Midterm

The following problems are due at the beginning of class on Tuesday October 27th.

(1) Consider the following linear program:

$$\begin{array}{ll} \text{maximize} & z = 3x_1 + 4x_2 \\ \text{subject to} & -x_1 + x_2 \geq -1 \\ & -x_1 + x_2 \leq 1 \\ & x_1 \leq 3 \\ & x_1, x_2 \geq 0. \end{array}$$

(a) Graph the feasible region.
(b) Solve the linear program via the null-space formulation of the simplex algorithm. Trace the progress of the algorithm on your graph and clearly identify the null-space matrix, reduced costs, search direction and maximum step length at each step.

(2) Solve the following linear program

$$\begin{array}{ll} \text{minimize} & z = -3x_1 - 2x_2 + x_3 \\ \text{subject to} & x_1 + x_2 + x_3 \leq 5 \\ & 3x_1 + 2x_3 \leq 5 \\ & 1 \leq x_1, x_2 \text{ free }, x_3 \geq 0. \end{array}$$

Express your answer in terms of the given variables and target function.

(3) Consider the following tableau

	x_1	x_2	x_3	x_4	x_5	x_6	
	0	−2	0	1	0	−3	4
x_3	0	3	1	2	0	4	1
x_1	1	−5	0	2	0	7	2
x_5	0	10	0	1	1	9	3

(a) What is the search direction associated to the nonbasic variable x_2?
(b) What is the maximum step length $\overline{\alpha}$ associated to the nonbasic variable x_6?

(4) Consider the following tableau

	x_1	x_2	x_3	x_4	
	−1	2	−5	−3	10
x_3	1	2	3	1	5
x_2	0	1	2	1	3

What is the reduced cost of the nonbasic variable x_1?

More on back!

(5) If possible, find a basic feasible solution to each of the following systems of linear inequalities:

(a)

$$\begin{aligned} x_1 - x_3 &\geq 1/2 \\ 2x_2 - x_3 &\geq 1 \\ x_1 - x_2 &\leq 0 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

(b)

$$\begin{aligned} -x_1 + 2x_2 &\geq 1 \\ x_1 - 3x_2 &\geq 2 \\ x_1 &\leq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$