MATH 171Q HONORS CALCULUS
Review Problems for Exam I

1. State

(a) the Well-Ordering Principle

(b) the Principle of Induction.

2. Define what it means to have

$$\lim_{x \to a} f(x) = l.$$

3. If $\lim_{x \to a} f(x) = 0$, and $g(x)$ is bounded for all x (that is, there is some M in $\mathbb{R}$ such that $|g(x)| \leq M$ for all x), show that $\lim_{x \to a} (fg)(x) = 0$.

4. Given an $\epsilon > 0$, show that there exists a number $M > 0$ such that for all $x > M$ we have

$$\left| \frac{x^2+1}{x^2-1} - 1 \right| < \epsilon \ .$$

That is, give an $(\delta - \epsilon)$ proof that $\lim_{x \to \infty} \dfrac{x^2+1}{x^2-1} = 1$.

5.

(a) State the binomial theorem for $(a+b)^n$ with n a natural number.

(b) What is the coefficient of x^{16} when one expands $\left(2 + \frac{x}{3}\right)^{20}$?

6. Find an example of each of the following functions.

(a) $f(x)$ has limits nowhere on $\mathbb{R}$.

(b) $f(x)$ has limits only at the irrationals in $(0, 1)$.

(c) $f(x)$ has limits only at one point of $\mathbb{R}$.

(d) $f(x)$ has limits nowhere on $\mathbb{R}$ but $f^2(x)$ has limits at all points of $\mathbb{R}$.

7. Prove that $1^3 + 2^3 + \cdots + n^3 = (n(n+1)/2)^2$.

8. Give an (δ-ϵ) proof that $\lim_{x \to 3} \frac{1}{x-1} = \frac{1}{2}$.

9. Assume that $\lim_{x \to 0} \frac{\sin x}{x} = \alpha$. (What is the $\lim_{x \to \infty} \frac{\sin x}{x}$?)

2(a) $\lim_{x \to 0^+} 2[x] \cos x$ (b) $\lim_{x \to 0^-} 2[x] \cos x$

(c) $\lim_{x \to 0} \frac{\sin^2 x + 3x^3}{x^2 + x^3}$ (d) $\lim_{x \to \infty} \frac{\sin^2 x + 3x^3}{x^2 + x^3}$

(e) $\lim_{x \to 2} \frac{x-2}{x^4 - 16}$ (f) $\lim_{x \to 0^-} \frac{\sin x}{|\sin x|}$; $\lim_{x \to 0^+} \frac{\sin x}{|\sin x|}$

(g) $\lim_{x \to 2^+} \left[\frac{4}{x}\right]$ (h) $\lim_{x \to 2^-} \left[\frac{4}{x}\right]$.

10. Prove that if $\lim_{x \to b} f(x)$ exists, then there is some $\delta > 0$ and some M for which $|f(x)| < M$ for all x such that $0 < |x - b| < \delta$.

11. Calculate the following limits.

(a) $\lim_{x \to 2}(4 - x^2)/(x^2 + 3x - 10)$

(b) $\lim_{x \to -2} \left(1/(x+2) - 4/(x^2 - 4)\right)$

(c) $\lim_{x \to -1} x/(x+1)^2$

(d) $\lim_{x \to 3}(x - 3)/(\sqrt{x} - \sqrt{3})$

(e) $\lim_{x \to \infty}(\sqrt{x^2 + x + 1} - x)$

(f) $\lim_{x \to -\infty} \sqrt{x^2 + x - 2}/(3x + 1)$

(g) $\lim_{x \to 1^+}([x] - x)/(x - 1)$

(h) $\lim_{x \to 1^-}([x] - x)/(x - 1)$

(i) $\lim_{x \to 0^+}(3^{1/x} - 1)/(3^{1/x} + 1)$

(j) $\lim_{x \to 0^-}(3^{1/x} - 1)/(3^{1/x} + 1)$