Name:__________________________________________

Student ID#:____________________________________

Instructor (Circle One):    ROGERS    HERMAN

• Show all your work, use backs of pages if necessary. Points may be deducted for correct answers with no justification.

• No calculators, books, notes, etc. are allowed on this exam.

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1. (12 pts) Evaluate the following determinant:

\[
\begin{vmatrix}
1 & 0 & -1 & 1 \\
0 & 1 & 2 & 0 \\
-3 & 2 & 0 & -2 \\
1 & 0 & 1 & -1 \\
\end{vmatrix}
\]
2. (15 pts) Let $A$ and $B$ be 3x3 matrices where $\det(A) = 2$ and $\det(B) = 3$. Evaluate the following determinants:

(a) $\det(-2AB^T)$

(b) $\det(A^{-1}BAB^{-1})$

(c) Find $\det(C)$, where $C$ is obtained from $A$ by multiplying the first column by 4 and swapping the 2nd and 3rd rows.
3. (15 pts) Let $A$ be the following matrix:

$$
A = \begin{bmatrix}
1 & 1 & -1 \\
2 & 3 & -2 \\
-1 & -1 & 0
\end{bmatrix}
$$

(a) Find $A^{-1}$

(b) Find the solution to $A\vec{x} = \vec{b}$, where $\vec{b} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$
4. (16 pts) Which of the following are subspaces of \( \mathbb{R}^3 \)?

(a) \( \{ (x, y, z) \mid 8x - 3y + 7z = 2 \} \)

(b) \( \{ (x, y, z) \mid 3x + 4y = 0 \} \)

(c) \( \{ (x, y, z) \mid z \geq 0 \} \)

(d) \( \{ (x, 3x, 2x - 3) \mid x \text{ arbitrary} \} \)
5. (15 pts) Find a linearly independent subset of the following set of vectors that spans the same subspace of $\mathbb{R}^3$.

\[
\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} \right\}
\]
6. (12 pts) Find a basis of the subspace of $\mathbb{R}^4$ defined by the equation

$$x_1 - 2x_2 + 3x_3 + x_4 = 0$$
7. (15 pts) Identify each of the given sets of vectors as one of the following:

I. Vectors do not span $\mathbb{R}^3$ and are linearly dependent
II. Vectors span $\mathbb{R}^3$ and are linearly dependent
III. Vectors do not span $\mathbb{R}^3$ and are linearly independent
IV. Vectors span $\mathbb{R}^3$ and are linearly independent

(a) \[
\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \right\}
\]

(b) \[
\left\{ \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 6 \end{bmatrix} \right\}
\]

(c) \[
\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} \right\}
\]