MTH 165: Linear Algebra with Differential Equations

2nd Midterm
April 5, 2012

NAME (please print legibly): ____________________________________________
Your University ID Number: ____________________________________________
Indicate your instructor with a check in the box:

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Time</th>
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<tbody>
<tr>
<td>Dan-Andrei Geba</td>
<td>MWF 10:00 - 10:50 AM</td>
</tr>
<tr>
<td>Ang Wei</td>
<td>MW 2:00 - 3:15 PM</td>
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• The presence of electronic devices (including calculators), books, or formula cards/sheets at this exam is strictly forbidden.

• Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

• Clearly circle or label your simplified final answers.

• You are responsible for checking that this exam has all 7 pages.

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<td><strong>TOTAL</strong></td>
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1. (10 points) Find the determinants of the matrices $A$, $B$, and $B^T A$, where

$$A = \begin{bmatrix}
1 & -1 & -1 & 1 \\
1 & 2 & 2 & 1 \\
-2 & 0 & 4 & 1 \\
0 & -2 & 3 & 4
\end{bmatrix}, \quad B = \begin{bmatrix}
-3 & 5 & 6 & -14 \\
0 & 2 & 13 & -156 \\
0 & 0 & -\frac{1}{3} & 0 \\
0 & 0 & 0 & 5
\end{bmatrix}.$$
2. (10 points) In each of the following, determine whether the subset $S$ is a subspace of the given vector space $V$:

i) $V = M_{2 \times 2}(\mathbb{R})$ and $S$ is the subset of all $2 \times 2$ invertible matrices;

ii) $V = P_2$, the vector space of real-valued polynomials of degree $\leq 2$, and

$$S = \{ax^2 + bx : a, b \in \mathbb{R}\}.$$
3. (10 points) Compute

\[ \text{span}\{(1, 0, -1), (2, 0, 4), (-5, 0, 2), (0, 0, 1)\} \]

in the vector space \( \mathbb{R}^3 \).
4. (10 points) Using the Wronskian, determine whether or not the functions

\[ f_1(x) = e^{2x}, \quad f_2(x) = e^{3x}, \quad f_3(x) = e^{-x} \]

are linearly independent on \( \mathbb{R} \).
5. (10 points) Let $S$ be the subspace of $\mathbb{R}^3$ that consists of all $(x, y, z)$ which satisfy the equation $x + 3y - 2z = 0$. Determine a basis for $S$ and find $\dim[S]$. 
6. (10 points) For the matrix

\[ A = \begin{bmatrix}
1 & 1 & -1 & 5 \\
0 & 2 & -1 & 7 \\
4 & 2 & -3 & 13
\end{bmatrix}, \]

find:

i) a basis and the dimension for colspace \((A)\);

ii) a basis and the dimension for nullspace \((A)\).