition of a deduction. Give a deduction of $\neg\mathbf{P}$ from the set

$$\{\neg(\mathbf{P} \wedge \mathbf{Q}), \mathbf{R} \rightarrow \mathbf{Q}, \mathbf{R}\}$$

4. Let σ be the set of wffs

$$\sigma = \bigcup_{i=1}^{\infty} \{(\mathbf{A}_1 \rightarrow \neg\mathbf{A}_i)\}$$

(here $\mathbf{A}_i$ are our sentence symbols). Is σ satisfiable?

5. Let $F : \{F, T\}^3 \longrightarrow \{F, T\}$ be the 3-place Boolean function such that $F(X_1, X_2, X_3) = T$ if and only if exactly two of the X_i are F. Find a wff α in the sentence symbols A_1, A_2, A_3, such that, for any truth assignment v on $\{A_1, A_2, A_3\}$, we have

$$\bar{v}(\alpha) = F(v(A_1), v(A_2), v(A_3)).$$

6 Three friends Pablo, Edvard, and Henri are talking to each other about the art collection of Leonardo. Pablo says: "Leonardo has at least four paintings of Rembrandt." Edvard says: "No, he has less than four paintings of Rembrandt." "According to me," says Henri, "Leonardo has at least one Rembrandt." **Question**: If you known that only one of the three friends is right, how many Rembrandts does Leonardo possess?

7. Suppose we are in a language with only the quantifier and one binary predicate E. For which wffs α below below do we have $\models \alpha$?

(1) $\forall\mathbf{v}_1\forall\mathbf{v}_2 E\mathbf{v}_1\mathbf{v}_2 \rightarrow E\mathbf{v}_3\mathbf{v}_4$ (note the lack of parentheses);
(2) $\exists\mathbf{v}_1 E\mathbf{v}_1\mathbf{v}_1$;
(3) $E\mathbf{v}_1\mathbf{v}_2 \rightarrow E\mathbf{v}_2\mathbf{v}_1$;
(4) $E\mathbf{v}_1\mathbf{v}_2 \vee \neg E\mathbf{v}_1\mathbf{v}_2$.