• Show all your work, use backs of pages if necessary. Points may be deducted for correct answers with no justification.

• No calculators, books, notes, etc. are allowed on this exam.
1. (14 pts) Find the solution of the initial value problem \( x + y e^{-x} \frac{dy}{dx} = 0 \), \( y(0) = -1 \)
2. (12 pts) Find the general solution of \( t^3 y' = 4t^2 y + 15t \), for \( t > 0 \).
3. (14 pts) Find the equation $y = f(x)$ of the curve that passes through the point $(0, 3)$ whose slope at each point $(x, y)$ is given by $\frac{2 \cdot x \cdot y^2}{1 + x^2}$. 
4. (10 pts) A ball of mass 4 kg is thrown straight downward with a speed of 5 m/s from a hot-air balloon 100 meters above the ground. Assume the air resistance force is proportional to the velocity cubed, and the magnitude of this force is 1 Newton when the speed is 2 m/s. Write the differential equation for the velocity of the body, and state initial conditions. (use gravitational constant \(g = 9.8 \text{ m/s}^2\) and indicate which direction is the positive direction for the velocity) **YOU DO NOT NEED TO SOLVE THE EQUATION**
5. (10 pts) Find all values of $r$ such that $y = t^r$ is a solution to the differential equation

$$t^2 y'' + 2ty' - 12y = 0$$

for $t > 0$. 
6. (12 pts) Put the following matrix into reduced row-echelon form.

\[
\begin{bmatrix}
1 & 2 & -1 & 3 \\
3 & 6 & 2 & 4 \\
-1 & -2 & 4 & -3
\end{bmatrix}
\]
7. (16 pts) Solve the following system of equations:

\[-x_1 + x_2 + 3x_3 = -2
2x_1 - 2x_2 + x_3 = 4
3x_1 - 3x_2 + 7x_3 = 6\]
8. (12 pts) For each of the following augmented matrices, determine whether the corresponding system of linear equations has no solutions, a unique solution, or infinitely many solutions.

(a) \[
\begin{bmatrix}
1 & -5 & | & 2 \\
0 & 1 & | & 3 \\
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
0 & 1 & 4 & | & 5 \\
0 & 0 & 1 & | & -2 \\
0 & 0 & 0 & | & 0 \\
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
1 & 3 & 7 & | & -2 \\
0 & 0 & 1 & | & 3 \\
0 & 0 & 0 & | & 6 \\
\end{bmatrix}
\]

(d) \[
\begin{bmatrix}
1 & -3 & 5 & 2 & | & 6 \\
0 & 0 & 1 & 3 & | & 0 \\
0 & 0 & 0 & 1 & | & 1 \\
\end{bmatrix}
\]