MTH 165: Linear Algebra with Differential Equations

1st Midterm
October 17, 2013

NAME (please print legibly): ________________________________

Your University ID Number: ________________________________

Indicate your instructor with a check in the box:

- Mijia Lai MWF 10:00 - 10:50 AM
- Eyvindur Ari Palsson MWF 10:00 - 10:50 AM
- Mihai Bailesteanu MW 2:00 - 3:15 PM

- The presence of electronic devices (including calculators, cell phones and iPods), books, or formula cards/sheets at this exam is strictly forbidden. If you have any forbidden materials with you, you must turn them in to a proctor before starting the exam. Failure to do so will be treated as an academic honors violation.

- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

- Clearly circle or label your simplified final answers.

- You are responsible for checking that this exam has all 7 pages.

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1. **(10 points)** Find the explicit solution for the initial value problem

\[
\frac{dy}{dx} - e^x y^2 = e^x, \quad y(0) = 1.
\]
2. (10 points) Find the general solution of the equation

\[ \frac{dy}{dt} + \frac{2t}{1 - t^2} y = 4t, \quad \text{for } -1 < t < 1. \]
3. **(10 points)** A 150 liter tank initially contains 60 L of solution in which there is dissolved 5 kg of salt. A solution containing $\frac{t}{t+20}$ kg/L of salt, where $t$ is the time in minutes from the initial state, flows into the tank at a rate of 9 L/min, and the well-stirred mixture flows out at a rate of 6 L/min. How much salt is in the tank just before the solution overflows?
4. (10 points) Consider the matrices

\[ A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 3 & 2 \end{bmatrix}. \]

Compute \((AB^T)^T\) if it is defined, else explain why it is not defined.
5. (10 points) Find the rank for the matrix

\[
A = \begin{bmatrix}
4 & -2 & 9 \\
-2 & -4 & 3 \\
3 & 1 & 8
\end{bmatrix}
\]

by computing its reduced row-echelon form.
6. (10 points) Solve the following linear system of equations:

\[
\begin{align*}
4x_1 - 5x_2 + 9x_3 + x_4 &= 15 \\
3x_1 + 2x_2 + x_3 - 5x_4 &= -6 \\
x_1 - 2x_2 + 3x_3 + x_4 &= 6
\end{align*}
\]