MTH 162: Calculus IIA

Midterm 2
April 4, 2013

NAME (please print legibly): ________________________________
Your University ID Number: ________________________________
Indicate your instructor with a check in the box:

<table>
<thead>
<tr>
<th>Instructor</th>
<th>MWF Time</th>
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<tbody>
<tr>
<td>Matthew Creek</td>
<td>MWF 1:00 - 1:50 PM</td>
</tr>
<tr>
<td>Mark Herman</td>
<td>MWF 10:00 - 10:50 AM</td>
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<tr>
<td>David Karapetyan</td>
<td>MWF 11:00 - 11:50 AM</td>
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<tr>
<td>Yoonbok Lee</td>
<td>MWF 9:00 - 9:50 AM</td>
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<tr>
<td>Meg Walters</td>
<td>TR 2:00 - 3:15 PM</td>
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</tbody>
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- There are no notes, textbooks, etc. allowed on this exam. The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.
- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Clearly circle or label your final answers.

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<th>QUESTION</th>
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1. (12 points)

Determine whether each of the sequences converges or diverges. If it converges, compute its limit. If it diverges, state whether it diverges to $+\infty$, $-\infty$, or neither.

(a) \[ \{\sin(n/2)\}_{n=1}^\infty \]

(b) \[ \{(1 + 5/n)^n\}_{n=1}^\infty \]
(c) \[\{(−1/3)^n\}_{n=1}^{\infty}\]
2. (28 points)

If the series converges, explain why and compute its sum. If the series diverges, state whether it diverges to \(+\infty\), \(-\infty\), or neither, and explain why.

(a) 

$$\sum_{n=1}^{\infty} a_n$$

where 

$$a_n = \begin{cases} 
(1 + 1/n)^n & 1 \leq n \leq 3 \\
0 & n \geq 4 
\end{cases}$$

(b) 

$$\sum_{n=1}^{\infty} [e^{-n} - e^{-(n+1)}]$$
(c) \[ \sum_{n=1}^{\infty} 18(1/3)^{n+1} \]

(d) \[ \sum_{n=1}^{\infty} (3/2)^n \]
3. (14 points) Use a convergence or divergence test to determine whether the following series converge or diverge. Make sure to state which test you are using and show all of your work.

(a) \[
\sum_{n=1}^{\infty} \frac{3}{n^2 + 1}
\]

(b) \[
\sum_{n=1}^{\infty} \frac{3^n}{n!}
\]
4. (21 points) For each of the following series, tell whether the series converges absolutely, converges conditionally, or diverges. Justify your answer. If your answer is conditionally convergent, you must explain why it is not absolutely convergent and how you know it converges conditionally.

(a) \[
\sum_{n=1}^{\infty} \frac{(-1)^n (3n^8 + 5)}{9n^8 + 1}
\]

(b) \[
\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[4]{n}^{3/4}}
\]
(c) \[ \sum_{n=1}^{\infty} \frac{\cos(n)}{n^2} \]
5. (10 points) Evaluate the following improper integral. Does it converge or diverge?

\[ \int_{0}^{3} \frac{dx}{x^2 - 6x + 5} \]
6. (15 points) Consider the curve $y = 1 + x^{3/2}$

(a) Find the arc length of the curve for $0 \leq x \leq 1$.

(b) Find the area of the surface obtained by rotating the segment of the curve $0 \leq x \leq 1$ around the $y$-axis.