MTH 235: Linear Algebra

1st Midterm Exam

October 9, 2009

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **You are responsible for checking that this exam has all 6 pages.**

Part A		
QUESTION	VALUE	SCORE
1	10	
2	10	
3	25	
4	15	
TOTAL	60	

Part A

1. (10 points) Determine whether the following are subspaces.

a) Let $A \in M_{n\times n}(F)$. The set $S = \{B \in M_{n\times n}(F) \mid AB + B = 0\}$.

b) $S = \{f \in P_3(F) \mid \text{f(x)=0 or } \deg f(x) = 3\}$.

2. (10 points) Let u, v, and w be distinct vectors of a vector space V. Show that if $\{u, v, w\}$ is a basis for V, then $\{u + v + w, v + w, w\}$ is also a basis for V.

3. (25 points)

a) Show that the collection $\beta = \{1, 1+x, 1+x+x^2\}$ forms a basis for $P_2(\mathbb{R})$.

b) Show that the transformation $T : P_2(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ defined by

$$T(f(x)) = \begin{pmatrix} f'(0) & 2f(1) \\ 0 & f''(3) \end{pmatrix}$$

is a linear transformation.

c) Consider the ordered basis

$$\gamma = \left\{ A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

of $M_{2\times 2}(\mathbb{R})$. What is $[T]_\beta^\gamma$?

d) Is T onto? Is T one-to-one?

e) State the "rank-nullity" (or dimension) theorem. Verify that T obeys the theorem.

4. (15 points) Circle **T**rue or **F**alse. Any "ambiguous" circles will be marked incorrect, so make sure your answer is clear.

T **F** A vector space may have more than one zero vector.

T **F** The zero vector is a linear combination of any nonempty set of vectors.

T **F** If S is a linearly independent set, then each vector of S is a linear combination of other vectors of S.

T **F** Subsets of linearly dependent sets are linearly dependent.

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T **F** The zero vector space has no basis.

T **F** If U and W are subspaces of a vector space V, then $U \cup W$ is a subspace of V.

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T **F** If $\text{span}(\{v_1, v_2, \ldots, v_n\}) = \mathbb{R}^n$ then $\{v_1, v_2, \cdots, v_n\}$ is a linearly independent set.

T **F** A vector space cannot have more than one basis.

T **F** If a vector space has a finite basis, then the number of vectors in every basis is the same.

T **F** If T is linear, then T preserves sums and scalar products.

T **F** If $T, U : V \to W$ are both linear and agree on a basis for V then $T = U$.

T **F** Suppose $T : V \to W$ is a linear transformation. If $T(x) = T(y)$ then $x = y$.

T **F** If $T : V \to W$ is a linear transformation and $v \in \ker(T)$ then $T(u + v) = T(u)$ for all $u \in V$.