MTH 165: Linear Algebra with Differential Equations

2nd Midterm
April 4, 2013

NAME (please print legibly): ________________________________
Your University ID Number: ________________________________
Indicate your instructor with a check in the box:

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Time</th>
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<tbody>
<tr>
<td>Dan-Andrei Geba</td>
<td>MWF 10:00 - 10:50</td>
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<tr>
<td>Giorgis Petridis</td>
<td>MWF 13:00 - 13:50</td>
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<tr>
<td>Eyvindur Ari Palsson</td>
<td>MW 14:00 - 15:15</td>
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• The presence of of electronic devices (including calculators), books, or formula cards/sheets at this exam is strictly forbidden.

• Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

• Clearly circle or label your simplified final answers.

• You are responsible for checking that this exam has all ?? pages.

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<th>QUESTION</th>
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<td>TOTAL</td>
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1. **(10 points)** Find the inverse of the matrix

\[
A = \begin{bmatrix}
-7 & -3 & 1 \\
2 & 1 & 0 \\
-28 & -13 & 3 \\
\end{bmatrix}.
\]
2. (10 points) Use cofactor expansion and/or row reduction to evaluate the determinant of the following matrix

\[
\begin{bmatrix}
1 & 2 & 2 & 4 \\
-2 & 2 & -2 & 2 \\
2 & 1 & -1 & -2 \\
-1 & -4 & 4 & 2
\end{bmatrix}
\]
3. (10 points) In each of the following, determine whether the subset $S$ is a subspace of the given vector space $V$:

i) $V = \mathbb{R}^4$ and $S = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 x_4 = 0\}$;

ii) $V = M_{2\times2}(\mathbb{R})$ and $S = \{A \in M_{2\times2}(\mathbb{R}) \mid A = 2A^T\}$. 

4. (10 points) Using the Wronskian, determine whether or not the functions

\[ f_1(x) = \sin x , \; f_2(x) = \sin 2x , \; f_3(x) = e^x \]

are linearly independent on \( \mathbb{R} \).
5. (10 points) Find a subset of

\[ S = \begin{bmatrix} 
3 & 2 \\
2 & 2 \\
2 & 1 \\
\end{bmatrix}, \begin{bmatrix} 
2 & 1 \\
2 & 2 \\
1 & 3 \\
\end{bmatrix}, \begin{bmatrix} 
4 & 3 \\
2 & 2 \\
3 & 4 \\
\end{bmatrix}, \begin{bmatrix} 
1 & 2 \\
3 & 3 \\
\end{bmatrix} \]

that forms a basis for the subspace of \( \mathbb{R}^4 \) generated by \( S \), i.e., \( \text{span} \, S \).
6. (10 points) For the matrix

\[
A = \begin{bmatrix}
3 & 1 & -3 & 11 & 10 \\
5 & 8 & 2 & -2 & 7 \\
2 & 5 & 0 & -1 & 14
\end{bmatrix},
\]

find a basis and the dimension for nullspace \( (A) \).