

MATH 230
STUDY GUIDE FOR MIDTERM 1

- (1) You should know how to do all of the homework problems from assignments 1 - 4. You will be asked a proof on the exam that will be the same as, or very similar to, one of the problems from the homework.

- (2) Find, and prove by induction, a formula for

$$1^3 + 4^3 + 7^3 + 10^3 + \cdots + (3n + 1)^3.$$

- (3) Give the prime factorization of each of the following integers.

- (a) 341
- (b) 343
- (c) 345
- (d) 347
- (e) 27!

(f) $\binom{46}{21} = \frac{46!}{21!25!}$

- (4) Let $n = 1260$.

- (a) Find the prime factorization of n .
- (b) Find the number of divisors of n , $d(n)$.
- (c) Find the sum of the divisors of n , $\sigma(n)$.
- (d) Find the smallest number with more divisors than n .

- (5) Let $a = 397$ and $b = 341$.

- (a) Find the greatest common divisor of a and b .
- (b) Do there exist natural numbers x and y such that $ax - by = 1$? If so, find such a pair (x, y) .
- (c) Do there exist natural numbers x and y such that $ax - by = 6$? If so, find such a pair (x, y) .
- (d) Do there exist natural numbers x and y such that $ay - bx = 1$? If so, find such a pair (x, y) .

- (6) Do the following calculations in modular arithmetic.

- (a) $1 + 2 + 3 + \cdots + 100 \pmod{101}$
- (b) $\frac{1}{8} \pmod{13}$
- (c) $\frac{3}{5} \pmod{31}$
- (d) $2^{14} \pmod{15}$
- (e) $3^{78} \pmod{17}$
- (f) $100! \pmod{103}$
- (g) $100! \pmod{105}$

- (7) Define $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$, so that F_n denotes the n^{th} Fibonacci number.

- (a) Show that if $m \mid n$, then $F_m \mid F_n$.
- (b) Show that if F_n is prime, then $n = 4$ or n is prime.
- (c) If $n \geq 3$ is prime, is F_n necessarily prime?