

MATH 165

Midterm

Thursday, April 3, 2008

Show all work (each step/computation) to receive full credit. No calculators. The exam contains 5 problems. Make sure it is complete.

No.	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

NAME : _____

SECTION : _____

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1. Compute the determinant of the following matrix:

$$\begin{bmatrix} 2 & 3 & 3 & 1 \\ 0 & 4 & 3 & -3 \\ 2 & -1 & -1 & -3 \\ 0 & -4 & -3 & 2 \end{bmatrix}$$

2. a) Let A and B be two 9×9 matrices, such that $\det A = 5$ and $\det B = 3$. Compute $\det(B^{-1}AB)$.

b) Let A be a 3×3 matrix such that $A^T = -A$ (i.e. A is skew-symmetric). Show that necessarily $\det A = 0$.

3. a) Let $S = \{A \in M_{2 \times 2}(\mathbb{R}) \mid \det A = 0\}$. Show that S is closed under scalar multiplication. Is S closed under matrix addition? Justify your answers.

b) Let $T = \{(a, 0, b, a - b + c) \mid a, b, c \in \mathbb{R}\}$. Show that T is a subspace of \mathbb{R}^4 . Find a basis and the dimension for T . Justify your answers.

4. Show that the functions

$$f_1(x) = e^x, \quad f_2(x) = \cos x, \quad f_3(x) = \sin x$$

are linearly independent on the interval $I = (-\infty, \infty)$, by computing their Wronskian.

5. Find a basis and the dimension for the column space of the following matrix:

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 2 \\ 3 & 4 & 9 & 0 & 7 \\ 2 & 3 & 5 & 1 & 8 \\ 2 & 2 & 8 & -3 & 5 \end{bmatrix}$$